Detecting Anomalous and Exceptional Behaviour on Credit Data by Means of Association Rules

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Abstract. Association rules is a data mining technique for extracting useful knowledge from databases. Recently some approaches has been developed for mining novel kinds of useful information, such us peculiarities, infrequent rules, exception or anomalous rules. The common feature of these proposals is the low support of such type of rules. Therefore, finding efficient algorithms for extracting them are needed.

The aim of this paper is three fold. First, it reviews a previous formulation for exception and anomalous rules, focusing on its semantics and definition. Second, we propose efficient algorithms for mining such type of rules. Third, we apply them to the case of detecting anomalous and exceptional behaviours on credit data.

Keywords: Data mining, association rules, exception rules, anomalous rules, fraud, credit.

1 Introduction

Association rules are one of the frequent used tools in data mining. They allow to identify novel, useful and comprehensive knowledge. The kind of knowledge they try to extract is the appearance of a set of items together in most of the transactions in a database. An example of association rule is "most of transactions that contain hamburger also contain beer", and it is usually noted $hamburger \rightarrow beer$. The intensity of the above association rule is frequently measured by the *support* and the *confidence* measures [1]. The support is the percentage of transactions satisfying both parts of the rule and the confidence measures the proportion of transactions that satisfying the antecedent, also satisfies the consequent. That is, the confidence gives an estimation of the conditional probability of the consequent given the antecedent [1]. There also exist many proposals imposing new quality measures for extracting semantically or even statistically different association rules [11]. In this line, the certainty factor [4] has some advantages over

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the confidence as it extracts more accurate rules and therefore, the number of mined rules is substantially reduced.

There are few approaches dealing with the extraction of unusual or exceptional knowledge that might be useful in some contexts. We focus in those proposals that allow to obtain some uncommon information, specially on exception and anomalous rules [20,3]. In general, these approaches are able to manage rules that, being infrequent, provide a specific domain information usually delimited by an association rule.

Previous approaches using data mining techniques for fraud detection try to discover the usual profiles of legitimate customer behaviour and then search the anomalies using different methodologies such us clustering [10]. The main scope of this paper is to apply such kind of "infrequent" rules to the case of detecting exceptional or anomalous behaviour automatically that could help for fraud detection, obtaining the common customer behaviour as well as some indicators (exceptions) that happen when the behaviour deviates from an usual one and the anomalous deviations (anomalies). For this purpose, we will perform several experiments in financial data concerning credits.

The structure of the paper is the following: next section offers a brief description of background concepts and related works on this topic. In section 3, we review previous proposals for mining exception and anomalous rules. Section 4 describes our proposal for mining exception and anomalous rules using the certainty factor. Section 5 presents the algorithm for extracting these kinds of rules and its application to the real dataset German-statlog about credits in a certain bank in section 6. Finally, section 7 contains the conclusions and some lines for future research.

2 Background Concepts and Related Work

2.1 Association Rules

Given a set I ("set of items") and a database D constituted by a set of transactions, each one being a subset of I, association rules [1] are "implications" of the form $A \to B$ that relate the presence of itemsets A and B in transactions of D, assuming $A, B \subseteq I$, $A \cap B = \emptyset$ and $A, B \neq \emptyset$.

The support of an itemset is defined as the probability that a transaction contains the itemset, i.e. $\mathrm{supp}(A) = |\{t \in D \,|\, A \subseteq t\}| \,/\, |D|$.

The ordinary measures to assess association rules are the *support* (the joint probability $P(A \cup B)$)

$$\mathrm{Supp}(A \to B) = \mathrm{supp}(A \cup B) \tag{1}$$

and the *confidence* (the conditional probability $P(B|A)$)

$$\mathrm{Conf}(A \to B) = \frac{\mathrm{supp}(A \cup B)}{\mathrm{supp}(A)}. \tag{2}$$

Given the minimum thresholds *minsupp* and $minconf$, that should be imposed by the user, we will say that $A \to B$ is *frequent* if $\mathrm{Supp}(A \to B) \geq minsupp$, and *confident* if $\mathrm{Conf}(A \to B) \geq minconf$.

