Coordinated Replenishment Policy for a Decentralized Assembly System Based on Supply Hub Under Stochastic Lead Times

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Abstract To analyze effects of coordination, we consider a decentralized assembly system, which consists of two suppliers, one supply hub and one manufacturer. We propose two kinds of component replenishment policy, including decentralized strategy and coordinated strategy. We compare both strategies, and show that cost advantage of the coordinated strategy than the decentralized strategy is mainly governed by the cost structure and the service level. We examine and discuss the relation between parameters and the optimal component replenishment policy.

Keywords Assembly system • Supply hub • Coordinated replenishment policy

1 Introduction

Supply hub is a new and effective model comes out in electronic industry and automobile industry (Gaonkar and Viswanadham 2001). Supply-Hub can realize centralized management, information transparency, and vertical and horizontal coordination of supply chain (Gong and Ma 2008). However, supply hub is only as a central warehouse in most industry practices, and there is little theoretical guidance for practical operation. This paper studies the coordination of a decentralized assembly system based on supply hub.

There are two streams of most relevant literature. One stream is on replenishment planning of multi-components. Song and Zipkin (2003) make a comprehensive survey of papers on the assembly system. Tang and Grubbström (2003) study

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the problem of planned lead times for two supporting-components with stochastic procurement lead times and fixed product demand. Chauhan et al. (2009) propose a model for supply planning of assembly systems with continuous random lead times for components. Some researchers study the problem with considering lot sizing. Mohebbi and Posner (1998) study the optimal lot sizing and reorder point in a single item inventory system under random lead time. Fujiwara and Sedarage (1997) extend the problem to assembly system, consider a multipart assembly system with stochastic part procurement lead times and constant demand, and solve the problem of simultaneously determining the order timing of parts and lot sizing. But, they don't consider the effect of service level and the inventory holding cost caused by components' earliness. Our problem setting extends Fujiwara and Sedarage's model, and considering the problem in a decentralized model.

Another stream of relevant literatures is on on-time delivery model with coordination. Grout (Grout and Christy 1996) proposes an effective incentive mechanism between manufacturer and supplier to improve the on-time delivery rate of procurement. Guiffrida and Nagi (2006) analyzes the effects of delivery window, under stochastic lead time. Li et al. (2011) studies the coordination of components' delivery quantity in a decentralized assembly system, and shows that the expected supply chain profit is larger when considering coordination of components' delivery quantity. Li considers the coordination of the decentralized assembly system with stochastic lead times, as our problem settings, and studies this problem in one single period model.

In this paper, we study the coordination of two suppliers, a manufacturer and a supply hub, under stochastic component replenishment lead times. We propose two replenishment policies, decentralized and coordinated replenishment policies. The main purpose of this paper is to show how to implement coordinated replenishment, and the advantages of the coordinated replenishment.

2 Model Description and the Decentralized Replenishment Policy

We consider a decentralized assembly system composed of two suppliers, one supply hub and one manufacturer. Both suppliers provide each component, and each of two components is assembled into a unit final product. The manufacturer assembles product according to customers' order. The supply hub is a coordinating organization between suppliers and manufacture, and inventory policy is *(Q, r)*-policy. The demand rate D for the final product is constant. The replenishment lead time y_i of component i, $i = (1,2)$ is stochastic. Let $f(\cdot)$ and $F(\cdot)$ are its probability density function and probability distribution function. Let S_i be the fixed replenishment cost for component i, h_i be the inventory holding cost per unit per time for component i, π be the backorder cost per unit per time for product. Let L_i be the lateness of component i, noting as $L_i = y_i - r_i/D$, and T_i be the earliness of component i, noting as $T_i = r_i/D - y_i$.

There two methods to decide components' replenishment policy, the decentralized and the coordinated replenishment policies. The first one is adopted in most practice of supply hub. Under this policy, supply hub makes replenishment plans for both components without considering the coordination, and decides the optimal Q_i and r_i to minimize supply chain total cost, under the constraint of customer service level P_0.

