Characterization Theorems for Revision of Logic Programs

Nicolas Schwind and Katsumi Inoue

National Institute of Informatics
2-1-2, Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
{schwind,inoue}@nii.ac.jp

Abstract. We address the problem of belief revision of logic programs, i.e., how to incorporate to a logic program $\mathcal{P}$ a new logic program $\mathcal{Q}$. Based on the structure of SE interpretations, Delgrande *et al.* [5] adapted the AGM postulates to identify the rational behavior of *generalized* logic program (GLP) revision operators and introduced some specific operators. In this paper, a constructive characterization of *all* rational GLP revision operators is given in terms of an ordering among propositional interpretations with some further conditions specific to SE interpretations. It provides an intuitive, complete procedure for the construction of all rational GLP revision operators and makes easier the comprehension of their semantic properties. In particular, we show that every rational GLP revision operator is derived from a propositional revision operator satisfying the original AGM postulates. Taking advantage of our characterization, we embed the GLP revision operators into structures of Boolean lattices, that allow us to bring to light some potential weaknesses in the adapted AGM postulates. To illustrate our claim, we introduce and characterize axiomatically two specific classes of (rational) GLP revision operators which arguably have a drastic behavior.

1 Introduction

Logic programs (LPs) are well-suited for modeling problems which involve common sense reasoning (e.g., biological networks, diagnosis, planning, etc.) Due to the dynamic nature of our environment, beliefs represented through an LP $\mathcal{P}$ are subject to change, i.e., because one wants to incorporate to it a new LP $\mathcal{Q}$. Since there is no unique, consensual procedure to revise a set of beliefs Alchourrón, Gärdenfors and Makinson [1] introduced a set of desirable principles w.r.t. belief change called *AGM postulates*. Katsuno and Mendelzon [14] adapted them for propositional belief revision and distinguished two kind of change operations, i.e., *revision* and *update* [13] characterized for each one of these change operations by a set of so-called *KM postulates*. Revision consists in incorporating a new information into a database that represents a static world, i.e., new and old beliefs describe the same situation but the new ones are more reliable. In the case of update, the underlying world evolves by the occurence of some events i.e., new and old beliefs describe two different states of the world.

Our interests focus here on the problem of revision of logic programs. Most of works dealing with belief change in logic programming are concerned with update

P. Cabalar and T.C. Son (Eds.): LPNMR 2013, LNAI 8148, pp. 485–498, 2013.

[20,2,8], and they do not lie into the AGM framework, particularly due to their syntactic, rule-based essence. Indeed, given the nonmonotonic nature of LPs the AGM/KM postulates can not be directly applied to logic programs. Still, the notion of *SE interpretations* [19] - initially introduced to characterize the strong equivalence between logic programs [16] - provide a monotonic semantical characterization of LPs. Then, based on these structures, Delgrande *et al.* [5,7] adapted the AGM/KM postulates in the context of logic programming. They proposed several revision operators and investigated their properties w.r.t. the adapted postulates. Their work covered a serious drawback in the field of belief revision in logic programming. However the constructive characterization of *all* rational belief revision operators remains an open issue.

In this paper, we consider the revision of *generalized* logic programs (GLPs), which is a very general form of programs. We provide a characterization theorem for the GLP revision operators, that is, a sound and complete model-theoretic construction of the rational LP revision operators (i.e., those which fully satisfy the adaptation of AGM postulates to LPs). Interestingly, our result shows that every rational LP revision operator is derived from a rational propositional revision operator (i.e., satisfying the KM postulates in the propositional setting). Our characterization makes easier the refined analysis of LP revision operators. Indeed, we can embed the GLP revision operators into structures of Boolean lattices, that allows us to bring out some potential weaknesses in the original postulates and pave the way for the discrimination of some rational GLP revision operators.

The next section introduces some preliminaries about belief revision in propositional logic and some necessary background on answer-set programs. Section 3 introduces the LP revision operators and some preliminary results. Section 4 provides the characterization theorem for GLP revision operators. In Section 5 we partition the class of GLP revision operators into subclasses of Boolean lattices, then we introduce and characterize axiomatically two specific classes of (rational) GLP revision operators, i.e., the *skeptical* and *brave* GLP revision operators. We conclude in Section 6 and propose some perspectives for further work. For space reasons, only proof sketches of some propositions are provided in an appendix.

2 Preliminaries

We consider a propositional language $\mathcal{L}$ defined from a finite set of propositional variables (also called *atoms*) $\mathcal{A}$ and the usual connectives. $\bot$ (resp. $\top$) is the Boolean constant always false (resp. true). An *interpretation* over $\mathcal{A}$ is a total function from $\mathcal{A}$ to $\{0, 1\}$. To avoid heavy expressions, an interpretation I is also viewed as the subset of atoms from $\mathcal{A}$ that are true in I. For instance, if $\mathcal{A} = \{p, q\}$, then the interpretation over $\mathcal{A}$ such that $I(p) = 1$ and $I(q) = 0$ is also represented as the set $\{p\}$. The set of all interpretations is denoted Ω. An interpretation I is a *model* of a formula $\phi \in \mathcal{L}$ iff it makes it true in the usual truth functional way. A *consistent* formula is a formula that admits a model. $mod(\phi)$ denotes the set of models of formula ϕ, i.e., $mod(\phi) = \{I \in \Omega \mid I \models \phi\}$.

2.1 Belief Revision in Propositional Logic

This section introduces some background on propositional belief revision. Basically, a revision operator $\circ$ is a mapping associating two formulae ϕ, ψ with a new formula, denoted $\phi \circ \psi$. The AGM framework [1] describes the standard principles for belief revision (e.g., consistency preservation and minimality of change), which capture changes occuring in a static domain. Katsuno and Mendelzon [13] equivalently rephrased the AGM postulates as follows:

Definition 1 (KM revision operator). *A KM revision operator $\circ$ is a propositional revision operator that satisfies the following postulates, for all formulae $\phi, \phi_1, \phi_2, \psi, \psi_1, \psi_2$:*

(R1) $\phi \circ \psi \models \psi$;
(R2) *If $\phi \wedge \psi$ is consistent, then $\phi \circ \psi \equiv \phi \wedge \psi$;*
(R3) *If ψ is consistent, then $\phi \circ \psi$ is consistent;*
(R4) *If $\phi_1 \equiv \phi_2$ and $\psi_1 \equiv \psi_2$, then $\phi_1 \circ \psi_1 \equiv \phi_2 \circ \psi_2$;*
(R5) $(\phi \circ \psi_1) \wedge \psi_2 \models \phi \circ (\psi_1 \wedge \psi_2)$;
(R6) *If $(\phi \circ \psi_1) \wedge \psi_2$ is consistent, then $\phi \circ (\psi_1 \wedge \psi_2) \models (\phi \circ \psi_1) \wedge \psi_2$.*

These so-called *KM postulates* capture the desired behavior of a revision operator, e.g., in terms of consistency preservation and minimality of change.