Definition 1. *[4] An association rule $A \rightarrow B$ is* strong *if it exceeds the minimum thresholds minsupp and $minconf$ imposed by the user, i.e. if $A \rightarrow B$ is frequent and confident.*

An alternative framework was proposed in [4] where the accuracy is measured by means of Shortliffe and Buchanan's certainty factors [17], as follows:

Definition 2. *[5] Let* $\mathrm{supp}(B)$ *be the support of the itemset B, and let* $\mathrm{Conf}(A \rightarrow B)$ *be the confidence of the rule. The* certainty factor *of the rule, denoted as $CF(A \rightarrow B)$, is defined as*

$$\begin{cases} \dfrac{\mathrm{Conf}(A \rightarrow B) - \mathrm{supp}(B)}{1 - \mathrm{supp}(B)} & \textit{if } \mathrm{Conf}(A \rightarrow B) > \mathrm{supp}(B) \\ \dfrac{\mathrm{Conf}(A \rightarrow B) - \mathrm{supp}(B)}{\mathrm{supp}(B)} & \textit{if } \mathrm{Conf}(A \rightarrow B) < \mathrm{supp}(B) \\ 0 & \textit{otherwise.} \end{cases} \tag{3}$$

The certainty factor yields a value in the interval [-1, 1] and measures how our belief that B is in a transaction changes when we are told that A is in that transaction. Positive values indicate that our belief increases, negative values mean that our belief decreases, and 0 means no change. Certainty factor has better properties than confidence and other quality measures (see [6] for more details), and helps to solve some of the confidence drawbacks [4,5]. In particular, it helps to reduce the number of rules obtained by filtering those rules corresponding to statistical independence or negative dependence.

Analogously, we will say that $A \rightarrow B$ is *certain* if $\mathrm{Supp}(A \rightarrow B) \geq minCF$, where $minCF$ is the minimum threshold for the certainty factor given by the user. The definition for strong rules can be reformulated when using CF as a rule which must be frequent and certain.

Definition 3. *[4] An association rule $A \rightarrow B$ is* very strong *if both rules $A \rightarrow B$ and $\neg B \rightarrow \neg A$ are strong.*

In addition, the certainty factor has the following property $CF(A \rightarrow B) = CF(\neg B \rightarrow \neg A)$, which tell us that when using the certainty factor, a strong rule is also very strong [4].

2.2 Related Works

The common denominator when mining association rules is their high support. Usually the mining process, as for instance Apriori [1], uses a candidate generation function which exploits the downward closure property of support (also called anti-monotonicity) which guarantees that for a frequent itemset all its subsets are also frequent. The problem here is that exception and anomalous rules are infrequent rules, and therefore such property cannot be used. In the literature we can find different approaches utilizing infrequent rules for capturing a novel type of knowledge hidden in data.

Peculiarity rules are discovered from the data by searching the relevant data among the peculiar data [26]. Roughly speaking, peculiar data is given by the attributes which contain any peculiar value. A peculiar value will be recognized when it is very different from the rest of values of the attribute in the data set. Peculiarity rules are defined as a new type of association rule representing a kind of regularity hidden in a relatively small number of peculiar data.

Infrequent rules are rules that do not exceed the minimum support threshold. They have been studied mainly for intrusion detection joint with exceptions [25,27]. There exists some approaches for mining them: in [19] the authors modify the known *Lambda* measure for obtaining more interesting rules using some pruning techniques. In [27] infrequent items are obtained first and then some measures are used for mining the infrequent rules. In particular, they used correlation and interest measures together with an incremental ratio of conditional probabilities associated to pairs of items. In [8] the infrequent rules are extracted using a new structure called co-occurrence transactional matrix instead of new interest measures.

Exception rules were first defined as rules that contradict the user's common belief [20]. In other words, for searching an exception rule we have to find an attribute that changes the consequent of a strong rule [23,12,22].

We can find two different ways of mining exception rules: direct or indirect techniques. The formers are in most of the cases highly subjective as the set of user's beliefs is compared to the set of mined rules [18,15,13]. The indirect techniques use the knowledge provided by a set of rules (usually strong rules) and then the exception rules are those that contradict or deviate this knowledge [22,25]. Good surveys on this topic can be found in [9,24,7].

Anomalous rules are in appearance similar to exception rules, but semantically different. An anomalous association rule is an association rule that appears when the strong rule "fails". In other words, it is an association rule that complement the usual behaviour represented by the strong rule [3]. Therefore, the anomalous rules will represent the unusual behaviour, having in general low support.

