Possibilistic DL-Lite

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Abstract. *DL-Lite* is one of the most important fragment of description logics that allows a flexible representation of knowledge with a low computational complexity of the reasoning process. This paper investigates an extension of *DL-Lite* to deal with uncertainty associated with objects, concepts or relations using a possibility theory framework. Possibility theory offers a natural framework for representing uncertain and incomplete information. It is particularly useful for handling inconsistent knowledge. We first provide foundations of possibilistic *DL-Lite*, denoted by π-*DL-Lite*, where we present its syntax and its semantics. We then study the reasoning tasks and show how to measure the inconsistency degree of a knowledge base using query evaluations. An important result of the paper is that the extension of the expressive power of *DL-Lite* is done without additional extra computational costs.

1 Introduction

Description Logics (DLs, for short) [2] are well-known logics based on first order logic, introduced for representing knowledge. Nowadays, DLs have regained an important place in various domain areas and especially in the Semantic Web. DLs provide the foundations of the Web Ontology Language (*OWL*). According to *W3C* [1] three profiles of *OWL2* are proposed as sub-languages of the full *OWL2* language, to offer important advantages in particular application scenarios. One of these profiles is *OWL2-QL* dedicated to applications that use huge volumes of data where query answering is the most important reasoning task. *OWL2-QL* is based on *DL-Lite* which is a family of tractable DLs investigated by [4]. Indeed, Knowledge Bases (KB) consistency and all DLs standard reasoning services are polynomial for combined complexity (*i.e.* the overall size of the KB) [1]. In these logics, the most important task of reasoning is answering complex queries (especially conjunctive queries) where the reasoning complexity is in *LogSpace* for data complexity (*i.e.* the size of the data) [1].

Now, in real world applications, knowledge is usually affected with uncertainty and imprecision. Recently, several works have been proposed to deal with probabilistic and non-probabilistic uncertainty [8] on one hand and to deal with fuzzy information [11] on the other hand. A particular attention was given to fuzzy

[1] http://www.w3.org/TR/owl2-overview/

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extensions of DLs (*e.g.* [18,3]) and *DL-Lite* (*e.g.* [19,12]). Besides, some works are devoted to possibilistic extensions of DLs (*e.g.* [10,8,14]) which are basically based on standard DLs reasoning services. However, there is no work on possibilistic extension of *DL-Lite* and there is no work that has been proposed to extend query answering within a possibility theory setting. This paper concerns the development of uncertainty-based *DL-Lite* using possibility theory. Possibility theory [9] is a very natural framework to deal with ordinal and qualitative uncertainty. It deals with non-probabilistic information as it is particularly appropriate when the uncertainty scale only reflects a priority relation between different pieces of information.

Possibilistic Description Logics (*Possibilistic-DLs* for short) are frameworks introduced to deal with uncertainty and to ensure reasoning under inconsistent KB. Originally, the use of possibility theory to extend DLs has been proposed by [10] then has been discussed by [8]. In these works the syntax and the semantics of DLs has been extended in the possibility theory framework by attaching to every axiom a confidence degree to encode its certainty. This confidence degree first reflects to what extent an axiom can be considered as certain (priority, important, etc) in the available knowledge. And then it is used to determine the inconsistency of a KB and to ensure inference services. However, there are no algorithms to compute inconsistency of a *Possibilistic-DLs* KB. In addition, only some inference services have been defined. Such limitation has constituted the main topics of the works proposed by [17,16] where the authors first redefine the syntax and semantics of *Possibilistic-DLs*, and then investigate several inference services that can be done on a *Possibilistic-DLs* KB. Furthermore, they provided an algorithm to compute inconsistency degree and possibilistic inference services. It has been shown that checking the consistency degree and several inference services can be done with classical DLs reasoning services through consistent sub-sets of the *Possibilistic-DLs* KB. An implementation of a reasoner called *DL-Poss*, has been provided in [13]. A deeper discussion on *Possibilistic-DLs* has been provided in [14]. Finally, it is important to point out that another method has been introduced in [6,15] for checking inconsistency of *possibilistic-DLs* as a direct extension of the tableau algorithm.

