RealTrac Technology Overview

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Abstract. This paper presents the overview of the RealTracTM technology developed by the RTL-Service ltd. It is based on the nanoLOC (IEEE 802.15.4a) radio standard. The RealTracTM technology features the local positioning system including the possibility of data transfer and voice communication. Radio access is provided by gateway units connected by wired network to a system server. Repeater units are used to increase the radio coverage area. Both gateway and repeater units serve as access points in a system. Channels for voice communication are supported by the Asterisk PBX software installed at the system server. Mobile handheld units periodically enter into active state and initiate the time-of-flight (ToF) ranging. Access points measure received signal strength (RSS) of the incoming radio signal. ToF and RSS data is processed by the server using a particle filter within localization algorithms. The following information is taken into consideration: ToF, RSS, structure of the building, air pressure value and inertial measurement unit data. The developed protocols for the communication in the system are discussed as well.

Keywords: local positioning system, indoor navigation, RTLS, Bayesian filter, particle filter, nanoLOC, IEEE 802.15.4a, time-of-flight, received signal strength, LOS, NLOS, RealTrac.

1 Introduction

Recently, research and business communities show a great interest in the Local Positioning System (LPS) technology. Unlike the Global Positioning Systems (GPS, Galileo, GLONASS, QZSS), LPS systems allow indoors localization. In order to locate an object within a certain area, a wireless infrastructure needs to be installed. As a rule, indoors, the localization accuracy depends on the spatial density of anchor nodes. Those anchors are used to measure distances to mobile nodes.

Real-time positioning systems are based on wireless networks which can utilize different methods of distance measurement: Time-of-Flight (ToF), Angleof-Arrival (AoA) and Received Signal Strength (RSS). Methods based on Time-of-Arrival (ToA), Time-Difference-of-Arrival (TDoA), Round-Trip-Time (RTT) are referred to the ToF methods group. First two methods (ToA and TDoA) do require system time synchronization between all nodes in the system, whereas the RTT method does not. Obviously, distance measurements based on the RSS are relatively inaccurate, especially in a case of substantial distances between nodes. However, knowing of the RSS value is very important for applications with roomlevel accuracy indoors: room walls create a drop in a signal strength, which is used to reliably determine the room where the mobile object is in. This technique is widely used in RSS patterns methods [1,2].

Along with the localization capability, wireless networks provide data communications channels between nodes. Commercial LPSs use various radio technologies: Wi-Fi, ZigBee, UWB, nanoLOC, NFC RFID, etc. This paper presents an overview of the RealTracTMtechnology developed by the RTL-Service ltd. It is based on the nanoLOC (IEEE 802.15.4a) radio standard. The RealTracTMtechnology combines good data transfer rates with low power consumption of radio devices, location estimation and voice communication feature at the same time.

The rest of this paper is organized as follows. Section 2 describes architecture of the RealTracTM technology; technical characteristics of devices used; data transfer protocols for communication between radio modules in the system and between a client and a server. Section 3 is devoted to the applied location estimation algorithms based primarily on the particle filter. ToF and RSS values, building structure, constraints on object velocity and data acquired from the embedded inertial measurement unit (IMU) are taken into a consideration. The opportunity of using the precise air pressure sensor is utilized for the floor identification and for the estimation of the relative height. Those features are based on the atmospheric pressure data of all devices in the system. Section 4 concludes the development work and briefly describes possible applications of the described technology and defines future development directions.

2 RealTracTMTechnology Description

2.1 Network Structure and Operation Algorithms

The RealTracTMradio system is based on the nanoLOC communication standard (introduced by Nanotron Technologies GmbH, Germany). The main feature of this standard is the automatic distance measurement using the Time-of-Flight method. The measured distances between fixed nodes and a mobile node are used to estimate the location of the mobile object. The brief summary of the nanoLOC radio specifications [3] is presented in Table 1.

The RealTracTMsystem components can be divided into two parts: hardware and software. The hardware part includes all physical devices (intercoms, gateways, repeaters, servers, switches, etc.). All devices are uniquely identified by their MAC addresses. The software part includes client software communicating with a system server.

