Chapter 1

Introduction

Und überall hingen, lagen und standen Uhren.
Da gab es auch Weltzeituhren in Kugelform,
welche die Zeit für jeden Zeitpunkt der Erde anzeigten.
[...]
"Vielleicht", meinte Momo,
"braucht man dazu eben so eine Uhr."
Meister Hora schüttelte lächelnd den Kopf.
"Die Uhr allein würde niemand nützen.
Man muß sie auch lesen können."
Michael Ende, Momo

Clocks were standing or hanging wherever Momo looked
– not only conventional clocks but spherical timepieces
showing what time it was anywhere in the world
[...]
"Perhaps one needs a watch like yours to recognize them by"
said Momo.
Professor Hora smiled and shook his head.
"No, my child, the watch by itself would be no use for anyone.
You have to know how to read it as well."[1]
Michael Ende, Momo

The objective of this book is to bring into accordance the two obviously completely contrary concepts of time and Petri nets. Introduced by C. A. Petri

[1]Trans. J. Maxwell Brownjohn (Doubleday & Company Inc., New York, and Penguin Books Ltd., 1984).

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in [Pet62], Petri nets can be used to study concurrency in the sense of causal independence, but they do not directly deal with time; time is involved only implicitly through the causal relationships. However, the explicit indication of time is indispensable for a great variety of real problems. Even qualitative studies of strongly time-dependent systems are very inexact if time is included only implicitly through causality. The question arises, whether one should at all try to describe and analyze such systems using Petri nets and whether it is even possible.

The first publications in which time and Petri nets are connected, were already published a good ten years after the introduction of Petri nets. As expected, time attributes were first assigned to transitions (cf. [Mer74], [Ram74], [MF76] etc.). After occasional studies in the 1970s and some articles in the 1980s (cf. [BM83], [Sta87], [PZ89] etc.) an avalanche of new time-dependent extensions of the classic Petri net followed in the 1990s. Times in the form of durations or intervals were assigned to places, tokens, and input and output arcs. Furthermore, various different firing rules were defined: earliest possible or latest possible firing, firing single transitions or firing in maximal-step mode, firing with certain probabilities, etc. These extensions arose from practical considerations. With some of these newly developed Petri nets, the originally central idea of concurrency can be found merely as simultaneity, but their practical significance is immense.

Classic Petri nets, i.e., Petri nets without any explicit indication of time, are excellent means for the depiction, simulation and analysis of systems of the most diverse origin. In the description of systems by means of classic Petri nets, events are mostly modeled as transitions. The pre- and postconditions of the events are represented by places and are considered as fulfilled if those places contain any tokens. Directed arcs connect all places which are preconditions of an event with the transition which models this event. Directed arcs analogously connect every transition with all places which represent postconditions for the event modeled by the transition. It is possible to refine these models as and when required by adding time to different elements of the net.

In a classic Petri net, any possible situation is expressed by the assignment of tokens to places, i.e., by a marking. Obviously this is a *discrete* description, whereas a situation in a time-dependent Petri net is a snapshot of a marking at a particular point of time. This means that in Petri nets with continuous time, the situation depends on a discrete parameter (the marking) as well as a continuous one (time). Therefore time-dependent Petri nets are a *hybrid* means of describing a system.

When defining a time-dependent Petri net the following generally need to be fixed:

- the type of time extension: time interval or duration.
- the manner of time modeling: continuous or discrete.
- the specific type of elements of the net to which the time extension is assigned: places, transitions or arcs.
- the firing rule.

When defining a firing rule, there are time-independent characteristics that need to be determined, such as:

- solution of conflicts.
- concurrency of a transition with itself (also called auto-concurrency or self-concurrency).

as well as the following specifications:

- firing mode: whether the transitions fire in single firing mode or in sets (steps).
- at what time the transitions fire: compulsive firing immediately after being enabled, compulsive firing at the latest possible moment after being enabled, without compulsion to fire, or according to a random distribution.

The monograph [Sta95] describes these fundamental construction principles and introduces basic classes of time-dependent Petri nets. According to these modeling specifications we systematize the basic classes of time-dependent Petri nets in the chronological order of their emergence.

The seminal studies of Merlin ([Mer74] in January) and Ramchandani ([Ram74] in February) about the combination of time and Petri nets appeared almost simultaneously. Merlin uses Petri nets to study the formal analysis and synthesis of recoverability of communication protocols. He extends Petri nets to overcome certain practical restrictions by introducing time in the following way: An interval $[a_t, b_t]$ is assigned to every transition t. The firing rule is modified in relation to time: An enabled transition t cannot fire immediately after being enabled. At least a_t time units must pass before t may fire and it must fire no later than b_t time units after being enabled, except if it becomes disabled in the meantime. The times a_t and b_t are considered relative to the latest enabling of t. Firing itself takes no time. Merlin called these time-dependent nets *Time Petri nets*.