An important question addressed in this paper is : "How one can extend the expressive power of *DL-Lite,* to deal with possibilistic uncertain information, without increasing the computational cost?". This paper provides a positive answer to this question. Such a good result is possible when we restrict ourself to *DL-Lite*. Note first that most extensions of possibilistic DLs [17,16,6] all need some extra computation costs. In these existing approaches, computing inconsistency degree comes down to achieve a number of calls (at least $log_2 N$ calls, where N is the size of the uncertainty scale) to the inconsistency checking in standard (without uncertainty) DLs.

This paper departs from existing approaches and follows another direction to achieve reasoning tasks in possibilistic *DL-Lite*. The idea is to modify the inconsistency computation algorithm used in standard *DL-Lite* by simply propagating the uncertainty degrees associated with axioms. In fact, we will see that the uncertainty propagation does not generate any extra computational cost, and hence the computational complexity of possibilistic *DL-Lite* is the same as the one of *DL-Lite*.

The rest of this paper is organized as follows: Section 2 briefly recalls preliminaries on *DL-Lite*. Section 3 rephrases possibility theory framework over *DL-Lite* interpretations. Section 4 discusses the possibilistic extension of *DL-Lite*, denoted π-*DL-Lite*, where we present its syntax and its semantics. Section 5 introduces the so-called π-*negated* closure of a π-*DL-Lite* knowledge bases. Section 6 gives a method to compute inconsistency of the π-*DL-Lite* KB using query evaluations. Section 7 studies deferent possibilistic inferences. Section 8 concludes the paper.

2 DL-Lite Logic

The vocabulary of DLs is based on concepts which correspond to unary predicates to denote sets of individuals, and roles, which correspond to binary predicates, to denote binary relations among individuals. A description language is characterized by a set of constructs used to build complex concepts and roles form atomic ones and it is employed to structure a domain of interest. Each description language allows different sets of constructs. A DLs knowledge base is specified through several inclusions between concepts and roles.

In this paper, we focus on *DL-Lite* one of the most important fragment of DLs. For sake of simplicity, we only consider $DL\text{-}Lite^{H}_{core}$ (originally $DL\text{-}Lite_{R}$) that underlies *OWL2-QL* language as *DL-Lite* logic. For more details about the different logics in *DL-Lite* family see [1]. However, results of this paper are valid for other logics of the *DL-Lite* family.

The language of $DL\text{-}Lite_{core}$ is the core language for $DL\text{-}Lite_{R}$ and it is ensured by a description language defined as follow [5]:

$$\begin{array}{lllll} B \longrightarrow & A \mid \exists R & \qquad & C \longrightarrow & B \mid \neg B \\ R \longrightarrow & P \mid P^{-} & \qquad & E \longrightarrow & R \mid \neg R \end{array}$$

where A is an atomic concept, P is an atomic role, Concepts B (*resp.* C) are called basic (*resp.* complex) concepts and roles R (r*esp.* E) are called basic (*resp.* complex) roles. Note that *DL-Lite* language does not allows the use of the conjunctive and the disjunctive operators. However, one can easily add conjunctions (*resp.* disjunction) in the right-hand side (*resp.* left-hand side) of inclusion axioms. Indeed, the conjunction of the form $B \sqsubseteq C_1 \sqcap C_2$ is equivalent to the pair of inclusion axioms $B \sqsubseteq C_1$ and $B \sqsubseteq C_2$, while the disjunction of the form $B_1 \sqcup B_2 \sqsubseteq C$ is equivalent to the pair of inclusion axioms $B_1 \sqsubseteq C$ and $B_2 \sqsubseteq C$.

A *DL-Lite* knowledge base is a pair $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ where $\mathcal{T}$ is a TBox and $\mathcal{A}$ is an ABox. The $DL\text{-}Lite_{core}$ TBox is constituted by a finite set of i*nclusion axioms* of the form $B \sqsubseteq C$. Let use a_i and a_j to denote two individuals (constants), the

DL-Lite$_{core}$ ABox is constituted by a finite set of *membership assertion* on atomic concepts and on atomic roles of the form $A(a_i)$ and $P(a_i, a_j)$. The *DL-Lite$_R$* extends *DL-Lite$_{core}$* with the ability of specifying inclusion assertions between roles of the form $R \sqsubseteq E$. For more detailed description on *DL-Lite* family, see [5].

