# **Multi Criteria Decision Making Related to Services**

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**Abstract.** A lot of data mining techniques are develop to handle large data sets. When applied on small data sets however they perform poorly. More often than not conclusions have to be drawn from relatively small data sets due to various reasons. Rough sets approximations can be applied in such situations since they do not need a critical amount of data in order to provide reliable results.

**Keywords:** Rough sets, evaluations, intelligent systems.

#### **1 Introduction**

Missing and contradictory data has been omitted nearly without hesitation from scientific investigations a few decades ago being regarded as a distraction. Obviously this implies partially correct conclusions since a lot of interesting dependencies can not be reviled. Use of Boolean logic in particular limits system's responses to true or false and cannot therefore recognize other occurrences like f. ex partially correct or incomplete information about services. Boolean logic appears to be quite sufficient for most everyday reasonings, but it is certainly unable to provide meaningful conclusions in presence of inconsistent and/or incomplete input, [4]. This problem can be resolved by applying methods from the theory of rough sets approximations.

In this work we are focussing on services' evaluations being subjects of cooperative decision making.

#### **2 Background**

A lattice is a partially ordered set, closed under least upper and greatest lower bounds. The least upper bound of  $x$  and  $y$  is called the join of  $x$  and  $y$ , and is sometimes written as  $x + y$ ; the greatest lower bound is called the meet and is sometimes written as *xy* , [6].

A *context* is a triple  $(G, M, I)$  where G and M are sets and  $I \subset G \times M$ . The elements of *G* and *M* are called *objects* and *attributes* respectively [1], [6], and [15].

For 
$$
A \subseteq G
$$
 and  $B \subseteq M$ , define  
\n $A' = \{m \in M \mid (\forall g \in A) \ gIm\}, B' = \{g \in G \mid (\forall m \in B) \ gIm\}$ 

where  $\overrightarrow{A}$  is the set of attributes common to all the objects in  $\overrightarrow{A}$  and  $\overrightarrow{B}$  is the set of objects possessing the attributes in *B* .

A *concept* of the context  $(G, M, I)$  is defined to be a pair  $(A, B)$  where  $A \subset G$ ,  $B \subset M$ ,  $A' = B$  and  $B' = A$ . The *extent* of the concept  $(A, B)$  is *A* while its *intent* is *B* . A subset *A* of *G* is the extent of some concept if and only if  $A^{\prime\prime} = A$  in which case the unique concept of the which *A* is an extent is  $(A, A)$ . The corresponding statement applies to those subsets  $B \in M$  which is the intent of some concepts.

The set of all concepts of the context  $(G, M, I)$  is denoted by  $B(G, M, I)$ .  $\langle B(G,M,I);\leq \rangle$  is a complete lattice and it is known as the *concept lattice* of the context  $(G, M, I)$ .

From classical stand point of view a concept is well defined by a pair of intention and extension. Existence of well defined boundaries is assumed and an extension is uniquely identified by a crisp set of objects. In real life situations one has to operate with concepts having grey/gradual boundaries, like f. ex. partially known concepts, [16], undefinable concepts, and approximate concepts, [9].

Rough Sets were originally introduced in [14]. The presented approach provides exact mathematical formulation of the concept of approximative (rough) equality of sets in a given approximation space. An *approximation space* is a pair  $A = (U, R)$ ,

where *U* is a set called universe, and  $R \subset U \times U$  is an indiscernibility relation.

Equivalence classes of *R* are called *elementary sets* (atoms) in *A* . The equivalence class of *R* determined by an element  $x \in U$  is denoted by  $R(x)$ . Equivalence classes of  $R$  are called *granules* generated by  $R$ .

Attributes reduction stands for removal of attributes that do not effect the primary system. Rough sets attribute analysis is usually applied in the process of establishing the relative importance of an attribute and consecutively remove it if it contains redundant information.

Data analysis with various applications is well presented in [1], [2], [7], [8], [13]. Multi-criteria methods for project evaluation are applied in [5], [11].

## **3 Summarized Assessments**

Services are evaluated by experts where their summarized assessments are denoted by  $v$  - very high level of success,  $s$  - high level of success,  $m$  - moderate level of success,  $l$  - low level of success, and  $u$  - unknown level of success. A concept lattice relating services and criteria evaluations is presented in Fig. 1. A graphical representation of rough sets approximations can also be seen in Fig. 2.



**Fig. 1.** A concept lattice



**Fig. 2.** Approximations

Below we present experts evaluations of services with respect to some criteria and outcomes.

The abbreviation  $P$  stands for a service and  $Cr$  for a criteron, where

Cr 1 - a nonempty cell indicates that two experts' evaluations found place within the lower approximation

Cr 2 - a nonempty cell indicates that two experts' evaluations found place within that part of the set that does not involve the lower approximation

Cr 3 - a nonempty cell indicates that two experts' evaluations found place within that part of the upper approximation that does not involve the lower approximation

Cr 4 - a nonempty cell indicates that two experts' evaluations found place outside of the upper approximation

- ∗ the first and the sec cond evaluations are compared
- - the second and the third evaluations are compared
- $x -$  the third and the forth evaluations are compared

	Cr <sub>1</sub>	$\rm Cr$ $2$	Cr <sub>3</sub>	Cr <sub>4</sub>
$P_1$	$\ast$	$\bullet$		X
P <sub>2</sub>	$\ast$	$\bullet$		X
$P_3$	$\ast$			X
$P_4$	$\ast$			X
P <sub>5</sub>	$\ast$			X
$P_6$	$\ast$			$\mathbf x$
P <sub>7</sub>		X		$\ast$
$P_8$		X	٠	$\ast$
P <sub>9</sub>		X	$\bullet$	$\ast$
$P_10$		$\mathbf X$		$\ast$
P <sub>11</sub>		$\mathbf X$	$\bullet$	$\ast$
P <sub>12</sub>	X		٠	$\ast$
P <sub>13</sub>	X		$\bullet$	$\ast$
P <sub>14</sub>	X		$\bullet$	$\ast$
P <sub>15</sub>	X		٠	$\ast$

**Table 1.** An illustrative decision table

The indiscernable sets are  $P_1 = \{ P1, P2, P3, P4, P5, P6, P7 \}, P_2 = \{ P8, P$ 9, P 10, P 11 },  $P_3 =$  { P 12, P 13, P 14, P 15 }. The set { P 1, P 2, P 4, P 5, P 12, P 13, P 14, P 15 } is a rough set because it can not be presented as an union of  $P_1$  and  $P_3$ . The upper and lower approximations of are = { \* *R* P 1, P 2, P 3, P 4, P 5, P 6, P 7, P 8, P 9, P 10, P 11, P 12, P 13, P 14, P 15 and  $R_* = \{ P 12, P 13, P 14, P 15 \}$ .

### **4 Conclusion**

Services' assessment is on many occasions forced to extract information from imperfect, imprecise, and incomplete data. Therefore, precise reasoning rules are difficult and some times impossible to use. Applying rough sets approximations facilitates a balance between accuracy and precision.

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