Timed Petri nets were introduced by Ramchandani to model speeds of operations or of parts of processes. He assigns to every transition t a duration d_t. In a Timed Petri net if a transition t is enabled, it has to fire immediately. Thereby a maximum number of (just) enabled transitions is always fired, i.e., they are fired according to the *maximal-step rule*. The firing of a transition t lasts d_t time units and cannot be interrupted.

Petri nets with indication of time in the places were introduced by Sifakis [Sif77]. He assigns a minimum retention time d_p to every place p. If a token reaches place p at time τ, it must remain there for at least d_t time units, before a transition is allowed to fire using this token. The transitions in these nets fire according to the maximal-step rule.

Finally, *Petri nets with time-dependent arcs* were introduced at the beginning of the 1980s in [Wal82]. A period of time τ_k is associated with each arc k. A transition t can fire in such a net if the number of tokens on every input place p of t is at least as great as the multiplicity of the arc (t, q). These tokens must remain on p for $\tau_{(p,t)}$ time units before they are used for firing. Then, $\tau_{(t,q)}$ time units after the firing of transition t to each post-place q of t, the number of tokens corresponding to the multiplicity of the arc (t, q) is added.

In the course of the last two decades all kinds of variations and combinations of the time-dependent Petri nets specified above have arisen. Some of them can be translated into each other. The introduction of each new class is nevertheless justified if it is a natural means of specifying practical systems. A study about the expressive power of Time Petri nets and their extensions as well as a comparison to timed automata is done in [BCH$^+$13].

In addition to Petri nets with deterministic time extensions, time-dependent Petri nets in which firing occurs according to a random distribution over time have been examined, see [MBC+96], [BK02] etc. These nets are called stochastic Petri nets. An essential difference between stochastic and non-stochastic Petri nets is the relationship between the state spaces of the time-dependent Petri net and that of the underlying timeless Petri net: A stochastic Petri net has the same state space as the underlying Petri net. The state space of a deterministic time-dependent Petri net on the other hand in general comprises only part of the state space of the underlying timeless Petri net. This alone leads to different analysis algorithms. For the analysis of deterministic Petri nets, for example, it is essential to try to determine the state space.

In this book we will look at the first three classes of Petri nets with time extensions mentioned above, Time Petri nets, Timed Petri nets and Petri nets with retention time in the places. We will introduce new methods of analysis for these types of nets. Petri nets with time-dependent arcs can be reduced to these three classes. An important aspect of the analysis is the description of the state space. We can consider a complete, reduced, or parametric state space. Fundamental properties such as boundedness and liveness are defined anew, under consideration of the time specification, and methods of analysis are introduced. The behavior of time-dependent and timeless nets is compared and algorithms for qualitative and quantitative analysis are presented. Applications of these nets are mentioned together with their use in specification and analysis.

This book is structured as follows: In Chapter 2 classic Petri nets without time are introduced. In the subsequent chapters Time Petri nets and Timed Petri nets as well as Petri nets with time-dependent places (Petri nets with time windows) are studied.

Notation

In this book the set of all natural numbers is denoted by $\mathbb{N}$ whereas $\mathbb{N}^+$ stands for the set of all natural numbers excluding 0. $\mathbb{Q}_0^+$ stands for the non-negative rational numbers and $\mathbb{R}_0^+$ for the set of non-negative real numbers.

$\mathbb{N}^n$ denotes the n-ary Cartesian product over the set of natural numbers $\mathbb{N}$.

Let u and v be vectors of dimension n. Then u is less than or equal to v ($u \leq v$) if every component of u is less than or equal to the corresponding component of v. The sum $u + v$ of the two vectors u and v is also a vector of dimension n, whose components are the sums of the corresponding components of u and v. The difference $u - v$ is defined analogously.

Let A be a finite set. Then A^* is the set of all finite words (sequences) over A. ε is the empty word. $\ell(w)$ denotes the length of the word w. A^+ stands for the set $A^* \setminus \{\varepsilon\}$. The number of elements of A is denoted by $|A|$.

For an arbitrary function $f : A \longrightarrow B$ the set A is called the domain and B the codomain of f.

An n-ary function f is called arithmetical or number-theoretical if its domain is the set $\mathbb{N}^n$ and its codomain is the set $\mathbb{N}$.

Furthermore W^D is defined as the set of all functions with domain D and codomain W.

Finally, the real number r rounded down is written as $\lfloor r \rfloor$ and r rounded up as $\lceil r \rceil$.