KM revision operators can be characterized in terms of total preorders over interpretations. Indeed, each KM revision operator corresponds to a faithful assignment [13]:

Definition 2 (Faithful assignment). *A* faithful assignment *is a mapping which associates with every formula ϕ a preorder $\leq_\phi$ over interpretations*[1] *such that for all interpretations I, J and all formulae ϕ, ϕ_1, ϕ_2, the following conditions hold:*

(a) *If $I \models \phi$ and $J \models \phi$, then $I \simeq_\phi J$;*
(b) *If $I \models \phi$ and $J \not\models \phi$, then $I <_\phi J$;*
(c) *If $\phi_1 \equiv \phi_2$, then $\leq_{\phi_1} = \leq_{\phi_2}$.*

Theorem 1 ([14]). *A revision operator $\circ$ is a KM revision operator if and only if there exists a faithful assignment associating every formula ϕ with a total preorder $\leq_\phi$ such that for all formulae ϕ, ψ, $mod(\phi \circ \psi) = \min(mod(\psi), \leq_\phi)$.*

KM revision operators include the class of distance-based revision operators (see, for instance, [4]), i.e., those operators characterized by a distance between interpretations:

Definition 3 (Distance-based revision operator). *Let d be a distance between interpretations*[2]*, extended to a distance between every interpretation I and*

[1] For each preorder $\leq_\phi$, $\simeq_\phi$ denotes the corresponding indifference relation and $<_\phi$ the corresponding strict ordering.
[2] Actually, a pseudo-distance is enough, i.e., triangular inequality is not mandatory.

every formula ϕ *by* $d(I,\phi) = min\{d(I,J) \mid J \models \phi\}$ *if* ϕ *is consistent,* 0 *otherwise. The distance-based revision operator* $\circ^d$ *is defined for all formulae* ϕ, ψ *by* $mod(\phi \circ^d \psi) = min(mod(\psi), \leq^d_\phi)$ *where the preorder* $\leq^d_\phi$ *induced by* ϕ *is defined for all interpretations* I, J *by* $I \leq^d_\phi J$ *iff* $d(I,\phi) \leq d(J,\phi)$.

Theorem 2. *Every distance-based revision operator is a KM revision operator, i.e., it satisfies the postulates (R1 - R6).*

Usual distances are d_D, the drastic distance ($d_D(I,J) = 1$ iff $I \neq J$), and d_H the Hamming distance ($d_H(I,J) = n$ if I and J differ on n variables). Noteworthy, the faithful assignment corresponding to the revision operator based on the drastic distance d_D (so-called drastic revision operator) associates with every formula a (unique) two-level preorder:

Definition 4 (Drastic revision operator). *The* drastic revision operator*, denoted* $\circ_D$*, is the revision operator based on the drastic distance.*

Likewise, the revision operator based on Hamming distance corresponds to the well-known Dalal revision operator [4]:

Definition 5 (Dalal revision operator). *The* Dalal revision operator*, denoted* $\circ_{Dal}$*, is the revision operator based on the Hamming distance.*

2.2 Logic Programming

In this section, we define the syntax and semantics of generalized logic programs. We use the same notations as in [5]. A *generalized logic program* (GLP) is a finite set of rules of the form

$$a_1; \ldots; a_k; \sim b_1; \ldots; \sim b_l \leftarrow c_1, \ldots, c_m, \sim d_1, \ldots, \sim d_n,$$

where $k, l, m, n \geq 0$.

Each a_i, b_i, c_i, d_i is either one of the constant symbols $\bot$, $\top$, or an atom from $\mathcal{A}$; $\sim$ is the negation by failure; ";" is the disjunctive connective, "," is the conjunctive connective of atoms. The right-hand and left-hand sides of r are respectively called the head and body of r. For each rule r, we define $H(r)^+ = \{a_1, \ldots, a_k\}$, $H(r)^- = \{b_1, \ldots, b_l\}$, $B(r)^+ = \{c_1, \ldots, c_m\}$, and $B(r)^- = \{d_1, \ldots, d_n\}$. For the sake of simplicity, a rule r is also expressed as follows:

$$H(r)^+; \sim H(r)^- \leftarrow B(r)^+, \sim B(r)^-.$$

A logic program is interpreted through its preferred models based on the answer set semantics. A *(classical) model* X of a GLP $\mathcal{P}$ (written $X \models \mathcal{P}$) is an interpretation from Ω that satisfies all rules from $\mathcal{P}$ according to the classical definition of truth in propositional logic. $mod(\mathcal{P})$ will denote the set of all models of a GLP $\mathcal{P}$. An *answer set* X of a GLP $\mathcal{P}$ is a minimal (w.r.t. set inclusion) set of atoms from $\mathcal{A}$ that is a model of the program $\mathcal{P}^X$, where $\mathcal{P}^X$ is called the *reduct* of $\mathcal{P}$ relative to X and is defined as $\mathcal{P}^X = \{H(r)^+ \leftarrow B(r)^+ \mid r \in \mathcal{P}, H(r)^- \subseteq X, B(r)^- \cap X = \emptyset\}$. The classical notion of equivalence between programs corresponds to the correspondence of their answer sets.

