Logic Aggregation

Xuefeng Wen and Hu Liu

Institute of Logic and Cognition & Department of Philosophy,
Sun Yat-sen University, Guangzhou, 510275, China
wxflogic@gmail.com, liuhu2@mail.sysu.edu.cn

Abstract. We study the possibility and impossibility of aggregating logics, which may come from different sources (individuals, agents, groups, societies, cultures). A logic is treated as a binary relation between sets of formulas and formulas (or a set of accepted arguments). Logic aggregation is treated as argument-wise. We prove that certain logical properties can be preserved by some desired aggregation functions, while some other logical properties cannot be preserved together under non-degenerate aggregation functions, as long as some natural conditions for the aggregation function are satisfied. We compare our framework of logic aggregation with other aggregation frameworks, including preference aggregation and judgment aggregation.

1 Motivations

Judgment aggregation [15] is to study how to aggregate individual judgments on logically correlated propositions to collective judgments. Since it is both a generalization of preference aggregation in social choice theory, and closely related to deliberative democracy in political science, as well as belief merging in informatics, it has been quickly developed in the past decade. For an up-to-date survey of it, refer to [14] and [11] (more technical). This paper is a further development of judgment aggregation, by setting up a framework called logic aggregation, or aggregation of logics. The research issue of logic aggregation is: given a set of logics, which may come from different sources (individuals, agents, groups, societies, cultures, etc.), how to aggregate these logics into one by some generally acceptable methods. Why is this problem interesting? Here are several motivations for studying logic aggregation.

Firstly, when a pluralistic view on logic is taken, the problem of logic aggregation arises naturally. All present research in judgment aggregation presumes a unified underlying logic, though it need not be the classical logic (see [2]). Not only different individuals have the same logic, but also the logic underlying the collective judgments is the same as those of individuals. But different sources may use different logics in judgments. This could be true for aggregating information from distributed systems, which may come from different domains and use different logics in representing their knowledge. It could also be the case for judgment aggregation in a situation of cross-cultural communication, where individuals or groups from different cultures may have different reasoning patterns.

D. Grossi, O. Roy, and H. Huang (Eds.): LORI 2013, LNCS 8196, pp. 282–295, 2013.

Moreover, even if different sources use a unified logic, it is not necessary for the collective to use the same logic. In other words, it is unjustified to presume the collective rationality to be the same as the normal rationality for individuals.

Secondly, logic aggregation provides a framework that may avoid some philosophical difficulties in judgment aggregation. Most research in judgment aggregation adopts a proposition-wise approach, i.e., aggregation of sets of judgments is reduced to aggregation of propositions. When the standard independence condition is assumed, the collective judgment of a proposition does not depend on individual judgments on other propositions. Though independence is natural in preference aggregation, it is controversial in judgment aggregation, because different propositions may have relevance in content apart from pure logical correlations. In particular, some propositions may be premises or reasons for others. From a deliberative point of view, proposition-wise aggregation with independence is undesirable. There are several approaches to this problem. One is to keep proposition-wise aggregation but weaken or generalize the notion of independence [17,4]. The other is to give up the proposition-wise aggregation completely and adopt a holistic method, like distance-based aggregation [18]. Logic aggregation is a compromised approach, going from proposition-wise aggregation to argument-wise aggregation. To realize it, we treat a logic as a set of (accepted) arguments. Then logic aggregation boils down to aggregation of sets of arguments, where the aggregation is argument-wise – a proposition is considered together with its premises (reasons) in aggregation.

Thirdly, logic aggregation opens the door for exploring more notions of rationality and collective rationality. In judgment aggregation, consistency is often required to be preserved from individual judgment sets to the collective one. But consistency is only one property in logic. There are other interesting properties in logics that can be considered, for example, transitivity (a.k.a. cut). In other words, going from judgment aggregation to logic aggregation, we are able to consider more notions of rationality and collective rationality. Unlike [10], which studies different rationality constraints for aggregation in different languages, we explore different rationality in the same language.

Last but not the least, logic aggregation is more or less an application of graph aggregation proposed in [5]. Graph aggregation is in effect the aggregation of arbitrary binary relations on a given set. Though it generalizes preference aggregation, where the binary relation is an ordering, it is too abstract to illustrate interesting applications. Since a logic can also be treated as a binary relation (between sets of formulas and formulas), logic aggregation can be roughly embedded in graph aggregation and thus provides an interesting instantiation of the latter. See Section 5 for more on this.

The problem of aggregating logics has been touched in judgment aggregation before [1,16]. But it was discussed in particular cases. In this paper, we propose the problem explicitly and study it generally in the framework of logic aggregation. The rest of the paper is organized as follows. In Section 2, we introduce the general notion of logic and some typical properties for logic. Section 3 presents the framework of logic aggregation, for which we prove some possibility and impossibility results in Section 4. Section 5 compares logic aggregation with other aggregation frameworks, including preference aggregation and judgment aggregation. The conclusion section indicates some future works.

2 Logics

A logic used to be treated as a set of formulas (which are valid in the logic). This view has been proved to be too narrow since the emergence of numerous non-classical logics, in particular, logics without valid formulas at all, such as Kleene's three-valued logic. Now a logic is usually considered to be a consequence relation (either syntactically or semantically defined), which we will adopt in this paper.

Let $\mathcal{L}$ be a fixed language, namely a set of sentences, which has at least three elements. It can be either finite or infinite. Generally, a *logic* for $\mathcal{L}$ is a binary relation $\vdash$ between $\wp(\mathcal{L})$ and $\mathcal{L}$, where $\wp(\mathcal{L})$ is the power set of $\mathcal{L}$. A pair $(\Sigma, \varphi) \in \wp(\mathcal{L}) \times \mathcal{L}$ is called an *argument* in $\mathcal{L}$, with Σ the set of premises and φ the conclusion. An argument with empty premise is called a *judgment*. An argument (Σ, φ) is called *valid* (or *accepted*) in a logic $\vdash$, if $(\Sigma, \varphi) \in \vdash$, which is often denoted by $\Sigma \vdash \varphi$ instead. Thus, a logic is treated as the set of all valid (or accepted) arguments (rather than formulas) in it. As we do not specify how validity is syntactically or semantically defined, as in standard logic textbooks, we do not distinguish between validity and acceptance of an argument. We assume that any binary relation between $\wp(\mathcal{L})$ and $\mathcal{L}$ is a logic. Instead of $\emptyset \vdash \varphi$, we write $\vdash \varphi$. By $\Sigma \vdash \Delta$, we mean $\Sigma \vdash \varphi$ for all $\varphi \in \Delta$.[1] By Σ, φ (or φ, Σ) and Σ, Σ' occurring on the left hand side of $\vdash$ or $\subseteq$, we mean $\Sigma \cup \{\varphi\}$ and $\Sigma \cup \Sigma'$, respectively. The following are typical properties of logics considered in the literature (see [8] for example).