3 Previous Approaches for Discovering Exception and Anomalous Rules

Exception rules were first defined as rules that contradict the user's common belief [20]. For mining this type of rules we will follow the notation by means of a set of rules which has been considered in [2]. An exception rule is defined joint with the strong rule that represents the common belief. Formally we have two rules noted by (*csr*,*exc*) where *csr* stands for *common sense rule* which is equivalent to the definition of strong rule; and *exc* represents the exception rule:

$$X \text{ strongly implies the fulfilment of } Y, \text{ (and not } E) \qquad (csr)$$
$$\text{but, } X \text{ in conjunction of } E \text{ implies } \neg Y. \qquad (exc)$$

For instance, if X represents antibiotics, Y recovery and E staphylococcus, it could be found the following exception rule [3]:

"with the help of *antibiotics*, the patient tends to *recover*,
unless *staphylococcus* appears",

in this case the combination of staphylococcus with antibiotics leads to death. This example shows how the presence of E changes the usual behaviour of rule $X \to Y$, where the value of Y is the patient recovery meanwhile $\neg Y$ is the patient death.

The problem description for exception rules extraction were first presented as obtaining a set of pairs of rules (common sense rule + exception rule) by Suzuki et al. in [21] composed by $(X \to y, X \wedge E \to y')$ where y and y' are two different values of the same item, and X, E are two itemsets. But for mining them they define a third rule for achieving more reliable results. This rule is called *reference rule*, ref for short, and setted as $E \to y'$ that must have low confidence.

Hussain et al. present a different approach also based on a triple (csr, ref, exc) as we show in Table 1 but instead of using the confidence for the exception rule $X \wedge E \to \neg Y$ they define a measure based on the difference of relative information of exc respect to csr and ref. Although the reference rule is defined in [12] as $E \to \neg Y$ with low support and/or low confidence, they check whether $E \to Y$ is a strong rule [12], which is an equivalent condition.

Table 1. Schema for mining exception rules given by Hussain et al.

$X \to Y$	Common Sense rule	(high *supp* and high $conf$)
$X \wedge E \to \neg Y$	Exception rule	(low *supp* and high $conf$)
$E \to \neg Y$	Reference rule	(low *supp* and/or low $conf$)

There are other proposals [24,13] that differ from those presented by Suzuki and Hussain et al. but we focus on these because their formulation are nearer to our proposal. In addition these two approaches not only find the unusual or contradictory behaviour of a strong rule, but also the 'agent' that causes it, represented by E.

Following the schema in Table 1, several types of knowledge can be discovered by adjusting the three involved rules in the triple (csr,exc,ref). This is the case of Berzal et al. approach in [3] and [2], where they capture anomalous knowledge.

An *anomalous* rule is an association rule that is verified when the common rule fails. In other words, it comes to the surface when the dominant effect produced by the strong rule is removed [3]. Table 2 shows its formal definition, where the more confident the rules $X \wedge \neg Y \to A$ and $X \wedge Y \to \neg A$ are, the stronger the anomaly is. In this approach, there is no imposition over the support of the anomalous and the reference rules.

An example of anomalous rule will be: *"if a patient have symptoms X then he usually has the disease Y; if not, he has the disease A"*. Anomalous rules have different semantics than exception rules, trying to capture the deviation from the common sense rule (i.e. from the usual behaviour). In other words: when X,

Table 2. Schema for mining anomalous rules given by Berzal et al.

$X \to Y$	Common Sense rule	(high *supp* and high $conf$)
$X \wedge \neg Y \to A$	Anomalous rule	(high $conf$)
$X \wedge Y \to \neg A$	Reference rule	(high $conf$)

then we have either Y (usually) or A (unusually). In this case A is not an agent like E, but it is the alternative behaviour when the usual fails.

In both cases, exception and anomalous rules, the reference rule acts as a pruning criterion to reduce the high number of obtained exceptions or anomalies. On the contrary, our approach will reduce the number of exceptions and anomalies by means of a stronger measure than the confidence.

4 Our Proposal for Mining Exception and Anomalous Rules

This section presents alternative approaches for mining exception and anomalous rules.

4.1 Our Approach for Exception Rules

For the case of exceptions, we offer an alternative approach that does not need the imposition of the reference rule, and we use the certainty factor instead of the confidence for validating the pair of rules (*csr*,*exc*).