It is known that when the internal service level SL_i is high enough, such as $SL \in (0.99,0.99)$, the probability of both components' tardiness is very small that can be ignored. So, the expected average total cost per unit time is:

$$E[TC] = \sum_{i=1}^{2} h_i \left[\frac{D}{Q_i} \left(\frac{(Q_i - DE(L_i))^2}{2D} + Q_i E(T_i) \right) + \frac{D}{Q_j} \frac{DE^2(L_j)}{2} \right] + \sum_{i=1}^{2} \frac{D}{Q_i} S_i + \pi \sum_{i=1}^{2} \frac{D}{Q_i} \frac{DE^2(L_i)}{2}. \tag{1}$$

where, the first term is the holding cost per unit time, and the second term is the fixed replenishment cost per unit time, and the third term is the backorder cost per unit time.

Obviously, service level P satisfies $P = F_1(r_1/D)F_2(r_2/D)$ under this policy.

The optimization problem (P1) is thus expressed as:

$$\begin{aligned} \text{P1}: \quad & \text{minimize} \quad E[TC] \\ & \text{subject to} \quad F_1(r_1/D)F_2(r_2/D) \geq P_0 \end{aligned}$$

Proposition 1. *Under decentralized replenishment policy, the expected average supply chain total cost per unit time E[TC] is joint convex in (Q_1,Q_2,r_1,r_2). With the constraint of customer service level, the optimal solution of (Q_1^*,Q_2^*) satisfies the first order condition, and the optimal solution of (r_1^*,r_2^*) satisfies the K-T condition.*

Proof. First, we get the optimal value of Q_i for any giving r_i, for it has no concern with the constraint in P1. It is obviously that $E[TC]$ is joint convex in (Q_1,Q_2), and we can get the optimal value from the first order condition, where

$$Q_1^* = \sqrt{\frac{2\left[S_1 D + (h_1 + h_2 + \pi) D^2 E^2(L_1)/2\right]}{h_1}};$$

$$Q_2^* = \sqrt{\frac{2\left[S_2 D + (h_1 + h_2 + \pi) D^2 E^2(L_2)/2\right]}{h_2}}.$$

Substituting above optimal solution into objective function $E[TC]$, we get:

$$E[TC] = \sqrt{2h_1\left[S_1 D + (h_1 + h_2 + \pi)D^2E^2(L_1)/2\right]} + h_1 D[E(T_1) - E(L_1)] \\ + \sqrt{2h_2\left[S_2 D + (h_1 + h_2 + \pi)D^2E^2(L_2)/2\right]} + h_2 D[E(T_2) - E(L_2)].$$

So, the optimization problem P1 can be expressed as,

$$\begin{cases} \min E[TC] \\ g(r_1, r_2) = F_1(r_1/D)F_2(r_2/D) - P_0 \geq 0 \end{cases}$$

Second, we search the optimal value of r_i with the K-T condition. It is easy to see that \$E[TC]\$ in above problem is joint concave in r_i, and r_i^* is the K-T point of this optimization problem.

3 Coordinated Replenishment Policy

Under this policy, supply hub adopts a common replenishment quality Q to coordinate suppliers' replenishment, to minimize the cost caused by cross replenishment. To simplify analysis, we let $Q_1 = Q_2 = Q$.

Let L be the tardiness of assembly, thus $L = \max(L_1, L_2)$. The expected average tardiness time for assembling product per cycle is given as $E(L)$. Thus the expected average total cost per unit time, $E[TC]$ can be expressed as:

$$E[TC] = \sum\nolimits_{i=1}^{2} h_i \frac{D}{Q}\left[\frac{(Q - DE(L))^2}{2D} + Q(E(L) - E(L_i) + E(T_i))\right] \\ + \sum\nolimits_{i=1}^{2} S_i \frac{D}{Q} + \pi \frac{D}{Q}\frac{DE^2(L)}{2}. \quad (2)$$

where, the first term is the average inventory holding cost per unit time, the second term is the fixed replenishment cost per unit time, the third term is the average backorder cost per unit time.

Obviously, customer service level satisfies $P = \min\{F_1(r_1/D), F_2(r_2/D)\}$.