- Non-triviality: A logic $\vdash$ for $\mathcal{L}$ is *non-trivial* if $\vdash \neq \wp(\mathcal{L}) \times \mathcal{L}$, i.e., a logic is non-trivial, if it does not accept all arguments in the language.
- Consistency: For $\mathcal{L}$ with negation $\neg$, a logic $\vdash$ for $\mathcal{L}$ is *consistent* if there is no $\varphi \in \mathcal{L}$ such that $\vdash \varphi$ and $\vdash \neg\varphi$.

For classical logic, non-triviality and consistency boil down to the same notion. But for non-classical logics, they are not the same. It is well known that paraconsistent logics can be inconsistent but non-trivial.

- Reflexivity: A logic $\vdash$ for $\mathcal{L}$ is *reflexive* if for all $\varphi \in \mathcal{L}$, $\varphi \vdash \varphi$.

This is also known as *restricted reflexivity*. A stronger version of reflexivity is as follows.

- Strong reflexivity: A logic $\vdash$ for $\mathcal{L}$ is *strongly reflexive* if for all $\Sigma, \varphi \subseteq \mathcal{L}$, $\varphi \in \Sigma$ implies $\Sigma \vdash \varphi$.

[1] Note that this is different from the standard multi-conclusion consequence, where $\Sigma \vdash \Delta$ means $\Sigma \vdash \varphi$ for *some* $\varphi \in \Delta$.

It is easily seen that strong reflexivity can be derived from reflexivity plus monotonicity given below.

– Monotonicity: A logic $\vdash$ for $\mathcal{L}$ is *monotonic* if for all $\Sigma, \Sigma', \varphi \subseteq \mathcal{L}$, $\Sigma \vdash \varphi$ implies $\Sigma, \Sigma' \vdash \varphi$.

Monotonicity is not as uncontroversial as reflexivity. In common sense reasoning, monotonicity is not obeyed, which motivates the branch of nonmonotonic reasoning. The following restricted version of monotonicity is weaker but less controversial.

– Cautious monotonicity: A logic $\vdash$ for $\mathcal{L}$ is *cautiously monotonic* if for all $\Sigma, \varphi, \varphi' \subseteq \mathcal{L}$, $\Sigma \vdash \varphi$ together with $\Sigma \vdash \varphi'$ imply $\Sigma, \varphi' \vdash \varphi$.
– Transitivity: A logic $\vdash$ for $\mathcal{L}$ is *transitive* if for all $\Sigma, \Sigma', \varphi, \varphi' \subseteq \mathcal{L}$, $\Sigma \vdash \varphi$ and $\varphi, \Sigma' \vdash \varphi'$ imply $\Sigma, \Sigma' \vdash \varphi'$.

Transitivity is also known as *cut* in proof theory. It is crucial in composing a valid argument (proof) from other valid arguments (proofs). Many logics (such as relevant logic, linear logic, nonmonotonic logic) lack monotonicity, while preserving transitivity.

– Compactness: A logic $\vdash$ for $\mathcal{L}$ is *compact* if for all $\Sigma, \varphi \subseteq \mathcal{L}$, if $\Sigma \vdash \varphi$ then there is a finite $\Sigma' \subseteq \Sigma$ such that $\Sigma' \vdash \varphi$.

In some literature, compactness refers to the following stronger property, which we call m-compactness.

– M-compactness: A logic $\vdash$ for $\mathcal{L}$ is *m-compact* if for all $\Sigma, \varphi \subseteq \mathcal{L}$, $\Sigma \vdash \varphi$ iff there is a finite $\Sigma' \subseteq \Sigma$ such that $\Sigma' \vdash \varphi$.

It is easily seen that m-compactness is actually the conjunction of compactness and monotonicity.[2]

– Formality: A logic $\vdash$ for $\mathcal{L}$ is *(universally) formal* (a.k.a. *structural*) if for all $\Sigma, \varphi \subseteq \mathcal{L}$, for all substitution σ, $\Sigma \vdash \varphi$ implies $\Sigma^\sigma \vdash \varphi^\sigma$, where φ^σ is the substitution of φ by σ and $\Sigma^\sigma = \{\psi^\sigma \mid \psi \in \Sigma\}$.[3]

Formality, which is realized by the substitution rule (or in effect by axiom schemes), is included in most logics, since it is the mechanism for a logic to be characterized by a finite set of arguments (or, axioms and rules). Universal formality, however, also restricts the power of logic to cover many valid arguments in natural language. For instance, "all bachelors are unmarried" is not valid in standard logic, since it is not true by its form but rather by the meanings of the expressions in it, unless we formalize 'bachelors' as a compound predicate. For this reason, defining a relativized formality as follows is reasonable.

[2] In abstract algebraic logic theory [12], pioneered by Tarski, only a binary relation $\vdash$ that satisfies reflexivity, monotonicity, and transitivity can be called a logic. In Tarski's original theory of logical consequence, the three minimal properties of a logic are reflexivity, transitivity, and m-compactness.

[3] As we do not specify the language $\mathcal{L}$ in our general framework, substitution here is underspecified. It can be defined precisely as long as the language $\mathcal{L}$ is specified.

Definition 1 (Formality). *Given $\mathcal{A} \subseteq \mathcal{L}$, a logic $\vdash$ for $\mathcal{L}$ is $\mathcal{A}$-*formal *if for all $\Sigma, \varphi \subseteq \mathcal{A}$, for all substitution σ, $\Sigma \vdash \varphi$ implies $\Sigma^\sigma \vdash \varphi^\sigma$, where $\Sigma^\sigma = \{\psi^\sigma \mid \psi \in \Sigma\}$. A logic for $\mathcal{L}$ is* formal *if it is $\mathcal{L}$-formal.*

For similar reasons, we propose relativized completeness and disjunctiveness as follows.