The network diagram of the system is presented in Fig. 1. The radio coverage area is formed by access points (AP). They operate in either gateway or repeater

Parameter	Value							
Frequency range	2.4 2.48 GHz, ISM, unlicensed							
Frequency band	80 MHz, 1 channel (optional: 3 channels of							
	22 MHz)							
RF signal encoding	chirp modulation							
Bit rate	1 Mbit/s							
TX power	100 mW (20 dBm), software control							
Medium access method	primarily CSMA (TDMA is possible)							
The accuracy of ranging	up to 1 meter							
The method of ranging	propagation delay, round-trip time							
Range of reliable connection between	up to 1500 meters (outdoors, directed an-							
access points	tennas), 50-70 meters (indoors, through							
	several walls)							
Range of reliable connection between	up to 400 meters (outdoors), 50 meters (in-							
an access point and a mobile node	doors, through several walls)							

Table 1. NanoLOC radio standard specificatio	ns
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mode. The gateways are connected to the server via Ethernet cable network (via switches). They act as a bridge between the wired and wireless segments of the system. All data packets from the wireless part of the network are redirected into the wired part and vice versa. When the AP is not connected by a wired connection to the Ethernet network, it switches to the repeater mode automatically. In this particular mode it retransmits all incoming broadcast traffic back into the wireless segment, therefore increasing the radio coverage area. The AP works as a reference point in positioning (as an anchor) in both gateway and repeater modes.

A mobile handheld device is called intercom, since it provides voice communication. The intercom is set into a power saving mode for the larger part of a duty cycle. In the active state it is in the listening mode and collecting the information regarding neighbor units (MAC addresses of gateways, receivers and other intercom units). The intercom receives copies of the own packets resented by repeaters and obtains their MAC addresses. The list of neighbors is constantly updated. Normally this list is never empty for duty cycles less than 10 seconds. For the larger periods some entries become outdated and they are deleted from the list.

The intercom performs ranging measurements to several anchors just after waking up. The ranging results (if any) are broadcasted by the intercom in a so-called blink packet. This packet is received by gateway(s) and then redirected to the system server. If there are no gateways around, this packet is delivered to the system server by repeaters, which retransmit the received broadcast packet.

The server analyzes the blink packet and issues a set of commands to a number of gateways to execute additional ranging to intercoms if needed. There was developed number of adaptive algorithms used to determine the required number of additional ToF distance measurements to satisfy the sufficient accuracy. This centralized control of ranging queue increases the efficiency of CSMA algorithm [4].



Fig. 1. RealTracTM technology network diagram

For large covered areas the server may control several non-intersecting radio zones and assign the dedicated queue to each radio zone. All the obtained ranging data is processed and the location of the mobile node is determined.

The RealTracTM technology does not directly limit the maximum number of used mobile devices. This number depends on the intercoms duty cycle and on the volume of generated data traffic in the wireless segment.

The maximum payload size of the nanoLOC data frame is 128 bytes. Therefore the largest frame transmission, including a preamble, duration of data radio frame, inter-frame gap and the acknowledgement frame, lasts approximately 1.4 ms at 1 Mbps (chirp/symbol duration is 1 us) speed. The NanoLOC radio uses RTT method for the ranging. To calculate the distance, three data frames together with following acknowledgements should be sent. This cycle lasts approximately 3-7 ms. The average value of the time differences between the data frame and the acknowledgement frame at local and remote sides corresponds to the measured distance.

Single location estimation assumes up to 4-5 ranging procedures to be executed. Accordingly, the system is able to process not more than 20-25 locations per 1 second taking into consideration a certain reserved space in the radio bandwidth. The system server can adjust the intercoms duty cycle in the range between 0.3 second to 3 minutes by sending a distinct command.

The simplified rule for the maximum quantity of nodes calculation is the following. If the intercoms duty cycle is 1 second, then the network can operate only with 20 intercom units; if duty cycle is 10 seconds, maximum units quantity increases up to 200. For 1200 mobile nodes the duty cycle is 20 seconds, however location estimations are done only once per 3 minutes.

2.2 Voice Communication

The intercom unit supports voice communication feature. That allows to make phone calls to other intercoms and to the system server in full duplex mode. Also it can be used like a radio set in half-duplex mode.

The bitrate of the one-way voice channel is approximately 8 kbps. Uncompressed voice is sampled at 16 bit quantization depth and frequency of 8 kHz. The hardware compression according to G.729A is applied. Sound packets by 100 bytes length are generated every 20 ms (50 packets per 1 second). In order to increase the reliability of voice channels additional redundancy was implemented. Sound packets might contain up to 5 sound fragments by 20 ms of the compressed sound: 1 actual fragment and 4 outdated. Redundancy level adaptively decreases in conditions of low packet loss and increases in environment with strong noise in 2.4 GHz band.