As usual in DLs, the *DL-Lite* semantics is given by an interpretation $I = (\Delta, .^I)$ which consists of a non-empty domain Δ and an interpretation function $.^I$. The function $.^I$ assigns to each individual a an element $a^I \in \Delta^I$, to each atomic concept A a subset $A^I \subseteq \Delta^I$ and to each atomic role P a subset $P^I \subseteq \Delta^I \times \Delta^I$ over the domain. Furthermore, the interpretation function $.^I$ is extended to complex concepts and roles (*e.g.* $(P^-)^I = \{(y, x) \in \Delta^I \times \Delta^I | (x, y) \in P^I\}$ and $(\exists R)^I = \{x \in \Delta^I | \exists y \in \Delta^I \textit{ such that } (x, y) \in R^I\}$).

For the TBox, we say that an interpretation I is a model of an inclusion axiom, denoted by $I \models B \sqsubseteq C$ (*resp.* $I \models R \sqsubseteq E$) iff $B^I \subseteq C^I$ (*resp.* $R^I \subseteq E^I$). For the ABox, we say that an interpretation I is a model of membership assertion, denoted by $I \models A(a_i)$ (*resp.* $I \models P(a_i, a_j)$) iff $a_i^I \in A^I$ (*resp.* $(a_i^I, a_j^I) \in P^I$). Note that we only consider *DL-Lite* with unique name assumption (*i.e.* $a_i \neq a_j$ where $i \neq j$). Thus, I is a model of knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, denoted by $I \vDash \mathcal{K}$, iff $I \vDash \mathcal{T}$ and $I \vDash \mathcal{A}$. A KB $\mathcal{K}$ is said to be consistent (or satisfiable) if it admits at least one model.

3 Possibility Distribution over DL-Lite Interpretations

Possibility theory (e.g. [9]) offers an important framework for representing and reasoning with uncertain, partial and inconsistent pieces of information. In what follows, we rephrase possibility theory framework over *DL-Lite* interpretations. Let $\mathcal{L}$ be a finite *DL-Lite* description language, Ω be a universe of discourse and $I = (\Delta, .^I) \in \Omega$ be a *DL-Lite* interpretation.

3.1 Possibility Distribution

A possibility distribution is considered as one of the main block of possibility theory. It is a mapping, denoted by π, from the universe of discourse Ω to the unit interval $[0, 1]$. It assigns to each interpretation $I \in \Omega$ a possibility degree $\pi(I) \in [0, 1]$ that represents its compatibility or consistency relative to the available knowledge. When $\pi(I) = 1$, we say that I is totally possible and it is fully consistent with the available knowledge. When $\pi(I) = 0$, we say that I is impossible and it is fully inconsistent with the available knowledge. Then, two special cases exist: a total ignorance when $\forall I \in \Omega, \pi(I) = 1$ and a complete knowledge when $\exists I' \in \Omega, \pi(I') = 1$ and $\forall I \in \Omega, I' \neq I, \pi(I) = 1$. By convention, a possibility distribution π is said to be normalized if there exists at least one totally possible interpretation, namely $\exists I \in \Omega, \pi(I) = 1$, otherwise, we say that π is sub-normalized. For two events I and I', we say that I is more consistent or compatible than I' if $\pi(I) > \pi(I')$.

3.2 Possibility and Necessity Measures

Let us consider φ be a subset of Ω. Let $\neg\varphi$ be the complementary of φ, namely $\neg\varphi = \Omega \setminus \varphi$. In standard possibility theory, given a possibility distribution π, one can define two measures from 2^{Ω} to the interval $[0, 1]$ which discriminate between the plausibility and the certainty of a subset φ. These two measure are:

Possibility Measure. A possibility measure, denoted by Π, is a function of the form $\Pi(\varphi) = max\{\pi(I) : I \in \varphi\}$

$\Pi(\varphi)$ evaluates to what extent the subset φ is compatible with the available knowledge encoded by π. When $\Pi(\varphi) = 1$, we say that φ is certainty true if $\Pi(\neg\varphi) = 0$ and we say that the φ is somewhat certain if $\Pi(\neg\varphi) \in]0, 1[$. When $\Pi(\varphi) = 1$ and $\Pi(\neg\varphi) = 1$, we say that there is a total ignorance about φ. The possibility measure satisfies the following properties for normalized possibility distributions:

$$\forall\varphi \in \Omega, \forall\psi \in \Omega, \Pi(\varphi \cup \psi) = max(\Pi(\varphi), \Pi(\psi))$$

$$\forall\varphi \in \Omega, \forall\psi \in \Omega, \Pi(\varphi \cap \psi) \leq min(\Pi(\varphi), \Pi(\psi))$$