Definition 2 (Syntactical completeness). *For $\mathcal{L}$ with negation*[4] $\neg$ *and nonempty $\mathcal{A} \subseteq \mathcal{L}$, a logic $\vdash$ for $\mathcal{L}$ is $\mathcal{A}$-*complete *if for all $\varphi \in \mathcal{A}$, either $\vdash \varphi$ or $\vdash \neg\varphi$. A logic $\vdash$ for $\mathcal{L}$ is* (syntactically) complete *(a.k.a.* negation complete*) if it is $\mathcal{L}$-complete.*

Definition 3 (Disjunction property). *For $\mathcal{L}$ with disjunction*[5] $\vee$ *and $\mathcal{A} \subseteq \mathcal{L}$ including at least one formula of the form $\varphi \vee \psi$, a logic $\vdash$ for $\mathcal{L}$ is $\mathcal{A}$-*disjunctive *if for all $\varphi \vee \psi \in \mathcal{A}$, $\vdash \varphi \vee \psi$ implies $\vdash \varphi$ or $\vdash \psi$. A logic $\vdash$ for $\mathcal{L}$ has the* disjunction property *if it is $\mathcal{L}$-disjunctive.*

It is well known that intuitionistic logic has the disjunction property. The following two properties are usually assumed for logics.

Definition 4 (Conjunction property). *For $\mathcal{L}$ with conjunction $\wedge$, a logic $\vdash$ for $\mathcal{L}$ is* conjunctive *if for all $\Sigma, \varphi, \psi \subseteq \mathcal{L}$, $\Sigma \vdash \varphi \wedge \psi$ iff $\Sigma \vdash \varphi$ and $\Sigma \vdash \psi$.*

Definition 5 (Confluency). *A logic $\vdash$ for $\mathcal{L}$ is* confluent *if for all $\Sigma, \Sigma', \varphi \subseteq \mathcal{L}$, $\Sigma \vdash \varphi$ and $\Sigma' \vdash \varphi$ imply $\Sigma, \Sigma' \vdash \varphi$.*

Note that monotonicity implies confluency but not vice versa. Finally, we introduce a property which is more familiar in informal logic than in formal logic.

Definition 6 (Non-tautologicity). *A logic $\vdash$ for $\mathcal{L}$ is* non-tautological *if for all $\Sigma, \varphi \subseteq \mathcal{L}$, $\Sigma \vdash \varphi$ implies $\varphi \notin \Sigma$.*

Non-tautologicity is usually not required in formal logic. Indeed, if a logic is reflexive or monotonic, then it can not be non-tautological. But non-tautologicity is rather plausible for natural language arguments, where begging the question is not allowed. In other words, a good argument should not contain its conclusion as one of its premises. To save monotonicity, we could slightly restrict it as: $\Sigma, \varphi' \vdash \varphi$ whenever $\Sigma \vdash \varphi$ and $\varphi' \neq \varphi$.

For brevity, we use the following notations in the sequel:

- $\mathbf{L}$: the set of all logics for $\mathcal{L}$, namely, $\mathbf{L} = \wp(\wp(\mathcal{L}) \times \mathcal{L})$.
- $\mathbf{L}_{cc}$: the set of all consistent and complete logics for $\mathcal{L}$.
- $\mathbf{L}_{cj}$: the set of all conjunctive logics for $\mathcal{L}$.
- $\mathbf{L}_{nt}$: the set of all non-tautological logics for $\mathcal{L}$.

[4] For our purpose, it need not be interpreted as the standard negation. It can be any unary connective or operator.

[5] For our purpose, it need not be interpreted as the standard disjunction. It can be any binary connective or operator.

3 Social Logic Function

Let $N = \{1, \ldots, n\}$ be a finite set of agents (groups, societies, cultures) with at least three members. A profile $\Vdash = (\vdash_1, \ldots, \vdash_n)$ is a vector of logics in $\mathbf{L}$. Analogously, we write $\Vdash'$ for the profile $(\vdash'_1, \ldots, \vdash'_n)$. Let $N^{\Vdash}_{\Sigma,\varphi}$ be the set of agents who accept the argument (Σ, φ) in the profile $\Vdash$, namely, $N^{\Vdash}_{\Sigma,\varphi} = \{i \in N \mid \Sigma \vdash_i \varphi\}$. We write $N^{\Vdash}_{\varphi}$ for $N^{\Vdash}_{\emptyset,\varphi}$ and $N^{\Vdash}_{\varphi,\psi}$ for $N^{\Vdash}_{\{\varphi\},\psi}$, respectively. Let $N^{\Vdash}_{\Sigma,\Delta} = \bigcap_{\varphi\in\Delta} N^{\Vdash}_{\Sigma,\varphi}$, namely, the set of agents who accept the arguments (Σ, φ) for all $\varphi \in \Delta$ in $\Vdash$. Instead of $F(\Vdash)$, we write $\vdash_F$, and analogously, we write $\vdash'_F$ for $F(\Vdash')$.

Definition 7 (Social logic function). *A* social logic function *(SLF) (for n logics) for $\mathcal{L}$ is a map $F : \mathbf{L}^n \to \mathbf{L}$.*

Some natural desiderata for SLFs borrowed from social choice theory are listed below.

Definition 8 (Unanimity). *An SLF $F : \mathbf{L}^n \to \mathbf{L}$ is* unanimous, *if for all $\Sigma, \varphi \subseteq \mathcal{L}$, for all profiles $\Vdash$ in $\mathbf{L}^n$, $\Sigma \vdash_i \varphi$ for all $i \in N$ implies $\Sigma \vdash_F \varphi$, i.e., if an argument is accepted by all individuals, then it is collectively accepted.*

The following is the counterpart of groundedness proposed in [20].

Definition 9 (Groundedness). *An SLF $F : \mathbf{L}^n \to \mathbf{L}$ is* grounded*, if for all $\Sigma, \varphi \subseteq \mathcal{L}$, for all profiles $\Vdash$ in $\mathbf{L}^n$, $\Sigma \vdash_F \varphi$ implies $\Sigma \vdash_i \varphi$ for some $i \in N$, i.e., if an argument is collectively accepted, then it must be accepted by one of the individuals. An SLF is* ungrounded *if it is not grounded.*

Unanimity and groundedness of F determine the lower and upper bound of $\vdash_F$, respectively. More precisely, if F is unanimous and grounded then for all profiles $\Vdash$, $\bigcap_{i\in N} \vdash_i \subseteq \vdash_F \subseteq \bigcup_{i\in N} \vdash_i$. We call an SLF *bounded* if it is both unanimous and grounded. If we restrict arguments to judgments, we get weak unanimity and weak groundedness, respectively.