As it was mentioned above, data and voice packets from handheld devices are directed to the system server for communication and data acquisition, processed and stored (see Fig. 2). Voice communication is provided by the open source PBX Asterisk software, featuring SIP telephony functionality. Software PBX is responsible for redirecting calls to other intercom units and external soft-phone clients. If the server is equipped with the special telephony adapter, then calls can be redirected even to POTS or cellular phone network. Any client software which supports G.729A codec might be used both on handheld computers and laptops to accept and place voice calls from and to intercoms. If the soft-phone does not support G.729A codec, Asterisk should be configured to transcode the voice traffic.



Fig. 2. The RealTracTM technology communication diagram

When the intercom unit is moving while the voice call, system server provides automatic roaming within the network. The server sends sound packets through the most appropriate gateway unit (the nearest to the intercom).

2.3 INCP

The RealTracTMuses the unique protocol INCP (Inter-Nano-Com Protocol), developed for the communication between devices and software modules. The asymmetrical protocol recognizes a dedicated system server with the certain roles and several clients with a limited number of functions. Every INCP message is encapsulated in either UDP packet (over Ethernet network) or nanoLOC frame (over wireless network).

The brief information on INCP headers is presented in Table 2.

Offset	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	
0	Ver[4]									Len[12]							
2	Type[8]									Hops[8]							
4	Packat ID [29]																
6	$1 \operatorname{acket}[52]$																
8	Reserved								B R R R DevType[4]								
10	TxPower[8]									RxPower[8]							
12																	
14	DeviceID[48]																
16																	
18	Message payload[]																

 Table 2. INCP message structure

Some fields in this table need to be explained. Due to the broadcast nature of wireless traffic, INCP messages may be delivered to the server via different routes. Consequently, the server may receive several copies of the same INCP message. Data fields *PacketID* and *DeviceID* provide a way to distinguish replicas of different INCP messages from each other. On one hand, the system server must recognize duplicates of the original message and prevent from repeated processing. On the other hand, those replicas may be useful for discovering network structure of the system. *PacketID* is a 32 bit integer number, which is unique across messages of certain device during certain interval (at least 10 sec). RealTracTMdevices use internal clock counter as a *PacketID* value. *DeviceID* is a 48 bit integer number, which corresponds to MAC address of the given device. The system server inserts the MAC address of the destination device into this field.

The message contains other fields, which are used for investigation process of the network structure in general and for the position calculation of the origin device – in particular: *Hops*, *RxPower*, *TxPower*. The value in the *Hops* field is incremented when the message passes gateway or repeater units on its way to the system server. Accordingly, the value 0 may be found in the messages originated by gateways. The value 1 may be found in a message from a mobile device when it is located in the range of a certain gateway. The values greater than 1 may be found in a message from a device, which is out of a range of any gateway, and thus the message was relayed by repeaters at least once.

The value in the TxPower field corresponds to the RF output power of the origin device. The value in the RxPower field corresponds to the received signal strength at the first repeater or gateway. These two values help to evaluate path loss between two devices.

All the INCP message features may be separated into three categories.

- Voice communication (CALL, BYE, SND_PACKET, BANDWIDTH_REQUEST).
- Firmware updates (TFTP_PACKET).
- Status acquisition and configuration (ALIVE, PARAMETER).

The ALIVE message may contain series of blocks, for example: *LOCATION*, *VERSION*, *CONFIG*, *STATE* and *IMU* blocks. *LOCATION* block is used to deliver ToF, RSS and IMU data to the server.

Some parameters of the device may be changed by the system server. It may set RF output power, status reporting interval, ring and voice volume by sending the PARAMETER message. Also, the server may turn the device off or suspend / resume IMU function.

2.4 RealTracTMAPI

The RealTracTM server can communicate with software clients through web protocols (see Fig. 2). The Real Time Location System Communication Protocol (RtlsCP) – the public API, was developed for this purpose. Requests and responses format is based on JSON notation.

RtlsCP covers most common features of all RTL systems and provides necessary services for handling the RealTracTMhardware and visualization. The common API provides the following data.

- Anchors and mobile nodes list.
- Anchors and mobile nodes real time locations data.
- Anchors and mobile nodes parameters and statuses.
- KML files for visualization.

KML formatted files were used in RtlsCP since KML become natively be supported by both Google Maps and Google Earth applications. RtlsCP can operate with either relative (x, y, height) or absolute (latitude, longitude, altitude) coordinates.

3 Positioning Technique Overview

3.1 Localization Server

All the measured data, including ToF, RSS, air pressure and IMU data is processed by the localization server. Initially, the air pressure data is used for altitude calculation. Next, the floor in a building is determined and the corresponding 2D map is identified. After that, all measurements and the 2D map are used for the estimation of a mobile node position (x, y coordinates) on the specified floor. Finally mobile node 3D location (x,y,z) is sent to software clients through the web using RtlsCP.