The optimization problem (P2) is thus expressed as:

$$\text{P2}: \quad \begin{array}{ll} \text{minimize} & E[TC] \\ \text{subject to} & \min\left\{F_1(r_1/D), F_2\left(r_2/D\right)\right\} \geq P_0 \end{array}$$

Proposition 2. *Under coordinated replenishment policy, the expected average supply chain total cost per unit time $E[TC]$ is convex in Q, but is not joint convex in r_1 and r_2. However, there still exists a unique global optimal solution that minimizes the expected average supply chain total cost per unit time.*

Proof. The proof is similar to the proof of proposition 1, which is omitted.

To get optimal solutions of r_1 and r_2, a two-dimensional search for the optimal r_1 and r_2 is used when function $F_1(r_1/D)F_2(r_2/D) - SL^2 = 0$ is not satisfied, and a one-dimensional search for the optimal solution is used when above function is satisfied.

4 Number Illustration

In section, DRP is short for decentralized replenishment policy, and CRP is short for coordinated replenishment policy. Let component procurement lead times be exponentially distributed with parameter λ_i, ($i = 1$, $i = 2$), and $\lambda_1 = 25$, $\lambda_2 = 20$, $S_2 = 200$, $D = 250$, $\pi = 500$, $h_1 = 30$ and $h_2 = 20$.

We first analyze the effect of service level constraint P_0. Result shows that supply chain total cost per unit time in DRP and CRP first unchanges as P_0 increases, and then increase as P_0 increases. The knee point of supply chain total cost per unit time in CRP is much higher than that in DRP. Supply chain total cost per unit time in CRP is always lower than that in DRP.

Second, we analyzed the impact of parameters on DRP and CRP, with a given service level (95 %) and fixed S_2 (200) and h_2 (20), as shown in Table 1.

Results show that whether CRP can save supply chain cost is decided by parameters h_1 and S_i. In appropriate combinations, CRP has a distinguished cost advantage than DRP, or supply chain cost under CRP is higher than that under DRP.

Cost advantage of CRP first increases as $\frac{S_i}{h_i}$ increases, and then decreases, it also increases as λ_i and D increase, but decreases as π increases. However, the impact of π on cost saving of CRP is very small when customers' service level is high.

Table 1 The impact of parameters on DRP and CRP

Parameters		TC^d	TC^c	$\frac{TC^d - TC^c}{TC^d}$ (%)	Parameters		TC^d	TC^c	$\frac{TC^d - TC^c}{TC^d}$ (%)
S_1	8	3,840.0	3,866.3	−0.68	λ_1	15	4,196.9	3,907.5	6.90
	10	3,880.1	3,877.1	0.08		20	4,459.9	4,114.7	7.41
	50	4,355.2	4,084.7	6.21		30	4,959.5	4,530.0	8.66
	200	5,219.8	4,744.3	9.11		35	5,201.1	4,738.0	8.90
	400	5,936.8	5,453.2	8.15	λ_2	15	4,499.6	4,183.1	7.04
	800	6,951.1	6,578.4	5.36		25	4,915.6	4,461.6	9.24
	2,000	8,963.9	8,992.5	−0.32		30	5,110.3	4,601.3	9.96
h_1	1	2,284.6	2,362.9	−3.43	π	300	4,712.2	4,318.0	8.37
	5	2,827.0	2,662.0	5.84		400	4,712.7	4,320.1	8.33
	10	3,302.2	3,018.5	8.59		600	4,713.5	4,324.3	8.26
	20	4,061.8	3,689.9	9.15	D	150	3,288.3	3,069.9	6.64
	50	5,870.1	5,510.1	6.13		200	4,019.5	3,715.4	7.57
	150	10,725.2	10,748.6	−0.22		300	5,380.1	4,901.6	8.89

5 Conclusion

Supply hub is a popular way to management material flows for giant assemblers. How to coordinate material flows and the coordination efficiency based on supply hub have not been covered thoroughly. This paper considers an assembly system with two-component suppliers, one supply hub and one manufacturer, and discusses two different component-replenishment policies with the constraint of customer service level. We explain that how supply hub operator coordinates components' replenishment under random lead times, and analyze the advantage when considering coordination. Our result show that coordinated replenishment policy can get more cost saving when customer service level is much higher. In addition, cost advantage of coordinated replenishment policy is affected by system parameters, especially components' characters. When the difference of components' characters is too large, supply chain total cost under coordinated replenishment policy may be higher than decentralized replenishment policy.

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