SE interpretations are semantic structures characterizing *strong equivalence* between logic programs [19], they provide a monotonic semantic foundation of logic programs under answer set semantics. An SE interpretation over $\mathcal{A}$ is a pair (X, Y) of interpretations over $\mathcal{A}$ such that $X \subseteq Y$. An *SE model* (X, Y) of a logic program $\mathcal{P}$ is an SE interpretation over $\mathcal{A}$ that satisfies $Y \models \mathcal{P}$ and $X \models \mathcal{P}^Y$, where $\mathcal{P}^Y$ is the reduct of $\mathcal{P}$ relative to Y. For the sake of simplicity, set-notations will be dropped within SE interpretations, e.g., the SE interpretation $(\{p\}, \{p, q\})$ will be simply denoted (p, pq). Through their SE models, logic programs are semantically described in a stronger way than through their answer sets, as shown in the following example which belongs to [5]:

Example 1. Let $\mathcal{P} = \{p; q \leftarrow \top\}$ and $\mathcal{Q} = \{p \leftarrow\sim q, q \leftarrow\sim p\}$. Then $AS(\mathcal{P}) = AS(\mathcal{Q}) = \{\{p\}, \{q\}\}$, that is, they admit the same answer sets, however their SE models differ: $SE(\mathcal{P}) = \{(p, p), (q, q), (p, pq), (q, pq), (pq, pq)\}$, while $SE(\mathcal{Q}) = \{(p, p), (q, q), (p, pq), (q, pq), (pq, pq), (\emptyset, pq)\}$.

A program $\mathcal{P}$ is *consistent* if $SE(P) \neq \emptyset$. Two programs $\mathcal{P}$ and $\mathcal{Q}$ are said to be *strongly equivalent*, denoted $\mathcal{P} \equiv_s \mathcal{Q}$, whenever $SE(\mathcal{P}) = SE(\mathcal{Q})$. We also write $\mathcal{P} \subseteq_s \mathcal{Q}$ if $SE(\mathcal{P}) \subseteq SE(\mathcal{Q})$. Two programs are equivalent if they are strongly equivalent, but the other direction does not hold in general. Note that Y is an answer set of $\mathcal{P}$ iff $(Y, Y) \in SE(\mathcal{P})$ and no $(X, Y) \in SE(\mathcal{P})$ with $X \subset Y$ exists. We also have $(Y, Y) \in SE(\mathcal{P})$ iff $Y \in mod(\mathcal{P})$. A set of SE interpretations S is *well-defined* if for every interpretation X, Y with $X \subseteq Y$, if $(X, Y) \in S$ then $(Y, Y) \in S$. Every GLP has a well-defined set of SE models. Moreoever, from every well-defined set S of SE models, one can build a GLP P such that $SE(P) = S$ [10,3].

3 Logic Program Revision Operators

Given the nonmonotonic nature of answer-set programs, Delgrande *et al.* [5] pointed out that the rational behavior of revision operators for logic programs cannot be expressed using the original KM postulates (cf. Definition 1). Therefore, they proposed an adaptation of these postulates in the context of logic programming using the characterization of logic programs through their SE models. To this end, they first defined the operation of *expansion* of two logic programs:

Definition 6 (Expansion operator [5]). *Given two programs $\mathcal{P}, \mathcal{Q}$, the* expansion *of $\mathcal{P}$ by $\mathcal{Q}$, denoted $\mathcal{P} + \mathcal{Q}$ is any program $\mathcal{R}$ such that $SE(\mathcal{R}) = SE(\mathcal{P}) \cap SE(\mathcal{Q})$.*

Though the expansion of logic programs trivializes the result whenever the two input logic programs admit no common SE models, this operation is of interest in its own right. Indeed, it has be shown that if $\mathcal{P}$ and $\mathcal{Q}$ are GLPs then there exists a construction of a logic program $\mathcal{P} + \mathcal{Q}$ that is also a GLP [6].

Expansion of programs corresponds to the model-theoretical definition of expansion expressed through KM postulates. Delgrande *et al.* rephrased the full set of KM postulates in the context of GLPs. Beforehand, we define a logic program revision operator as a simple function, that considers two GLPs (the original one and the new one) and returns a revised GLP:

Definition 7 (LP revision operator). *A* LP revision operator $\star$ *is a mapping associating two GLPs $\mathcal{P}, \mathcal{Q}$ with a new GLP, denoted $\mathcal{P} \star \mathcal{Q}$.*

Definition 8 (GLP revision operator [5]). *A* GLP revision operator $*$ *is an LP revision operator that satisfies the following postulates, for all GLPs $\mathcal{P}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{Q}, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{R}$:*

(RA1) $\mathcal{P} * \mathcal{Q} \subseteq_s \mathcal{Q}$*;*
(RA2) *If $\mathcal{P} + \mathcal{Q}$ is consistent, then $\mathcal{P} * \mathcal{Q} \equiv_s \mathcal{P} + \mathcal{Q}$;*
(RA3) *If $\mathcal{Q}$ is consistent, then $\mathcal{P} * \mathcal{Q}$ is consistent;*
(RA4) *If $\mathcal{P}_1 \equiv_s \mathcal{P}_2$ and $\mathcal{Q}_1 \equiv_s \mathcal{Q}_2$, then $\mathcal{P}_1 * \mathcal{Q}_1 \equiv \mathcal{P}_2 * \mathcal{Q}_2$;*
(RA5) $(\mathcal{P} * \mathcal{Q}) + \mathcal{R} \subseteq_s \mathcal{P} * (\mathcal{Q} + \mathcal{R})$*;*
(RA6) *If $(\mathcal{P} * \mathcal{Q}) + \mathcal{R}$ is consistent, then $\mathcal{P} * (\mathcal{Q} + \mathcal{R}) \subseteq_s (\mathcal{P} * \mathcal{Q}) + \mathcal{R}$.*

Delgrande *et al.* proposed in [5] a specific revision operator that is inspired from Satoh's propositional revision operator [18], i.e., it is based on the set containment of SE interpretations. This operator satisfies postulates (RA1 - RA5). Though it seems to have a good behavior on some instances, this operator does not satisfy (RA6), so that it does not fully respect the principle of minimality of change (see [12], Section 3.1 for details on this postulate). However, the whole set of postulates is consistent, as they later introduce the so-called *cardinality-based revision operator* [6] that reduces to the Dalal revision operator over propositional models[3], and that satisfies all the postulates (RA1 - RA6):