Definition 10 (Weak unanimity). *An SLF $F : \mathbf{L}^n \to \mathbf{L}$ is* weakly unanimous, *if for all $\varphi \in \mathcal{L}$, for all profiles $\Vdash$ in $\mathbf{L}^n$, $\vdash_i \varphi$ for all $i \in N$ implies $\vdash_F \varphi$, i.e., if a judgment is accepted by all individuals, then it is collectively accepted.*

Definition 11 (Weak groundedness). *An SLF $F : \mathbf{L}^n \to \mathbf{L}$ is* weakly grounded*, if for all $\varphi \in \mathcal{L}$, for all profiles $\Vdash$ in $\mathbf{L}^n$, $\vdash_F \varphi$ implies $\vdash_i \varphi$ for some $i \in N$, i.e., if a judgment is collectively accepted, then it must be accepted by one of the individuals.*

The following fact should be easily verified. Recall that $\mathbf{L}_{cc}$ is the set of all consistent and complete logics.

Proposition 1. *$F : \mathbf{L}^n_{cc} \to \mathbf{L}_{cc}$ is weakly unanimous iff it is weakly grounded.*

Definition 12 (IIA). *An SLF F is* independent of irrelevant arguments *(IIA), if for all $\Sigma, \varphi \subseteq \mathcal{L}$, for all profiles $\Vdash$ and $\Vdash'$, $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash'}_{\Sigma,\varphi}$ implies that $\Sigma \vdash_F \varphi$ iff $\Sigma \vdash'_F \varphi$, i.e., the collective acceptance of an argument only depends on the individual acceptance of this argument.*

Independence is the most controversial property in social choice, particularly in judgment aggregation. Since we lift aggregation from proposition-wise to argument-wise, independence in logic aggregation is more justified than that in judgment aggregation. Of course, we can still ask the reasons for an argument, just as we can ask the reasons for a proposition. But we can then take the reasons of an argument to be the premises of the argument and form a meta-argument. Thus, an abstract argument-wise aggregation framework is applicable unless we lift the level of arguments constantly.

Definition 13 (Neutrality). *An SLF F is* neutral *(for arguments) if for all* $\Sigma, \Sigma', \varphi, \varphi' \subseteq \mathcal{L}$, for all profiles $\Vdash$, $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash}_{\Sigma',\varphi'}$ implies that $\Sigma \vdash_F \varphi$ iff $\Sigma' \vdash_F \varphi'$, i.e. if two arguments receive the same individual acceptance, their collective acceptances are also the same. In other words, all arguments are treated equal.

We define two weak versions of neutrality as follows, which seem to have no counterparts in the literature.

Definition 14 (C-Neutrality). *An SLF F is* neutral for conclusions *if for all $\Sigma, \varphi, \varphi' \subseteq \mathcal{L}$, for all profiles $\Vdash$, $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash}_{\Sigma,\varphi'}$ implies that $\Sigma \vdash_F \varphi$ iff $\Sigma \vdash_F \varphi'$.*

Definition 15 (P-Neutrality). *An SLF F is* neutral for premises *if for all $\Sigma, \Sigma', \varphi \subseteq \mathcal{L}$, for all profile $\Vdash$, $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash}_{\Sigma',\varphi}$ implies that $\Sigma \vdash_F \varphi$ iff $\Sigma' \vdash_F \varphi$.*

Proposition 2. *An IIA SLF is neutral iff it is both C-neutral and P-neutral.*

Proof. The direction from left to right is obvious. For the other direction, suppose F is both C-neutral and P-neutral. Given a profile $\Vdash$, suppose $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash}_{\Sigma',\varphi'} =_{df} C$ and $\Sigma \vdash_F \varphi$. We need to show that $\Sigma' \vdash_F \varphi'$. Consider a profile $\Vdash'$ such that $N^{\Vdash'}_{\Sigma,\varphi} = N^{\Vdash'}_{\Sigma,\varphi'} = N^{\Vdash'}_{\Sigma',\varphi'} = C$. Since $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash'}_{\Sigma,\varphi}$, by IIA it follows from $\Sigma \vdash_F \varphi$ that $\Sigma \vdash'_F \varphi$, which implies $\Sigma \vdash'_F \varphi'$ by C-neutrality of F. It in turn implies $\Sigma' \vdash'_F \varphi'$ by P-neutrality of F. By IIA again, we have $\Sigma' \vdash_F \varphi'$.

We call an SLF *systematic* if it is both IIA and neutral.

Definition 16 (N-monotonicity). *An SLF F is* n-monotonic *if for all all $\Sigma, \varphi \subseteq \mathcal{L}$, for all profiles $\Vdash$, $N^{\Vdash}_{\Sigma,\varphi} \subseteq N^{\Vdash}_{\Sigma',\varphi'}$ and $\Sigma \vdash_F \varphi$ imply $\Sigma' \vdash_F \varphi'$, i.e., compared to a collectively accepted argument, any argument with the same or additional acceptance will also be collectively accepted.*

It is a bit surprising that this natural property had not been proposed before, until its first presence in [6]. Note that n-monotonicity is different from the standard notion of monotonicity, which involves two profiles rather than one. It is easily seen that n-monotonicity implies neutrality but not vice versa.

Definition 17 (Dictatorship). *An SLF F is* dictatorial *if there exists an $i \in N$ such that for all profiles $\Vdash$, $\vdash_F = \vdash_i$, i.e. the social logic is always the same as i's logic.*

We say that a profile $\Vdash$ satisfies a property P if $\vdash_i$ satisfies P for all $i \in N$. The following notion is adapted from [10].

Definition 18 (Collective rationality, Robustness). *An SLF F is* collectively rational for a property P *if for all profiles $\Vdash$, $\vdash_F$ satisfies P whenever $\Vdash$ satisfies P. In this case, we also say that P* is robust under F.

We intentionally give another name for collective rationality. With the name of collective rationality, preserving certain logical properties are considered to be 'rational' (and thus desired) for an aggregation function. But there is no reason to assume that these properties on the individual level should also be satisfied on the social level. If we take a serious *social* view on logic, the fact that some logical properties are not preserved under aggregation does not mean that the aggregation is not rational. It only indicates that the rationality or logic on the social level is different. By using the name of robustness, we can compare different logical properties under the framework of social choice and provide a new perspective on logic.

The main question we are to address is: what logical properties are robust? More precisely, which logical properties can be preserved under the desired social logic functions?

4 Some Possibility and Impossibility Results

First we give some easy possibility results.