The first reason which motivates to reject the use of the reference rule is that it does not offer a semantic enrichment when defining exception rules. Second reason is that the reference rule should be defined in the *csr* antecedent's domain, because the definition of the exception rule does not make sense out of the dominance of X (the *csr* antecedent). Then, we reformulate the triple as follows.

Definition 4. *[7] Let X, Y and E be three non-empty itemsets in a database D. Let $D_X = \{t \in D : X \subset t\}$, that is, D_X is the set of transactions in D satisfying X. We define an* exception rule *as the pair of rules (csr, exc) satisfying the following two conditions:*

- $X \to Y$ *is frequent and certain in* D (*csr*)
- $E \to \neg Y$ *is certain in* D_X (*exc*)

where $\varphi \to \psi$ is a certain rule if it exceeds imposed threshold for the certainty factor.

With Definition 4 we achieve two important issues when mining exception rules: (1) to reduce the quantity of extracted pairs (*csr*, *exc*); (2) to obtain reliable exception rules.

We want to remark that we restrict to D_X when defining *exc* because we want that the exception rule is true in the dominance of the common sense rule antecedent. If we look again to the previous example, we can see that searching for exception rules is focused on finding the 'agent' E which, interacting with X, changes the usual behaviour of the common sense rule, that is, it changes the *csr* consequent. In addition, our definition can be formulated as the pair $(X \to Y,\ X \wedge E \to X \wedge \neg Y)$, but this choice for the *exc* is not allowed in usual definitions of association rules because antecedent and consequent are not disjoint. Nevertheless, by restricting to D_X our proposal coincides with the previous approach (without restricting to D_X) when using the confidence measure, i.e., $\mathrm{Conf}(X \wedge E \to \neg Y) = \mathrm{Conf}_X(E \to \neg Y)$.

4.2 Our Approach for Anomalous Rules

Our approach for extracting anomalous rules is based on the same two ideas we used for exception rules:

1. To define anomalous rules using the domain D_X.
2. To use the certainty factor instead of the confidence. The certainty factor reduces the number of common sense rules since it discards non-reliable rules and, as a consequence, the number of anomalous rules is also reduced.

In [7] there is an analysis of the reference rule taken in the approach of Berzal et al. This analysis concludes affirming that the increasing of $\mathrm{Conf}(X \wedge Y \to \neg A)$ is higher as $\mathrm{Supp}(X \to Y)$ increases. This leads to affirm that the reference rule condition depends on the following supports $\mathrm{Supp}(X \to Y) = \mathrm{supp}(X \cup Y)$ and $\mathrm{supp}(X \cup Y \cup A)$. This gives reason to propose an alternative formulation for anomalous rules changing the reference rule for a stronger condition (as we prove in Theorem 1) than the one given in [3,2].

Definition 5. *Let X, Y be two non-empty itemsets and A an item. We define an* anomalous rule *by the triple (csr, anom, ref) satisfying the following conditions:*

- *$X \to Y$ is frequent and certain (csr).*
- *$\neg Y \to A$ is certain in D_X (anom).*
- *$A \to \neg Y$ is certain in D_X (ref).*

Comparing our formulation with the one of Berzal et al., our approach is equivalent to that from a formal point of view if *anom* and *ref* are defined in D_X, because $A \to \neg Y$ is equivalent to $\neg\neg Y \to \neg A \equiv Y \to \neg A$.

The following theorem shows a relation between our definition for anomalous rules and the definition given by Berzal et al. [3], in other words, it shows that our approach is more restrictive than the one proposed in [3].

Theorem 1. *[7] Let X, Y and A be arbitrary itemsets. The following inequality holds*

$$\mathrm{Conf}(X \wedge A \to \neg Y) \leq \mathrm{Conf}(X \wedge Y \to \neg A) \quad (4)$$

if and only if

$$\mathrm{supp}(X \cup A) \leq \mathrm{supp}(X \cup Y).$$

Our proposal is similar and logically equivalent to that of Berzal et al. but it does not have the disadvantage that the confidence of the rule $X \wedge Y \to \neg A$ is affected by an increment when the support of $X \cup Y$ is high (see [7] for more details).