Definition 9 (Cardinality-based revision operator). *Given a program $\mathcal{P}$, let $\phi_{\mathcal{P}}, \psi_{\mathcal{P}}, \psi_{\mathcal{Q}}, \alpha_{(\mathcal{P},\mathcal{Q})}$ be propositional formulae satisfying $mod(\phi_{\mathcal{P}}) = \{X \mid (X, Y) \in SE(\mathcal{P})\}$, $mod(\psi_{\mathcal{P}}) = mod(\mathcal{P})$, $mod(\psi_{\mathcal{Q}}) = mod(\mathcal{Q})$ and $mod(\alpha_{(\mathcal{P},\mathcal{Q})}) = \{X \mid (X, Y) \in SE(\mathcal{Q}), Y \models \psi_P \circ_{Dal} \psi_{\mathcal{Q}}\}$. The cardinality-based revision operator, denoted $\star_c$, is defined for all programs $\mathcal{P}, \mathcal{Q}$ by $SE(\mathcal{P} \star_c \mathcal{Q}) = \{(X, Y) \mid Y \models \psi_{\mathcal{P}} \circ_{Dal} \psi_{\mathcal{Q}}, X \models \phi_{\mathcal{P}} \circ_{Dal} \alpha_{(\mathcal{P},\mathcal{Q})}\}$.*

Theorem 3 ([6]). *$\star_c$ is a GLP revision operator.*

In addition, we introduce below a simple LP revision operator which also satisfies the whole set of postulates (RA1 - RA6):

Definition 10 (Drastic LP revision operator). *The* drastic GLP revision operator $*_D$ *is defined for all GLPs $\mathcal{P}, \mathcal{Q}$ as $\mathcal{P} *_D \mathcal{Q} = \mathcal{P} + \mathcal{Q}$ if $\mathcal{P} + \mathcal{Q}$ is consistent, otherwise $\mathcal{P} *_D \mathcal{Q} = \mathcal{Q}$.*

Proposition 1. *$*_D$ is a GLP revision operator.*

[3] This definition is equivalent to the original one introduced in [6], reformulated here for space reasons.

Theorem 3 and Proposition 1 show that postulates (RA1 - RA6) form a consistent set of properties, but it is not known whether there exist more GLP revision operators than the cardinality-based and the drastic LP revision operators. Moreoever, the cardinality-based revision operator has a parsimonious behavior compared to the drastic LP revision operator; however, both are fully satisfactory in terms of revision principles; this raises the problem on how to discard some rational operators from others.

In the next section, we fill the gap and we give a constructive, full characterization of GLP revision operators. This allows us to get a clear, complete picture of the class of GLP revision operators.

4 Characterization of GLP Revision Operators

We now provide the main result of our paper, i.e., a characterization theorem for GLP revision operators. That is, we show that each GLP revision operator (i.e., each LP revision operator satisfying the postulates (RA1 - RA6)) can be characterized in terms of preorders over the set of all classical interpretations, with some further conditions specific to SE interpretations.

Definition 11 (LP faithful assignment). *A* LP faithful assignment *is a mapping which associates with every GLP $\mathcal{P}$ a preorder $\leq_\mathcal{P}$ over interpretations such that for every GLP $\mathcal{P}, \mathcal{Q}$ and every interpretation Y, Y', the following conditions hold:*

(1) *If $Y \models \mathcal{P}$ and $Y' \models \mathcal{P}$, then $Y \simeq_\mathcal{P} Y'$;*
(2) *If $Y \models \mathcal{P}$ and $Y' \not\models \mathcal{P}$, then $Y <_\mathcal{P} Y'$;*
(3) *If $\mathcal{P} \equiv_s \mathcal{Q}$, then $\leq_\mathcal{P} = \leq_\mathcal{Q}$.*

Definition 12 (Well-defined assignment). *A* well-defined assignment *is a pair (Φ, Ψ), where Φ is an LP faithful assignment and Ψ is a mapping which associates with every GLP $\mathcal{P}$ and every interpretation Y a set of interpretations $\Psi(\mathcal{P}, Y)$ (simply denoted $\mathcal{P}(Y)$), such that for all GLPs $\mathcal{P}, \mathcal{Q}$ and all interpretations X, Y, the following conditions hold:*

(a) *$Y \in \mathcal{P}(Y)$;*
(b) *If $X \in \mathcal{P}(Y)$, then $X \subseteq Y$;*
(c) *If $(X, Y) \in SE(\mathcal{P})$, then $X \in \mathcal{P}(Y)$;*
(d) *If $(X, Y) \notin SE(\mathcal{P})$ and $Y \models \mathcal{P}$, then $X \notin \mathcal{P}(Y)$;*
(e) *If $\mathcal{P} \equiv_s \mathcal{Q}$, then $\mathcal{P}(Y) = \mathcal{Q}(Y)$.*

We are ready to bring to light our main result:

Proposition 2. *An operator $\star$ is a GLP revision operator iff there exists a well-defined assignment (Φ, Ψ), where Φ associates with every GLP $\mathcal{P}$ a total preorder $\leq_\mathcal{P}$, Ψ associates with every GLP $\mathcal{P}$ and every interpretation Y a set of interpretations $\mathcal{P}(Y)$, such that for all GLPs $\mathcal{P}, \mathcal{Q}$, $SE(\mathcal{P} \star \mathcal{Q}) = \{(X, Y) \mid (X, Y) \in SE(\mathcal{Q}), \forall Y' \models \mathcal{Q}\ Y \leq_\mathcal{P} Y', X \in \mathcal{P}(Y)\}$.*

Note that there is no relationship between the mappings Φ, Ψ induced from a well-defined assignment, that is, each one of them can be defined in a completely independent way. Therefore, an interesting consequence from Theorem 1 and Proposition 2 is that every GLP revision operator is an extension of a (propositional) KM revision operator:

Definition 13 (Propositional-based LP revision operator). *Given a program $\mathcal{P}$, let $\psi_{\mathcal{P}}$ be any propositional formula such that $mod(\psi_{\mathcal{P}}) = mod(\mathcal{P})$. Let $\circ$ be a propositional revision operator and f be a mapping from Ω to 2^{Ω} such that for every interpretation Y, $Y \in f(Y)$ and if $X \in f(Y)$ then $X \subseteq Y$. The* propositional-based LP revision operator *w.r.t. $\circ$ and f, denoted $\star^{\circ,f}$, is defined for all GLPs $\mathcal{P}, \mathcal{Q}$ by $SE(\mathcal{P} \star^{\circ,f} \mathcal{Q}) = SE(\mathcal{P} + \mathcal{Q})$ if $\mathcal{P} + \mathcal{Q}$ is consistent, otherwise $SE(\mathcal{P} \star^{\circ,f} \mathcal{Q}) = \{(X,Y) \mid (X,Y) \in SE(\mathcal{Q}), Y \models \psi_{\mathcal{P}} \circ \psi_{\mathcal{Q}}, X \in f(Y)\}$.*