3.2 Location Algorithms Implementation

To combine all available sources of information for accurate positioning the Bayesian filtering [5] was used. The main goal of this method is to give an optimal estimation of a target location \hat{x}_t which is characterized by pdf $P(X_t)$. By Bayes theorem

$$P(X_{t_n}|Y_{t_1,\dots,t_n}) = \frac{P(Y_{t_n}|X_{t_n}) * P(X_{t_n})}{P(Y_{t_n})}.$$
(1)

In this equation a posterior probability $P(X_{t_n}|Y_{t_1,\ldots,t_n})$ represents the estimated pdf of the system state. The likelihood $P(Y_{t_n}|X_{t_n})$ refers to the measurements model. The vector of distances $R_n = [r_{B_1}, \ldots, r_{B_k}]$ and the vector of signal strengths $RSS_n = [rss_{B_1}, \ldots, rss_{B_k}]$ measured at each time moment t_n between the set of anchors B_1, \ldots, B_k and a mobile node (target) are used for the likelihood calculation. The prior $P(X_t)$ represents the target model. For the prior calculation it is possible to use the information about the motion model, IMU data, and structure of the building. The $P(Y_{t_n})$ is used as the normalization coefficient.

The filter is based on a recursive estimation of the system state X_{t_n} by noisy measurements Y_{t_n} at time t_n taking into account all previous measurements $Y_{t_1,\ldots,t_{n-1}}$. The system state X_{t_n} consists of the target coordinates x_n and velocity v_n . The Y_{t_n} refers to the data obtained from ranging measurements.

In the ongoing work particle filter was applied for the location calculation. Particle filter is the implementation of the Bayesian filtering using the Sequential Monte-Carlo Method. In this method system state is represented by the set of random samples or particles with corresponding weights. This particle system is located, weighted and propagated recursively according to the Bayesian rule [6].

At each moment the pdf is characterized by the set of particles $x_t^{(i)}$, i = 1..N, with weights $w_t^{(i)}$, i = 1..N, where particle $x_t^{(i)}$ corresponds to the system state and includes information about target coordinates and velocities, $w_t^{(i)}$ is the non-negative weight of the corresponding particle. The weights are normalized the way, so that $\sum w_t^{(i)} = 1$.

The algorithm of the particle filter consists of several phases: initialization, propagation, weights calculation, resampling, and state estimation. During the initialization the weights are uniformly distributed in the area of the intersection of circles corresponded to the measured distances (Fig. 3), thus $w^{(i)} = \frac{1}{N}$.

In propagation phase the position of a particle is calculated with the use of the following motion model equation

$$x_n = x_{n-1} + (v_{n-1} \cdot t), \tag{2}$$

where x_n and x_{n-1} denote to coordinates of the target location at the corresponding time moments n and n-1, v_{n-1} is the vector of the target velocity at the moment n-1, and t is the time interval between time moments nand n-1. The vector v of the target velocity consists of two components v_r , and v_{α} . The components change consequently that $v_{rn} = v_{rn-1} + \Delta v_r$, and



Fig. 3. Initial particles, generated inside the circles intersection area

 $v_{an} = v_{an-1} + \Delta v_{\alpha}$. $\Delta v_r = N(0, \sigma_{v_r})$ is normally distributed random noise with deviation σ_{v_r} , corresponding to the possible changes in speed of a target, and $\Delta v_{\alpha} = R(-\alpha, \alpha)$ is the uniformly distributed random noise, corresponding to the possible changes in target direction.

The information about v_r and v_{α} is gotten from IMU or is chosen corresponding to the motion model. The errors δv_r and δv_{α} are chosen empirically.

The structure of the building information is used for increasing positioning accuracy [7]. If the particle during its motion crosses the wall (which is impossible for real target motion), it is removed from the current set (see Fig. 4).



Fig. 4. Particle propagation restricted due to building structure

The source ToF and RSS data is used to recalculate the weight of each particle. The procedure of the weights calculation corresponds to the calculation of likelihood in (1). At this step it is possible to combine (or fuse) the results of different weighting algorithms. Let indexes A_1, \ldots, A_m correspond to different algorithms, and let $w_{A_j}^{(i)}$ be the weight of i-th particle calculated from algorithm A_j . Then after calculation of weights for each particle for each algorithm the final particle weight is calculated as

$$w^{(i)} = \frac{\prod_{k=1}^{m} w_{A_k}^{(i)}}{\sum_{j=1}^{N} \prod_{k=1}^{m} w_{A_k}^{(j)}}.$$
(3)

In the ongoing work the RSS pattern matching algorithm [8] was used as the basic algorithm for likelihood calculation $P(Y_t|X_t)$ since it demonstrates better accuracy in the case of the signal shadowing and NLOS conditions [1].