It can be proven that $\mathrm{Conf}_X(A \to B) = \mathrm{Conf}(X \wedge A \to B)$, but this is not true when using the certainty factor. This is due to the appearance of the consequent's support in D or D_X in the computation of certainty factor:

$$\begin{aligned} CF(X \wedge \neg Y \to X \wedge A) &\neq CF_X(\neg Y \to A) \\ CF(X \wedge A \to X \wedge \neg Y) &\neq CF_X(A \to \neg Y) \end{aligned} \tag{5}$$

because

$$\mathrm{supp}(X \wedge A) = \frac{|X \cap A|}{|D|} \neq \frac{|X \cap A|}{|X|} = \mathrm{supp}_X(A). \tag{6}$$

5 Algorithm

We have proposed new approaches using the certainty factor for mining exception rules as well as anomalous rules. Mining exceptions and anomalies associated to a strong rule offers a clarification about the agents that perturbs the strong rule's usual behaviour, in the case of exceptions, or the resulting perturbation, if we find anomalies.

The algorithm 1, called **ERSA** (Exception Rule Search Algorithm), is able to mine together the set of common sense rules in a database with their associated exceptions. For anomalous rules, **ERSA** can be modified into **ARSA** (Anomalous Rule Search Algorithm) only by changing step 2.2.1. The process is very similar, in this case we take $A \in I$ (we do not impose not to have attribute in common with the items in the *csr*), and then we compute the CFs for the anomalous and the reference rule.

In our implementation we only consider exceptions and anomalies given by a single item, for a simpler comprehension of the obtained rules. To mine the association rules we have used an itemset representation by means of BitSets. Previous works [14,16] have implemented the Apriori algorithm using a bit-string representation of items. Both obtained quite good results with respect to time. One advantage of using a bit-string representation of items is that it speeds up logical operations such as conjunction or cardinality.

The algorithm complexity depends on the total number of transactions n and the number of obtained items i having in the first part a theoretical complexity of $O(n2^i)$, but in the second part it also depends on the number of *csr* obtained (r). So, theoretically both **ARSA** and **ERSA** have $O(nri2^i)$. Although this is a high complexity, in the performed experiments with several real databases, the algorithm takes reasonable times. In fact, the two influential factors in the execution time are the number of *csr* extracted.

The memory consumption in both algorithms, **ARSA** and **ERSA**, is high because the vector of BitSets associated to the database is stored in memory, but for standard databases this fact does not represent any problem. For instance,

Algorithm 1. ERSA (Exception Rule Search Algorithm)

Input: Transactional database, *minsupp*, *minconf* or *minCF*
Output: Set of association rules with their associated exception rules.

1. **Database Preprocessing**
   1.1 Transformation of the transactional database into a boolean database.
   1.2 Database storage into a vector of BitSets.
2. **Mining Process**
   2.1 **Mining Common Sense Rules**
      Searching the set of candidates (frequent itemsets) for extracting the *csr*.
      Storing the indexes of BitSet vectors associated to candidates and their supports.
      csr extraction exceeding *minsupp* and *minconf*/*minCF* thresholds
      2.2.1 **Mining Exception Rules**
         For every common sense rule $X \to Y$ we compute the possible exceptions:
           For each item $E \subset I$ (except those in the common sense rule)
             Compute $X \wedge E \wedge \neg Y$ and its support
             Compute $X \wedge E$ and its support
             Using confidence:
               If $Conf(X \wedge E \to \neg Y) \geq minconf$ **then** we have an exception
             Using certainty factor:
               Compute $supp_X(\neg Y)$
               If $CF_X(E \to \neg Y) \geq minCF$ **then** we have an exception rule

database `Barbora`[1] used in the PKDD99 conference held in Prague [16] consists in 6181 transactions and 12 attributes (33 items). The required memory in this case for the vector of BitSets is 107 kb, and for 61810 transactions is 1.04 MB. More details about the algorithm can be found in [7].

6 Experimental Evaluation

The benchmark data set German-statlog, about credits and the clients having a credit in a German bank, from the UCI Machine Learning repository has been used to empirically evaluate the performance of **ERSA** and **ARSA** algorithms. It is composed of 1000 transactions and 21 attributes, from which 18 are categorical or numerical, and 3 of them are continuous. The numerical continuous attributes have been categorized into meaningful intervals.