$\star^{\circ,f}$ is said to be a propositional-based GLP revision operator *if $\circ$ is a KM revision operator (i.e., satisfying postulates (R1 - R6)).*

Proposition 3. *The classes of GLP revision operators and propositional-based GLP revision operators coincide.*

For every propositional revision operator $\circ$, let $GLP(\circ)$ denote the set of all propositional-based LP revision operators w.r.t. $\circ$. From Definition 13, it is easy to see that each propositional-based LP revision operator is built from a *unique* propositional revision operator, that is, for all propositional revision operators $\circ_1, \circ_2$, we have $\circ_1 \neq \circ_2$ if and only if $GLP(\circ_1) \cap GLP(\circ_2) = \emptyset$. Therefore, a direct consequence of Proposition 3 is that the class of GLP revision operators can be viewed as the partition $\{GLP(\circ) \mid \circ$ is a KM revision operator$\}$. Similarly, for each propositional revision operator $\circ$, for all propositional-based LP revision operators $\star^{\circ,f_1}, \star_2^{\circ,f_2}$, we have $\star^{\circ,f_1} \neq \star_2^{\circ,f_2}$ if and only if $f_1 \neq f_2$.

Note that the cardinality-based revision operator $\star_c$ (cf. Definition 9) corresponds to the propositional-based GLP revision operator $\star^{\circ_{Dal},f}$, where $\circ_{Dal}$ is the Dalal revision operator (cf. Definition 5) and f is defined for every interpretation Y as $f(Y) = \{X \mid X \subseteq Y, \exists Z \models \psi_{\mathcal{P}} \circ_{Dal} \psi_{\mathcal{Q}}, \forall X' \subseteq Y, \forall Z' \models \psi_{\mathcal{P}} \circ_{Dal} \psi_{Q}, d_H(X,Z) \leq d_H(X',Z')\}$. In addition, the drastic GLP revision operator (cf. Definition 10) corresponds to the propositional-based GLP revision operator $\star^{\circ_D,f}$, where $\circ_D$ is the drastic revision operator (cf. Definition 4) and f is defined for every interpretation Y as $f(Y) = 2^Y$.

Remark that in the case where $\mathcal{P}$ and $\mathcal{Q}$ have no common SE models, then a propositional-based GLP revision operator $\star^{\circ,f}$ gives preference to the second component of SE interpretations, that is driven by the choice of the underlying propositional revision operator $\circ$. However, one can directly see from Definition 13 that the first element of SE interpretations (that is specified using f) is totally unconstrained. We will show in the next section that this "freedom" on the choice of the first component of SE interpretations raises some issues for some subclasses of fully rational LP revision operators.

Our characterization theorem provides an intuitive construction of GLP revision operators and aids the analysis of their semantic properties, as it is illustrated in the next section.

5 GLP Revision Operators Embedded into Boolean Lattices

We now take a closer look to the set of GLP revision operators associated with each given KM revision operator. The characterization theorem provided in the previous section allows us to embed the subclass $GLP(\circ)$, for each KM revision operator $\circ$, into a structure of Boolean lattice[4].

Definition 14. *Let $\circ$ be a propositional revision operator. We define the binary relation $\preceq_\circ$ over $GLP(\circ)$ as follows: for all propositional-based LP revision operators $\star^{\circ,f_1}, \star^{\circ,f_2}$, $\star^{\circ,f_1} \preceq_\circ \star^{\circ,f_2}$ if and only for every interpretation Y, we have $f_2(Y) \subseteq f_1(Y)$.*

It can be easily checked that for each propositional revision operator $\circ$, $(GLP(\circ), \preceq_\circ)$ forms a Boolean lattice, that corresponds to the product of the Boolean lattices $\{(\mathbb{B}_Y, \subseteq) \mid Y \in \Omega\}$, where $\mathbb{B}_Y = \{Z \cup \{Y\} \mid Z \in 2^{2^Y \setminus Y}\}$. The following result shows that this lattice structure can be used to analyse the relative semantic behavior of GLP revision operators from $(GLP(\circ), \preceq_\circ)$.

Proposition 4. *Let $\circ$ be a KM revision operator. Then for all GLP revision operators $\star_1, \star_2 \in GLP(\circ)$, $\star_1 \preceq_\circ \star_2$ if and only if for all GLPs $\mathcal{P}, \mathcal{Q}$, we have $AS(\mathcal{P} \star_1 \mathcal{Q}) \subseteq AS(\mathcal{P} \star_2 \mathcal{Q})$.*

This result paves the way for the choice of a specific GLP revision operator depending on the desired "amount of information" provided by the revised GLP in terms of number of its answer sets. We illustrate this notion by considering two specific classes of GLP revision operators that correspond respectively to the suprema and infima of lattices $(GLP(\circ), \preceq_\circ)$ for all KM revision operators $\circ$.

Definition 15 (Skeptical GLP revision operators). *The* skeptical GLP revision operators, *denoted $\star_S^\circ$ are the propositional-based GLP revision operators $\star^{\circ,f}$ where f is defined for every interpretation Y by $f(Y) = 2^Y$.*

Definition 16 (Brave GLP revision operators). *The* brave GLP revision operators, *denoted $\star_B^\circ$ are the propositional-based GLP revision operators $\star^{\circ,f}$ where f is defined for every interpretation Y by $f(Y) = \emptyset$.*

For each propositional revision operator $\circ$, we have $\star_S^\circ = inf(GLP(\circ), \preceq_\circ)$ and $\star_B^\circ = sup(GLP(\circ), \preceq_\circ)$. We now illustrate how much the behavior of skeptical and brave GLP revision operators diverge through the following representative example:

Example 2. Consider $\circ_D$ the propositional drastic revision operator. Let $\mathcal{P} = \{p \leftarrow \top, q \leftarrow \top, \bot \leftarrow r\}$ and $\mathcal{Q} = \{\bot \leftarrow p, q, \sim r\}$. We have $AS(\mathcal{P}) = \{p, q\}$, $AS(\mathcal{Q}) = \{\emptyset\}$, $AS(\mathcal{P} \star_S^{\circ_D} \mathcal{Q}) = \{\emptyset\}$ and $AS(\mathcal{P} \star_B^{\circ_D} \mathcal{Q}) = \{\emptyset, \{p\}, \{q\}, \{r\}, \{pr\}, \{qr\}, \{pqr\}\}$.