Proposition 3. *(Strong) reflexivity is robust under any unanimous SLF $F : \mathbf{L}^n \to \mathbf{L}$.*

Proof. Let $F : \mathbf{L}^n \to \mathbf{L}$ be unanimous and $\Vdash$ satisfy (strong) reflexivity. Then (given $\varphi \in \Sigma$), for every $i \in N$, $\varphi \vdash_i \varphi$ ($\Sigma \vdash_i \varphi$). By unanimity, $\varphi \vdash_F \varphi$ ($\Sigma \vdash_F \varphi$).

Proposition 4. *Monotonicity is robust under any n-monotonic SLF $F : \mathbf{L}^n \to \mathbf{L}$.*

Proof. Let $F : \mathbf{L}^n \to \mathbf{L}$ be an n-monotonic SLF and $\Vdash$ satisfy monotonicity. Suppose $\Sigma \vdash_F \varphi$ and $\Sigma \subseteq \Sigma'$. Since $\Vdash$ is monotonic, $N^{\Vdash}_{\Sigma,\varphi} \subseteq N^{\Vdash}_{\Sigma',\varphi}$. Then by the n-monotonicity of F, we have $\Sigma' \vdash_F \varphi$.

Proposition 5. *$\mathcal{A}$-formality is robust under any n-monotonic SLF $F : \mathbf{L}^n \to \mathbf{L}$.*

Proof. Let $F : \mathbf{L}^n \to \mathbf{L}$ be an n-monotonic SLF and $\Vdash$ satisfy $\mathcal{A}$-formality. Suppose $\Sigma, \varphi \subseteq \mathcal{A}$ and $\Sigma \vdash_F \varphi$. For any substitution σ, we have $N^{\Vdash}_{\Sigma,\varphi} \subseteq N^{\Vdash}_{\Sigma^\sigma,\varphi^\sigma}$, since $\Vdash$ is $\mathcal{A}$-formal. It follows from the n-monotonicity of F that $\Sigma^\sigma \vdash_F \varphi^\sigma$.

Proposition 6. *M-compactness is robust under any n-monotonic SLF $F : \mathbf{L}^n \to \mathbf{L}$.*

Proof. Let $F : \mathbf{L}^n \to \mathbf{L}$ be an n-monotonic and grounded SLF and $\Vdash$ satisfy m-compactness. Since $\Vdash$ is monotonic, by Proposition 4, $\vdash_F$ is also monotonic. Thus it suffices to prove that $\vdash_F$ is compact. Suppose $\Sigma \vdash_F \varphi$. Since $\Vdash$ is compact, for each $i \in N^{\Vdash}_{\Sigma,\varphi}$, there is a finite $\Delta_i \subseteq \Sigma$ such that $\Delta_i \vdash_i \varphi$. Let $\Delta = \bigcup_{i \in N^{\Vdash}_{\Sigma,\varphi}} \Delta_i$. Then $\Delta \subseteq \Sigma$ is also finite. Since $\Vdash$ is monotonic, $\Delta \vdash_i \varphi$ for all $i \in N^{\Vdash}_{\Sigma,\varphi}$. Hence, $N^{\Vdash}_{\Sigma,\varphi} \subseteq N^{\Vdash}_{\Delta,\varphi}$. It follows from n-monotonicity of F that $\Delta \vdash_F \varphi$, as required.

Note that m-compactness can not be replaced by compactness in the above proposition. Actually, compactness alone is not robust even under the majority rule. Consider $\vdash_i = \{(\{p_i\}, p), (\{p_i \mid i \in \mathbb{N}\}, p)\}$ for $i \in N$. Then every $\vdash_i$ is compact. But by the majority rule, $\vdash = \{(\{p_i \mid i \in \mathbb{N}\}, p)\}$, which is not compact.

Proposition 7. *Cautious monotonicity is robust under any n-monotonic SLF* $F : \mathbf{L}^n_{cj} \to \mathbf{L}_{cj}$.

Proof. Recall that $\mathbf{L}_{cj}$ is the set of all conjunctive logics. Let $F : \mathbf{L}^n_{cj} \to \mathbf{L}_{cj}$ be an n-monotonic SLF and $\Vdash$ satisfy cautious monotonicity. Suppose $\Sigma \vdash_F \varphi$ and $\Sigma \vdash_F \varphi'$. Since $\vdash_F$ is conjunctive, we have $\Sigma \vdash_F \varphi \wedge \varphi'$. By the conjunction property of $\Vdash$, $N^{\Vdash}_{\Sigma,\varphi\wedge\varphi'} \subseteq N^{\Vdash}_{\Sigma,\varphi} \cap N^{\Vdash}_{\Sigma,\varphi'}$. By cautious monotonicity of $\Vdash$, $N^{\Vdash}_{\Sigma,\varphi} \cap N^{\Vdash}_{\Sigma,\varphi'} \subseteq N^{\Vdash}_{\Sigma\cup\{\varphi'\},\varphi}$. Hence, $N^{\Vdash}_{\Sigma,\varphi\wedge\varphi'} \subseteq N^{\Vdash}_{\Sigma\cup\{\varphi'\},\varphi}$. Since F is n-monotonic, it follows from $\Sigma \vdash_F \varphi \wedge \varphi'$ that $\Sigma, \varphi' \vdash_F \varphi$.

Now we give some impossibility results. First, an easy one, which says that n-monotonicity for non-trivial logics forces groundedness.

Proposition 8. *There is no n-monotonic and ungrounded SLF* $F : \mathbf{L}^n \to \mathbf{L}$ *that is collectively rational for non-triviality.*

Proof. Suppose $F : \mathbf{L}^n \to \mathbf{L}$ is n-monotonic and ungrounded that is collectively rational for non-triviality. Then there is a profile $\Vdash$ of non-trivial logics and $\Sigma, \varphi \subseteq \mathcal{L}$ such that no one accepts (Σ, φ) but $\Sigma \vdash_F \varphi$. Thus, for any $\Sigma', \varphi' \subseteq \mathcal{L}$, $N^{\Vdash}_{\Sigma,\varphi} = \emptyset \subseteq N^{\Vdash}_{\Sigma',\varphi'}$, which implies by n-monotonicity that $\Sigma' \vdash_F \varphi'$. Hence, $\vdash_F$ is trivial, contradicting the assumption.