After weights calculation the multinomial resampling algorithm was used [6] to avoid the degeneracy problem (in case if after several steps the weights of majority of particles become zero).

Finally, the system state is calculated as the weighted sum of resampled particles:

$$\hat{x} = \sum_{i=1}^{N} w^{(i)} x^{(i)}.$$
(4)

3.3 Pressure Sensor Usage

Every RealTracTM device is equipped with the Bosch BMP085 air pressure sensor. Therefore both stationary and mobile nodes measure absolute air pressure and send results to the system server.

The BMP085 features 4 sampling modes and has absolute accuracy ± 100 Pa and 1 Pa resolution of the output data. According to preliminary experiments with this sensor, which was set in high and ultra-high resolution modes, the peak-to-peak pressure values do not exceed the range of 25-30 Pa (2.5-3 m) [9]. Thus it is possible to use the sensor for floor identification in a multistory building and even for localization of an object on inter-floor places.

The applicability of the pressure sensor to measure the changes of relative height is demonstrated in Fig. 5. At zero time moment two devices were at the same height (on the floor). Approximately at the 43-th second one of the devices (altitude values marked as rhombi) was raised to the height of 1.5 m. Two curves were obtained by the simplest 5-point un-weighted smoothing. The altitude in meters was calculated using the international barometric formula with the measured pressure and the pressure at the sea level at 1013.25 hPa.



Fig. 5. Altitude changes registration

3.4 IMU-Based Localization

The developed solution assumes that an IMU could be installed in any point at the persons body. The IMU has 3-axis accelerometer and 3-axis gyroscope. The accelerometer is active on permanent basis. In order to preserve the power, the gyroscope is turned on only in case of the person motion or walking process is detected.

The IMU-based solution includes three modules: distance, orientation and trajectory calculation modules.

The first module calculates the total travelled distance by counting number of steps and estimation of each steps length. The number of steps is calculated with the high accuracy, more than 98%. Algorithms displayed excellent performance with no regard to orientation of the device, speed of walk or other person specific parameters. For step length estimation several methods were tested. The following formula [10] was used as the most efficient: $L_{step} = K \sqrt[4]{a_{max} - a_{min}}$. The calibration constant value was chosen empirically.

To represent the orientation of the device the quaternions were used. To avoid heavy calculations while getting the attitude, the complementary filter was applied [11]. The heading direction is calculated as a mean of 2-dimensional vector using quaternion observations.

The position update was performed at each foot step. When the new step is detected, the system acquires the step length and the step heading direction. Both the step length and the heading direction are served into the path restoration module. Those values are multiplied and new position point is added to the trajectory. Those calculations are performed by the internal MCU of handheld devices. Compressed trajectory data is sent to the server within the INCP ALIVE message.

During the experiments on evaluation of IMU accuracy, a researcher walked along the corridors of O-shaped building with a perimeter about 270 m and then returned to the starting point. The computed return position was compared with the real one. The parameter Return Position Error (RPE) for multiple tests did not exceed 3% of the total travelled distance.

4 Conclusion

The RealTracTM technology can be successfully applied for the Local Positioning Systems development and for providing voice communication over the wireless sensors network infrastructure. As an example, a pilot project in a collaboration with Intelmine ltd. (Russian Federation) was started in the year of 2012. It was devoted to the development of the positioning, data and voice communication system in mines. Specific restrictions on intrinsic safety were applied to the equipment. The first trial results at the coal mine "Polysayevskaya" displayed high reliability of the technology. The RealTracTM can also be successfully used in hospitals, in logistics for the large industrial areas, in hotel business, etc.

It should be noted that the current implementation of IMU does not require user calibration and works robust with any speed of walk. However, it accumulates yaw

error and this error could not be removed easily without additional information. Thus one of the directions of future development will be discovering new algorithms of the IMU-based data fusion with the RSS/ToF based data and the building structure in the localization engine. This work will also include further modification of the particle filter.

Another area for the development concerns the altitude estimation accuracy. The authors team plan to investigate cooperative algorithms for calculation of relative height providing accuracy better than 0.5 m.

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