For the experiments, we used a 1.73GHz Intel Core 2Duo notebook with 1024MB of main memory, running Windows XP using Java. Tables 3 and 4 show respectively the number of rules and the employed time when mining exception and anomalous rules using our algorithm. In this collection of experiments we impose as 3 the limit of the maximum number of items in the antecedent or the consequent of the *csr* in order to obtain more manageable rules.

Once the rules are obtained, an expert should clarify if some of them are really interesting. We highlight here some of them, that we think they are in some sense remarkable.

[1] `http://lispminer.vse.cz/download`

Table 3. Number of *csr*, *exc* and *anom* rules found for different thresholds in `German-statlog` database

minsupp	*minCF* = 0.8			*minCF* = 0.9			*minCF* = 0.95		
	csr	*exc*	*anom*	*csr*	*exc*	*anom*	*csr*	*exc*	*anom*
0.08	674	66	326	309	11	39	270	6	10
0.1	384	27	208	137	4	12	123	3	5
0.12	226	11	142	62	1	3	57	0	2

Table 4. Time in seconds for mining exception and anomalous rules for different thresholds in `German-statlog` database

minsupp	*minCF* = 0.8		*minCF* = 0.9		*minCF* = 0.95	
	ERSA	**ARSA**	**ERSA**	**ARSA**	**ERSA**	**ARSA**
0.08	137	139	116	116	115	116
0.1	73	71	64	63	63	64
0.12	43	43	38	38	38	38

"*IF* `present employment since 7 years` *AND* `status & sex = single male` *THEN* `people being liable to provide maintenance for = 1`$(Supp{=}0.105\,\&\,CF{=}0.879)$ *EXCEPT* when `Purpose = business` $(CF = 1)$".

Previous exception rule tell us that when the `Purpose = business` the previous *csr* changes its behaviour. We have also found anomalous rules as for instance

"*IF* `property = real estate` *AND* `number of existing credits on this bank = 1` *THEN* `age is in between 18 and 25` $(Supp = 0.082\ \&\ CF = 0.972)$ *OR* `property = car` (unusually with $CF_1 = 1$, $CF_2 = 1$)".

This common sense rule has an anomalous rule introduced by the clause *OR* indicating that this is the unusual behaviour of the *csr*. Like in this example, we have observed that many anomalous rules contain items that are complementary to the common sense rule consequent, that is, A and Y has the attribute in common, but they differ in the value. This is very useful in order to see what is the usual behaviour (strong association) and their anomalous or unusual behaviours.

7 Conclusions and Future Research

Mining exception or anomalous rules can be useful in several domains. We have analysed their semantics and formulation, giving a new proposal that removes the imposition of the reference rule for the case of exceptions. Relative to anomalous rules our approach uses a more restrictive reference rule. Our approaches are also sustained in using the certainty factor as an alternative to confidence, achieving a smaller and a more accurate set of exceptions or anomalies. We also provide efficient algorithms for mining these kinds of rules. These algorithms have been run in a database about credits, obtaining a manageable set of interesting rules that should be analysed by an expert.

For future works we are interested in the development of a new approach for searching exceptional and anomalous knowledge with uncertain data. The first idea is to smooth the definitions presented here by means of fuzzy association rules. Other interesting task concerns the search of exception or anomalous rules in certain levels of action.

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References

1. Agrawal, R., Manilla, H., Sukent, R., Toivonen, A., Verkamo, A.: Fast discovery of Association rule. In: Advances in Knowledge Discovery and Data Mining, pp. 307–328. AAA Press (1996)
2. Balderas, M.A., Berzal, F., Cubero, J.C., Eisman, E., Marín, N.: Discovering hidden association rules. In: KDD Workshop on Data Mining Methods for Anomaly Detection, Chicago, pp. 13–20 (2005)
3. Berzal, F., Cubero, J.C., Marín, N., Gámez, M.: Anomalous association rules. In: IEEE ICDM Workshop Alternative Techniques for Data Mining and Knowledge Discovery (2004)
4. Berzal, F., Delgado, M., Sánchez, D., Vila, M.A.: Measuring accuracy and interest of association rules: A new framework. Intelligent Data Analysis 66(3), 221–235 (2002)
5. Delgado, M., Marín, N., Sánchez, D., Vila, M.A.: Fuzzy association rules: General model and applications. IEEE Trans. on Fuzzy Systems 11(2), 214–225 (2003)
6. Delgado, M., Ruiz, M.D., Sánchez, D.: Studying interest measures for association rules through a logical model. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 18(1), 87–106 (2010)
7. Delgado, M., Ruiz, M.D., Sánchez, D.: New approaches for discovering exception and anomalous rules. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 19(2), 361–399 (2011)
8. Ding, J., Yau, S.S.T.: TCOM, an innovative data structure for mining association rules among infrequent items. Computers & Mathematics with Applications 57(2), 290–301 (2009)
9. Duval, B., Salleb, A., Vrain, C.: On the discovery of exception rules: A survey. Studies in Computational Intelligence 43, 77–98 (2007)
10. Fawcet, T., Provost, F.: Adaptative fraud detection. In: Data Mining and Knowledge Discovery, pp. 291–316 (1997)
11. Geng, L., Hamilton, H.J.: Interestingness measures for data mining: A survey. ACM Comput. Surv. 38(3), 9 (2006)
12. Hussain, F., Liu, H., Suzuki, E., Lu, H.: Exception rule mining with a relative interestingness measure. In: Terano, T., Liu, H., Chen, A.L.P. (eds.) PAKDD 2000. LNCS, vol. 1805, pp. 86–97. Springer, Heidelberg (2000)