This result is not as pessimistic as the usual impossibility results in social choice theory. It only indicates that n-monotonicity should be applied together with groundedness; otherwise, we may get a trivial logic by aggregation. The following results are more parallel with the usual impossibility results. The proofs are canonical, using the property of ultrafilters, which was first introduced in [7] for an alternative proof of Arrow's theorem, and later adapted and refined for other aggregation frameworks in the literature, including [9], [3], and [13] for judgment aggregation and [5] for graph aggregation.

Definition 19. *A group* $C \subseteq N$ *is a* winning coalition *of* (Σ, φ) *(under* F*), if for all profiles* $\Vdash$, $N^{\Vdash}_{\Sigma,\varphi} = C$ *implies* $\Sigma \vdash_F \varphi$.

The following lemma is easily verified.

Lemma 1. *Let F be an IIA SLF and $\mathcal{W}^F_{\Sigma,\varphi}$ the set of all winning coalitions of (Σ,φ) under F.*

1. *If there is a profile $\Vdash$ such that $N^{\Vdash}_{\Sigma,\varphi} = C$ and $\Sigma \vdash_F \varphi$, then C is a winning coalition of (Σ,φ).*
2. *For all profiles $\Vdash$, $\Sigma \vdash_F \varphi$ iff $N^{\Vdash}_{\Sigma,\varphi} \in \mathcal{W}^F_{\Sigma,\varphi}$.*

We often omit the superscript F in $\mathcal{W}^F_{\Sigma,\varphi}$ if F is clear from the context. The following lemma is adapted from [5].

Lemma 2. *Suppose F is IIA. Let $\mathcal{W}_{\Sigma,\varphi}$ be defined as above. Then*

1. *F is unanimous iff $N \in \mathcal{W}_{\Sigma,\varphi}$ for all $\Sigma,\varphi \subseteq \mathcal{L}$.*
2. *F is grounded iff $\emptyset \notin \mathcal{W}_{\Sigma,\varphi}$ for all $\Sigma,\varphi \subseteq \mathcal{L}$.*
3. *F is neutral iff $\mathcal{W}_{\Sigma,\varphi} = \mathcal{W}_{\Sigma',\varphi'}$ for all $\Sigma,\Sigma',\varphi,\varphi' \subseteq \mathcal{L}$.*

Proof. The first two clauses are obvious. For (3), suppose F is neutral and $N^{\Vdash}_{\Sigma,\varphi} \in \mathcal{W}_{\Sigma,\varphi}$. Let $\Vdash'$ be a profile such that $N^{\Vdash'}_{\Sigma',\varphi'} = N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash}_{\Sigma,\varphi}$. Since $N^{\Vdash}_{\Sigma,\varphi} \in \mathcal{W}_{\Sigma,\varphi}$, we have $\Sigma \vdash_F \varphi$, which implies $\Sigma \vdash'_F \varphi$ by IIA. It in turn implies $\Sigma' \vdash'_F \varphi'$ by neutrality. Thus, $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash'}_{\Sigma',\varphi'} \in \mathcal{W}_{\Sigma',\varphi'}$. Hence, $\mathcal{W}_{\Sigma,\varphi} \subseteq \mathcal{W}_{\Sigma',\varphi'}$. Similarly, we have $\mathcal{W}_{\Sigma',\varphi} \subseteq \mathcal{W}_{\Sigma,\varphi}$. For the other direction of (3), suppose $\mathcal{W}_{\Sigma,\varphi} = \mathcal{W}_{\Sigma',\varphi'}$ for all $\Sigma,\Sigma',\varphi,\varphi' \subseteq \mathcal{L}$ and $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash}_{\Sigma',\varphi'}$. Then $\Sigma \vdash_F \varphi$ iff $N^{\Vdash}_{\Sigma,\varphi} \in \mathcal{W}_{\Sigma,\varphi}$ iff $N^{\Vdash}_{\Sigma',\varphi'} \in \mathcal{W}_{\Sigma',\varphi'}$ iff $\Sigma' \vdash_F \varphi'$.

Now let's recall the definition of ultrafilters.

Definition 20. *(Ultrafilter) An* ultrafilter *$\mathcal{W}$ over N is a set of subsets of N satisfying the following conditions:*

1. *$\mathcal{W}$ is proper, i.e. $\emptyset \notin \mathcal{W}$;*
2. *$\mathcal{W}$ is closed under (finite) intersection, i.e. $C_1, C_2 \in \mathcal{W}$ implies $C_1 \cap C_2 \in \mathcal{W}$;*
3. *$\mathcal{W}$ is maximal, i.e. for all $C \subseteq N$, either $C \in \mathcal{W}$ or $\overline{C} \in \mathcal{W}$, where $\overline{C} = N - C$ is the complement of C.*

An ultrafilter $\mathcal{W}$ over N is principal *if $\mathcal{W} = \{C \subseteq N \mid i \in C\}$ for some $i \in N$.*

The following is a well-known fact of ultrafilters.

Lemma 3. *Any ultrafilter over a finite set is principal.*

Theorem 1. *For all nonempty $\mathcal{A} \subseteq \mathcal{L}$, any bounded and systematic SLF $F : \mathbf{L}^n \to \mathbf{L}$ that is collectively rational for transitivity and $\mathcal{A}$-completeness must be dictatorial.*

Proof. Let $F : \mathbf{L}^n \to \mathbf{L}$ be bounded (unanimous and grounded), systematic (IIA and neutral), and collectively rational for transitivity and $\mathcal{A}$-completeness. By Lemma 2, there is a set $\mathcal{W}$ of winning coalitions such that $\Sigma \vdash_F \varphi$ iff $N^{\Vdash}_{\Sigma,\varphi} \in \mathcal{W}$ for all $\Sigma,\varphi \subseteq \mathcal{L}$. By Lemma 3, it suffices to prove that $\mathcal{W}$ is an ultrafilter. First, $\mathcal{W}$ is proper by Lemma 2(2), since F is grounded. Second, suppose $C_1, C_2 \in \mathcal{W}$. Consider a transitive profile $\Vdash$ such that $C_1 = N^{\Vdash}_{\varphi}$, $C_2 = N^{\Vdash}_{\varphi,\psi}$, and $C_1 \cap C_2 = N^{\Vdash}_{\psi}$.