[4] A Boolean lattice is a partially ordered set $(E, \leq_E)$ which is isomorphic to the set of subsets of some set F together with the usual set-inclusion operation, i.e., $(2^F, \subseteq)$.

We provide an axiomatic characterization of each one of these two subclasses of GLP revision operators in order to get a clearer view of their general behavior. Each characterization theorem below is given in terms of answer sets of the revised program.

Proposition 5. *The skeptical GLP revision operators are the only GLP revision operators* $\star$ *such that for all GLPs* $\mathcal{P}, \mathcal{Q}$*, whenever* $\mathcal{P} + \mathcal{Q}$ *is inconsistent, we have* $AS(\mathcal{P} \star \mathcal{Q}) \subseteq AS(\mathcal{Q})$.

Proposition 6. *Given a program* $\mathcal{P}$*, let* ψ_P *be any propositional formula such that* $mod(\psi_{\mathcal{P}}) = mod(\mathcal{P})$*. The brave GLP revision operators are the only GLP revision operators* $\star^{\circ,f}$ *such that for all GLPs* $\mathcal{P}, \mathcal{Q}$*, whenever* $\mathcal{P} + \mathcal{Q}$ *is inconsistent, we have* $AS(\mathcal{P} \star^{\circ,f} \mathcal{Q}) = mod(\psi_{\mathcal{P}} \circ \psi_{\mathcal{Q}})$.

Remark that the drastic GLP revision operator (cf. Definition 10), i.e., the skeptical GLP revision operator based on the propositional drastic revision operator, is a specific case from the result given in Proposition 5 where $AS(\mathcal{P} \star_S^{\circ_D} \mathcal{Q}) = AS(\mathcal{Q})$ whenever $\mathcal{P} + \mathcal{Q}$ is inconsistent. In addition, the brave GLP revision operator based on the propositional drastic revision operator satisfies $AS(\mathcal{P} \star_B^{\circ_D} \mathcal{Q}) = mod(\mathcal{Q})$ whenever $\mathcal{P} + \mathcal{Q}$ is inconsistent.

Though they are rational LP revision operators w.r.t. the postulates (RA1 - RA6), skeptical and brave operators have a rather trivial, thus undesirable behavior. On the one hand, consider where p is believed to be true, then learned to be false. That is, $\{\bot \leftarrow p\} \subseteq \mathcal{P}$ and $\mathcal{Q} = \{p \leftarrow \top\}$. Then one obtains that $AS(\mathcal{P} \star_S^{\circ} \mathcal{Q}) \subseteq AS(\mathcal{Q})$, that is, $AS(\mathcal{P} \star_S^{\circ} \mathcal{Q}) = \{\{p\}\}$, i.e., for any such program $\mathcal{P}$, on learning that p is true the revision states that *only* p is true. On the other hand, brave operators only focus on classical models of logic programs $\mathcal{P}, \mathcal{Q}$ to compute $\mathcal{P} \star_B^{\circ} \mathcal{Q}$ (whenever $\mathcal{P} + \mathcal{Q}$ is inconsistent), thus they do not take into consideration the inherent, non-monotonic behavior of logic programs. As a consequence, programs $\mathcal{P} \star_B^{\circ} \mathcal{Q}$ will often admit many answer sets that are actually irrelevant to the input programs $\mathcal{P}$ and $\mathcal{Q}$. Stated otherwise, skeptical and brave GLP revision operators are dual sides of a "drastic" behavior for the revision. These operators are representative examples that provide some "bounds" of the complete picture of GLP revision operators $GLP(\circ)$, for each KM revision operator $\circ$. Discarding such drastic behaviors may call for additional postulates in order to capture more parsimonious revision procedures in logic programming, as for instance the cardinality-based revision operator (cf. Definition 9) which is neither brave nor skeptical. Stated otherwise, it seems necessary to refine the existing properties that every rational revision operator should satisfy so that the answer sets of the revised program $\mathcal{P} \star^{\circ,f} \mathcal{Q}$ fall "between" these two extremes (i.e., between $AS(\mathcal{Q})$ and $mod(\mathcal{P} \circ \mathcal{Q})$) in the sense of set inclusion.

6 Conclusion and Perspectives

In this paper, we pursued some previous work on revision of logic programs, where the adopted approach is based on a monotonic characterization of logic programs using SE interpretations. We considered the revision of generalized logic programs (GLPs) and characterized the class of rational GLP revision operators in terms of an ordering among classical interpretations with some further conditions specific to SE interpretations. The constructive characterization we provide facilitates the comprehension of their semantic properties by drawing a clear, complete picture of GLP revision operators. Interestingly, we showed that a GLP revision operator is an extension of a rational propositional revision operator, that is, each propositional revision operator corresponds to a specific subclass of GLP revision operators. Moreover, we showed that each one of these subclasses can be embedded into a Boolean lattice, which infimum and supremum, the so-called *skeptical* and *brave* GLP revision operators, have some drastic behavior.

This work can be extended into several directions in belief change theory for logic programming. Our results make easier the improvement of the current AGM framework in the context of logic programming. Indeed, though the subclasses of skeptical and brave GLP revision operators are fully satisfactory w.r.t. the AGM revision principles, their behavior is shown to be rather trivial. This may call for additional postulates which would aim to capture more parsimonious, balanced classes of GLP revision operators. Additionally, we will investigate the case of logic program *merging* operators (merging can be viewed as a multi-source generalization of belief revision). Indeed it is not even known whether there exists a fully rational merging operator, i.e., that satisfies the whole set of postulates proposed by Delgrande *et al.* [7] for logic programs merging operators.