13. Liu, H., Lu, H., Feng, L., Hussain, F.: Efficient search of reliable exceptions. In: Zhong, N., Zhou, L. (eds.) PAKDD 1999. LNCS (LNAI), vol. 1574, pp. 194–204. Springer, Heidelberg (1999)
14. Louie, E., Lin, T.Y.: Finding association rules using fast bit computation: Machine-oriented modeling. In: Ohsuga, S., Raś, Z.W. (eds.) ISMIS 2000. LNCS (LNAI), vol. 1932, pp. 486–494. Springer, Heidelberg (2000)
15. Padmanabhan, B., Tuzhilin, A.: A belief driven method for discovering unexpected patterns. In: Proceedings of the 4th International Conference on Knowledge Discovery and Data Mining, pp. 94–100 (1998)
16. Rauch, J., Šimunek, M.: An alternative approach to mining association rules. Studies in Computational Intelligence (SCI) 6, 211–231 (2005)
17. Shortliffe, E., Buchanan, B.: A model of inexact reasoning in medicine. Mathematical Biosciences 23, 351–379 (1975)
18. Silberschatz, A., Tuzhilin, A.: User-assisted knowledge discovery: how much should the user be involved. In: ACM-SIGMOD Workshop on Research Issues on Data Mining and Knowledge Discovery (1996)
19. Sim, A.T.H., Indrawan, M., Srinivasan, B.: Mining infrequent and interesting rules from transaction records. In: 7th WSEAS Int. Conf. on AI, Knowledge Engineering and Databases (AIKED 2008), pp. 515–520 (2008)
20. Suzuki, E.: Discovering unexpected exceptions: A stochastic approach. In: Proceedings of the Fourth International Workshop on RSFD, pp. 225–232 (1996)
21. Suzuki, E.: Undirected discovery of interesting exception rules. International Journal of Pattern Recognition and Artificial Intelligence 16(8), 1065–1086 (2002)
22. Suzuki, E.: Discovering interesting exception rules with rule pair. In: Proc. Workshop on Advances in Inductive Rule Learning at PKDD 2004, pp. 163–178 (2004)
23. Suzuki, E., Shimura, M.: Exceptional knowledge discovery in databases based on information theory. In: Proceedings of the Second International Conference on Knowledge Discovery and Data Mining, pp. 275–278. AAAI Press (1996)
24. Taniar, D., Rahayu, W., Lee, V., Daly, O.: Exception rules in association rule mining. Applied Mathematics and Computation 205, 735–750 (2008)
25. Yao, Y., Wang, F.Y., Zeng, D., Wang, J.: Rule + exception strategies for security information analysis. IEEE Intelligent Systems, 52–57 (2005)
26. Zhong, N., Ohshima, M., Ohsuga, S.: Peculiarity oriented mining and its application for knowledge discovery in amino-acid data. In: Cheung, D., Williams, G.J., Li, Q. (eds.) PAKDD 2001. LNCS (LNAI), vol. 2035, pp. 260–269. Springer, Heidelberg (2001)
27. Zhou, L., Yau, S.: Efficient association rule mining among both frequent and infrequent items. Computers & Mathematics with Applications 54(6), 737–749 (2007)