This is possible since by the transitivity of $\Vdash$, $C_1 \cap C_2 \subseteq N^{\Vdash}_{\psi}$. As C_1, C_2 are winning coalitions, we have $\vdash_F \varphi$ and $\varphi \vdash_F \psi$. It follows that $\vdash_F \psi$ by the collective rationality of F for transitivity. Hence, $C_1 \cap C_2 \in \mathcal{W}$. Finally, Let $C \subseteq N$. Consider an $\mathcal{A}$-complete profile $\Vdash$ such that $C = N^{\Vdash}_{\varphi}$ and $\overline{C} = N^{\Vdash}_{\neg\varphi}$, where $\varphi \in \mathcal{A}$. By the collective rationality of F for $\mathcal{A}$-completeness, $\vdash_F \varphi$ or $\vdash_F \neg\varphi$. Hence, $C \in \mathcal{W}$ or $\overline{C} \in \mathcal{W}$.

Theorem 2. *For all $\mathcal{A} \subseteq \mathcal{L}$ including at least one formula of the form $\varphi \vee \psi$ with $\varphi \neq \psi$, any bounded and systematic SLF $F : \mathbf{L}^n \to \mathbf{L}$ that is collectively rational for transitivity and $\mathcal{A}$-disjunctiveness must be dictatorial.*

Proof. The proof is almost the same as above, except the verification of the maximality of $\mathcal{W}$. Let $C \subseteq N$. Consider an $\mathcal{A}$-disjunctive profile $\Vdash$ such that $C = N^{\Vdash}_{\varphi}$, $\overline{C} = N^{\Vdash}_{\psi}$, and $N = N^{\Vdash}_{\varphi\vee\psi}$, where $\varphi \vee \psi \in \mathcal{A}$. By unanimity, $\vdash_F \varphi \vee \psi$. Since F is collectively rational for $\mathcal{A}$-disjunctiveness, we have $\vdash_F \varphi$ or $\vdash_F \psi$. Hence, $C \in \mathcal{W}$ or $\overline{C} \in \mathcal{W}$.

Here is a reformulation of the above two theorems: the conjunction of transitivity and $\mathcal{A}$-completeness ($\mathcal{A}$-disjunctiveness) is not robust under any bounded, systematic, and non-dictatorial social logic function.

For non-tautological logic, the above theorems can be strengthened by dropping neutrality in the assumption, due to the following lemma. Recall that $\mathbf{L}_{nt}$ is the set of all non-tautological logics.

Lemma 4. *Every unanimous and IIA SLF $F : \mathbf{L}^n_{nt} \to \mathbf{L}_{nt}$ that is collectively rational for both transitivity must be neutral.*

Proof. Let F be unanimous, IIA, and collectively rational for transitivity. Let C be a winning coalition of (Σ, φ). By Lemma 2(3), it suffices to prove that C is also winning coalition of (Σ', φ'). First, we prove that C is a winning coalition of (Σ, φ'). Let $\Vdash$ be a transitive profile such that $N^{\Vdash}_{\Sigma,\varphi} = N^{\Vdash}_{\Sigma,\varphi'} = C$ and $N^{\Vdash}_{\varphi,\varphi'} = N$. This is possible, since $\varphi' \notin \Sigma$ by non-tautologicity. Since C is a winning coalition of (Σ, φ), we have $\Sigma \vdash_F \varphi$. On the other hand, we have $\varphi \vdash'_F \varphi'$ by unanimity. It follows that $\Sigma \vdash_F \varphi'$ by transitivity. Hence, C is a winning coalition of (Σ, φ'). To prove that C is a winning coalition of (Σ', φ'), let $\Vdash$ be a transitive profile such that $N^{\Vdash}_{\Sigma,\varphi'} = N^{\Vdash}_{\Sigma',\varphi'} = C$ and $N^{\Vdash}_{\Sigma',\Sigma} = N$. This is possible, since $\varphi' \notin \Sigma$ by non-tautologicity. Since C is a winning coalition of (Σ, φ'), we have $\Sigma \vdash_F \varphi'$. On the other hand, we have $\Sigma' \vdash \Sigma$ by unanimity. It follows that $\Sigma' \vdash_F \varphi'$ by transitivity. Hence, C is a winning coalition of (Σ', φ').

Using the above lemma, by slightly modifying the proofs of Theorem 1 and Theorem 2, we obtain the following results.

Theorem 3. *For all nonempty $\mathcal{A} \subseteq \mathcal{L}$, any bounded and IIA SLF $F : \mathbf{L}^n_{nt} \to \mathbf{L}_{nt}$ that is collectively rational for transitivity and $\mathcal{A}$-completeness must be dictatorial.*

Theorem 4. *For all $\mathcal{A} \subseteq \mathcal{L}$ including at least one formula of the form $\varphi \vee \psi$ with $\varphi \neq \psi$, any bounded and IIA SLF $F : \mathbf{L}^n_{nt} \to \mathbf{L}_{nt}$ that is collectively rational for transitivity and $\mathcal{A}$-disjunctiveness must be dictatorial.*

In all the above theorems, transitivity can be replaced by confluency or the conjunction property, proofs of which are almost the same. Actually, all properties of the form $A \wedge B \rightarrow C$ can replace the role of transitivity in the above theorems. A parallel result in aggregation of general binary relations has already been obtained in [5].

5 Relating to Other Aggregation Frameworks

Preference Aggregation. If we treat a logic as a binary relation on formulas, instead of that between sets formulas and formulas, then a logic can be roughly regarded as a preference. The reflexivity and transitivity of preferences can be naturally assumed for logics. But the completeness of preferences are not suitable for logics. In other words, when logic is treated as a binary relation on formulas, a framework of general binary relation (as in [5]) or partial relation (as in [19]) is more suitable for logic aggregation than standard preference aggregation.

Conversely, preference aggregation can be embedded into logic aggregation, since preference aggregation can be embedded into judgment aggregation, and the latter can in turn be embedded into logic aggregation (see the next subsection).

Judgment Aggregation. A logic $\vdash$ can be regarded as a set of judgments $J = \{(\Sigma, \varphi) \mid \Sigma \vdash \varphi\} \cup \{\neg(\Sigma, \varphi) \mid \Sigma \nvdash \varphi\}$. The difference is that J is infinite if the language $\mathcal{L}$ is infinite, whereas in judgment aggregation a judgement set is usually finite. Regardless of this difference, logic aggregation can be turned to judgment aggregation of special propositions, where a proposition expresses whether an argument holds.