References

1. Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change: Partial meet contraction and revision functions. Journal of Symbolic Logic 50(2), 510–530 (1985)
2. Alferes, J.J., Leite, J.A., Pereira, L.M., Przymusinska, H., Przymusinski, T.C.: Dynamic updates of non-monotonic knowledge bases. Journal of Logic Programming 45(1-3), 43–70 (2000)
3. Cabalar, P., Ferraris, P.: Propositional theories are strongly equivalent to logic programs. Theory and Practice of Logic Programming 7(6) (2007)
4. Dalal, M.: Investigations into a theory of knowledge base revision: Preliminary report. In: Proc. of the 7th National Conference on Artificial Intelligence, AAAI 1988, pp. 475–479 (1988)
5. Delgrande, J.P., Schaub, T., Tompits, H., Woltran, S.: Belief revision of logic programs under answer set semantics. In: Proc. of the 11th International Conference on Principles of Knowledge Representation and Reasoning, KR 2008, pp. 411–421 (2008)
6. Delgrande, J.P., Schaub, T., Tompits, H., Woltran, S.: A general approach to belief change in answer set programming. CoRR, abs/0912.5511 (2009)
7. Delgrande, J., Schaub, T., Tompits, H., Woltran, S.: Merging logic programs under answer set semantics. In: Hill, P.M., Warren, D.S. (eds.) ICLP 2009. LNCS, vol. 5649, pp. 160–174. Springer, Heidelberg (2009)

8. Eiter, T., Fink, M., Sabbatini, G., Tompits, H.: On properties of update sequences based on causal rejection. Theory and Practice of Logic Programming 2(6), 711–767 (2002)
9. Eiter, T., Fink, M., Tompits, H., Woltran, S.: On eliminating disjunctions in stable logic programming. In: Proc. of the 9th International Conference on Principles of Knowledge Representation and Reasoning, KR 2004, pp. 447–458 (2003)
10. Eiter, T., Tompits, H., Woltran, S.: On solution correspondences in answer set programming. In: Proc. of the 19th International Joint Conference on Artificial Intelligence, IJCAI 2005, pp. 97–102 (2005)
11. Inoue, K., Sakama, C.: Negation as failure in the head. Journal of Logic Programming 35(1), 39–78 (1998)
12. Katsuno, H., Mendelzon, A.O.: A unified view of propositional knowledge base updates. In: Proc. of the 11th International Joint Conference on Artificial Intelligence, IJCAI 1989, pp. 1413–1419 (1989)
13. Katsuno, H., Mendelzon, A.O.: On the difference between updating a knowledge base and revising it. In: Proc. of the 2nd International Conference on Principles of Knowledge Representation and Reasoning, KR 1991, pp. 387–394 (1991)
14. Katsuno, H., Mendelzon, A.O.: Propositional knowledge base revision and minimal change. Artificial Intelligence 52(3), 263–294 (1992)
15. Konieczny, S., Pino Pérez, R.: Merging information under constraints: a logical framework. Journal of Logic and Computation 12(5), 773–808 (2002)
16. Lifschitz, V., Pearce, D., Valverde, A.: Strongly equivalent logic programs. ACM Transactions on Computational Logic 2(4), 526–541 (2001)
17. Lifschitz, V., Woo, T.Y.C.: Answer sets in general nonmonotonic reasoning (preliminary report). In: Proc. of the 3rd International Conference on Principles of Knowledge Representation and Reasoning, KR 1992, pp. 603–614 (1992)
18. Satoh, K.: Nonmonotonic reasoning by minimal belief revision. In: Proc. of the International Conference on Fifth Generation Computer Systems, FGCS 1988, pp. 455–462 (1988)
19. Turner, H.: Strong equivalence made easy: nested expressions and weight constraints. Theory and Practice of Logic Programming 3(4-5), 609–622 (2003)
20. Zhang, Y., Foo, N.Y.: Towards generalized rule-based updates. In: Proc. of the 15th International Joint Conference on Artificial Intelligence, IJCAI 1997, pp. 82–88 (1997)

Appendix: Proof Sketches

Proposition 2

(Only if part) In this proof, for every well-defined set of SE interpretations S, $\mathsf{glp}(S)$ denotes any GLP $\mathcal{P}$ such that $SE(\mathcal{P}) = S$. Let $*$ be a GLP revision operator. For every GLP $\mathcal{P}$, define the relation $\leq_{\mathcal{P}}$ over interpretations such that $\forall Y, Y' \in \Omega$, $Y \leq_{\mathcal{P}} Y'$ iff $Y \models \mathcal{P} * \mathsf{glp}(\{(Y,Y),(Y',Y')\})$. Moreover, for every GLP $\mathcal{P}$, $\forall Y \in \Omega$, let $\mathcal{P}(Y) = \{X \subseteq Y \mid (X,Y) \in SE(\mathcal{P} * \mathsf{glp}(\{(X,Y),(Y,Y)\}))\}$. We claim that $\leq_{\mathcal{P}}$ is a total preorder (this part of proof is similar to the one given for Theorem 11 in [15][5]).

[5] In the proof of Theorem 11 in [15], propositional merging operators are considered. Multi-sets of formulae (so-called *profiles*) are merged under a certain integrity constraint represented by a formula. This part of our proof is similar if one restricts ourselves to singleton profiles.