On the other hand, each set J of judgments together with the underlying logic $\vdash$ can be regarded as a new logic $\vdash' = \vdash \cup \{(\emptyset, p) \mid p \in J\}$. But notice that usually the judgments in J are not formal, in the sense that the substitution rule is not applicable to them in the new logic. In this way, judgment aggregation under a unified logic can be regarded as logic aggregation, where the individual logics are obtained from the unified logic augmented with the individual judgments as non-logical axioms. In this sense, judgment aggregation can be translated into logic aggregation. The relation between these two frameworks is just like that between object language and metalanguage. We can always turn a metalanguage into an object one and vice versa. The logics underlying the judgments expressed by metalanguage in judgment aggregation are turned into object language in logic aggregation, which helps to understand the logical properties better in judgment aggregation.

Graph Aggregation. Graph aggregation proposed in [5] is the aggregation of arbitrary binary relations on a given set V. If we take V to be the set of formulas, and assume compactness of logic, then a logic can be treated as a binary relation on V. An edge from vertex φ to ψ in a graph G represents an accepted argument from premise φ to conclusion ψ in the logic G. But notice that in graph aggregation V is usually assumed to be finite, whereas in logic aggregation the set of formulas are usually infinite. Besides this difference, vertices in a graph are independent of each other while a formula in a logic can be composed from other formulas. Hence, even for compact logics, graph aggregation is too abstract to express logic aggregation.

On the other hand, if we take a graph as a frame for modal logic, then a set of graphs define a logic. Thus the aggregation of sets of graphs can be transformed to the aggregation of modal logics. Moreover, graph aggregation is a special case of the aggregation of sets of graphs (aggregating singleton sets). In this way, graph aggregation can be transformed to logic aggregation.

6 Conclusion and Future Works

We propose a formal framework for logic aggregation, in which some possibility and impossibility results are proved. We also compare logic aggregation with other aggregation frameworks. Our contribution is mainly conceptual rather than technical. This is only a first step in applying the social choice framework to logic. There are a lot left to be explored.

Firstly, more general possibility and impossibility results can be explored under the framework we proposed. Secondly, the framework of logic aggregation itself can also be generalized. An immediate generalization is to take a substructural view on logic, where a logic is no longer a binary relation between sets of formulas and formulas, but between structures of formulas and formulas. Then more nonclassical logics, such as linear logic and Lambek calculus can be incorporated in logic aggregation. Thirdly, logic can be treated in a more functional or dynamic way, where a logic is a procedure (an algorithm, or a method) rather than reducing it to the input-output data (accepted arguments). This means that even if two algorithms produce the same set of valid arguments, they are different logics. Finally, the method of logic aggregation need not be argument-wise. Just as there are global or holistic methods in judgment aggregation, we can also explore global or holistic methods in logic aggregation, for example, the distance-based approach. If this approach is taken, we have to clarify what it means for two logics to be close, and how to define the distance between logics.

Acknowledgements. The paper was supported by the National Fund of Social Science (No. 13BZX066), Humanity and Social Science Youth foundation of Ministry of Education of China (No.11YJC72040001), and the Fundamental Research Funds for the Central Universities (No. 1309073). We would like to than the anonymous referees for their helpful comments.

References

1. Benamara, F., Kaci, S., Pigozzi, G.: Individual Opinions-Based Judgment Aggregation Procedures. In: Torra, V., Narukawa, Y., Daumas, M. (eds.) MDAI 2010. LNCS (LNAI), vol. 6408, pp. 55–66. Springer, Heidelberg (2010)

2. Dietrich, F.: A generalised model of judgment aggregation. Social Choice and Welfare 28(4), 529–565 (2007)
3. Dietrich, F., List, C.: Arrow's theorem in judgment aggregation. Social Choice and Welfare 29(1), 19–33 (2007)
4. Dietrich, F., Mongin, P.: The premiss-based approach to judgment aggregation. Journal of Economic Theory 145(2), 562–582 (2010)
5. Endriss, U., Grandi, U.: Graph Aggregation. In: Proceedings of the 4th International Workshop on Computational Social Choice (COMSOC 2012) (2012)
6. Endriss, U., Grandi, U., Porello, D.: Complexity of Judgment Aggregation: Safety of the Agenda. In: Proceedings of AAMAS 2010, pp. 359–366 (2010)
7. Fishburn, P.C.: Arrow's impossibility theorem: Concise proof and infinite voters. Journal of Economic Theory 2(1), 103–106 (1970)
8. Gabbay, D.M. (ed.): What is a Logical System? Oxford University Press (1995)
9. Gärdenfors, P.: A Representation Theorem for Voting with Logical Consequences. Economics and Philosophy 22(2), 181 (2006)
10. Grandi, U., Endriss, U.: Lifting Rationality Assumptions in Binary Aggregation. In: Proceedings of AAAI 2010, pp. 780–785 (2010)
11. Grossi, D., Pigozzi, G.: Introduction to Judgment Aggregation. In: Bezhanishvili, N., Goranko, V. (eds.) ESSLLI 2010/2011. LNCS, vol. 7388, pp. 160–209. Springer, Heidelberg (2012)
12. Jansana, R.: Propositional Consequence Relations and Algebraic Logic. In: Zalta, E.N. (ed.) The Stanford Encyclopedia of Philosophy. Spring 2011 edn. (2011), `http://plato.stanford.edu/archives/spr2011/entries/consequence-algebraic/`
13. Klamler, C., Eckert, D.: A simple ultrafilter proof for an impossibility theorem in judgment aggregation. Economics Bulletin 29(1), 319–327 (2009)
14. List, C.: The theory of judgment aggregation: an introductory review. Synthese 187(1), 179–207 (2012)
15. List, C., Pettit, P.: Aggregating Sets of Judgments: An Impossibility Result. Economics and Philosophy 18(1), 89–110 (2002)
16. Miller, M.K.: Judgment Aggregation and Subjective Decision-Making. Economics and Philosophy 24(2) (2008)
17. Mongin, P.: Factoring out the impossibility of logical aggregation. Journal of Economic Theory 141(1), 100–113 (2008), `http://linkinghub.elsevier.com/retrieve/pii/S0022053107001457`
18. Pigozzi, G.: Belief merging and the discursive dilemma: an argument-based account to paradoxes of judgment aggregation. Synthese 152(2), 285–298 (2006)
19. Pini, M.S., Rossi, F., Venable, K.B., Walsh, T.: Aggregating Partially Ordered Preferences. Journal of Logic and Computation 19(3), 475–502 (2009)
20. Porello, D., Endriss, U.: Ontology Merging as Social Choice. In: Leite, J., Torroni, P., Ågotnes, T., Boella, G., van der Torre, L. (eds.) CLIMA XII 2011. LNCS, vol. 6814, pp. 157–170. Springer, Heidelberg (2011)