Now we show that $SE(\mathcal{P} * \mathcal{Q}) = \{(X, Y) \mid (X, Y) \in SE(\mathcal{Q}), \forall Y' \models \mathcal{Q}, Y \leq_{\mathcal{P}} Y', X \in \mathcal{P}(Y)\}$. Let us denote S the latter set and first show the first inclusion $SE(\mathcal{P} * \mathcal{Q}) \subseteq_s \mathsf{S}$. Let $(X, Y) \in SE(\mathcal{P} * \mathcal{Q})$ and let us show that *(i)* $(X, Y) \in SE(\mathcal{Q})$, *(ii)* $\forall Y' \models \mathcal{Q}, Y \leq_{\mathcal{P}} Y'$ and that *(iii)* $X \in \mathcal{P}(Y)$. (i) is direct from (RA1). For (ii), let $Y' \models \mathcal{Q}$. Since $*$ returns a GLP, $SE(\mathcal{P} * \mathcal{Q})$ is well-defined. That is, since $(X, Y) \in SE(\mathcal{P} * \mathcal{Q})$, we also have $(Y, Y) \in SE(\mathcal{P} * \mathcal{Q})$. Therefore, $\mathcal{P} * \mathcal{Q} + \mathsf{glp}(\{(Y, Y), (Y', Y')\})$ is consistent. So by (RA5) and (RA6), $(Y, Y) \in \mathcal{P} * \mathsf{glp}(\{(Y, Y), (Y', Y')\})$. Hence, $Y \leq_{\mathcal{P}} Y'$. For (iii), since $(X, Y) \in SE(\mathcal{P} * \mathcal{Q})$, $SE(\mathcal{P} * \mathcal{Q}) + \mathsf{glp}(\{(X, Y), (Y, Y)\})$ is consistent, we have $(X, Y) \in SE(\mathcal{P} * \mathsf{glp}(\{(X, Y), (Y, Y)\}))$ by (RA5) and (RA6); hence, $X \in \mathcal{P}(Y)$. Let us now show the other inclusion $\mathsf{S} \subseteq_s SE(\mathcal{P} * \mathcal{Q})$. Assume $(X, Y) \in \mathsf{S}$. So $\forall Y' \models \mathcal{Q}$, $Y \leq_{\mathcal{P}} Y'$ and $X \in \mathcal{P}(Y)$. First, from the definition of $\mathcal{P}(Y)$ and by (RA1) and (RA3), we have $Y \in \mathcal{P}(Y)$. So $Y \in \mathsf{S}$. Since $\mathsf{S} \neq \emptyset$, $\mathcal{Q}$ is consistent, thus by (RA1) and (RA3) $\exists Y_* \models \mathcal{Q}$, $Y_* \in SE(\mathcal{P} * \mathcal{Q})$. Let $\mathcal{R}_\# = \mathsf{glp}(\{(X, Y), (Y, Y), (Y_*, Y_*)\})$. So $\mathcal{P} * \mathcal{Q} + \mathcal{R}_\#$ is consistent. Then by (RA5) and (RA6), $\mathcal{P} * \mathcal{Q} + \mathcal{R}_\# = \mathcal{P} * \mathcal{R}_\#$. We have to show that $(X, Y) \in \mathcal{P} * \mathcal{R}_\#$. Assume towards a contradiction that $(X, Y) \notin \mathcal{P} * \mathcal{R}_\#$. By (RA1) and (RA3) and since $(Y_*, Y_*) \in \mathcal{P} * \mathcal{R}_\#$, we have two remaining cases: (i) $\mathcal{P} * \mathcal{R}_\# = \mathsf{glp}(\{(Y_*, Y_*)\})$. Since $\mathcal{P} * \mathcal{R}_\# + \mathsf{glp}(\{(Y, Y), (Y_*, Y_*)\})$ is consistent, by (RA5) and (RA6) we get that $\mathcal{P} * \mathsf{glp}(\{(Y, Y), (Y_*, Y_*)\}) = \mathsf{glp}(\{(Y_*, Y_*)\})$. This contradicts $Y \leq_{\mathcal{P}} Y'$. (ii) $\mathcal{P} * \mathcal{R}_\# = \mathsf{glp}(\{(Y, Y), (Y_*, Y_*)\})$. Since $\mathcal{P} * \mathcal{R}_\# + \mathsf{glp}(\{(X, Y), (Y, Y)\})$ is consistent, by (RA5) and (RA6) we get that $\mathcal{P} * \mathsf{glp}(\{(X, Y), (Y, Y)\}) = \mathsf{glp}(\{(Y, Y)\})$. This contradicts $X \in \mathcal{P}_Y$.

It is harmless to verify that all conditions (1 - 3) of the faithful assignment and conditions (a - e) of the well-defined assignment are satisfied: conditions (a) and (b) are direct from the definition of $\mathcal{P}(Y)$, conditions (1), (2), (c) and (d) come from (RA2), and conditions (3) and (e) are derived from (RA4).

(If part) We consider a faithful assignment that associates with every GLP $\mathcal{P}$ a total preorder $\leq_{\mathcal{P}}$ and a well-defined assignment that associates with every GLP $\mathcal{P}$ and every interpretation Y a set $\mathcal{P}(Y) \subseteq \Omega$, such that $\forall \mathcal{P}, \mathcal{Q}$, $SE(\mathcal{P} * \mathcal{Q}) = \{(X, Y) \mid (X, Y) \in SE(\mathcal{Q}), \forall Y' \models \mathcal{Q}, Y \leq_{\mathcal{P}} Y', X \in \mathcal{P}(Y)\}$. Let $\mathcal{P}, \mathcal{Q}$ be two GLPs and $X, Y \in \Omega$. We have to show that $SE(P * Q)$ is well-defined. Let $(X, Y) \in SE(\mathcal{P} * \mathcal{Q})$. Since $SE(P * Q)$ is a set of SE interpretations, $X \subseteq Y$. Moreover, by condition (a) of the well-defined assignment, $Y \in \mathcal{P}(Y)$, so $Y \in SE(\mathcal{P} * \mathcal{Q})$. Hence, $SE(\mathcal{P} * \mathcal{Q})$ is well-defined.

It is harmless to verify that postulates (RA1 - RA6) are satisfied: (RA1) and (RA3) are obvious from the definition of $SE(\mathcal{P} * \mathcal{Q})$, (RA2) comes from conditions (1), (2), (c) and (d), (RA4) is derived from conditions (3) and (e), and (RA5) and (RA6) hold by definition. ■

Proposition 3

Consider beforehand that $\mathcal{P} + \mathcal{Q}$ is inconsistent (the other case is trivial from Proposition 2 and postulate (RA2)). When reducing the SE interpretations to their second components, the fact that the set of all classical models of $\mathcal{P} \star^{\circ, f} \mathcal{Q}$

corresponds to the models of $\psi_{\mathcal{P}} \circ \psi_{\mathcal{Q}}$ comes from the similarities between an LP faithful assignment (cf. Definition 11) and a faithful assignment (cf. Definition 2), and from Proposition 2 and Theorem 1. Regarding all first components of SE interpretations, the correspondence between f (cf. Definition 13) and $\mathcal{P}(Y)$ (cf. conditions (a - e) of the well-defined assignment in Definition 12) can be easily seen. ■