Necessity Measure. A necessity measure, denoted by N, is a function of the form $N(\varphi) = 1 - \Pi(\neg\varphi)$

$N(\varphi)$ evaluates to what extent φ is certainty entailed from available knowledge encoded by π. When $N(\varphi) = 1$, we say that φ is certain. When $N(\varphi) \in]0, 1[$, we say that φ is somewhat certain. When $N(\varphi) = 0$ and $N(\neg\varphi) = 0$, we say that there is a total ignorance. The necessity measure satisfies the following properties for normalized possibility distributions:

$$\forall\varphi \in \Omega, \forall\psi \in \Omega, N(\varphi \cap \psi) = min(N(\varphi), N(\psi))$$

$$\forall\varphi \in \Omega, \forall\psi \in \Omega, N(\varphi \cup \psi) \geq max(N(\varphi), N(\psi))$$

Now, clearly not all subsets of Ω represent axioms of *DL-Lite* language. For instance, assume that our vocabulary is composed of one concept A and two individuals a_1 and a_2. Assume that we have two interpretations $I_1 = \left(\Delta = \{a_1, a_2\}, .^{I_1}\right)$ and $I_2 = \left(\Delta = \{a_1, a_2\}, .^{I_2}\right)$ such that $A^{I_1} = \{a_1\}$ and $A^{I_2} = \{a_2\}$. Clearly, $\{I_1, I_2\}$ does not correspond to any axiom of our *DL-Lite* language, since $\{I_1, I_2\}$ intuitively encodes the axiom $A(a_1) \vee A(a_2)$, while the disjunction operator is not allowed in *DL-Lite* language.

In the following, possibility and necessity measures are assumed to only be defined over a *DL-Lite* language. Namely, if ϕ is an axiom, we define its associated possibility measures as: $\Pi(\phi) = \max_{I \in \Omega}\{\pi(I) : I \models \phi\}$ and its associated necessity measures as: $N(\phi) = 1 - \max_{I \in \Omega}\{\pi(I) : I \not\models \phi\}$ where $I \not\models \phi$ means that I is not a model of ϕ.

4 Possibilistic DL-Lite

In this section we go one step further in the definition of possibilistic extension of *DL-Lite*, denoted by π-*DL-Lite* by presenting its syntax and how to generate a possibility distribution associated with a π-*DL-Lite* KB.

4.1 Syntax

We consider $\mathcal{L}$ the description language *DL-Lite* recalled in Section 2.

Definition 1. *A π-DL-Lite KB $\mathcal{K} = \{\langle\phi_i, \alpha_i\rangle : 1, ..., n\}$ is a set of possibilistic axioms of the form $\langle\phi, \alpha\rangle$ where ϕ is an axiom expressed in $\mathcal{L}$ and $\alpha \in \,]0, 1]$ is the degree of certainty of ϕ.*

Only somewhat certain information (namely $\alpha > 0$) are explicitly represented in π-*DL-Lite* KB. $\langle\phi, \alpha\rangle$ means that the uncertainty degree of ϕ is at least equal to α. The higher is the degree α the more important is the axiom or the fact. The degree α can be associated either with an inclusion axiom between concepts or roles (TBox), or with facts (ABox). A π-*DL-Lite* $\mathcal{K}$ will also be represented by a couple $\mathcal{K} = \langle\mathcal{T}, \mathcal{A}\rangle$ where both elements in $\mathcal{T}$ and $\mathcal{A}$ may be uncertain. Note that, if we consider $\alpha = 1$ then we represent a classical *DL-Lite* KB: $\mathcal{K}^* = \{\phi_i : \langle\phi_i, \alpha_i\rangle \in \mathcal{K}\}$.

Example 1. Let $Teacher$, *PhdStudent* and *Student* be three atomic concepts and $TeachesTo$ be an atomic role. The following possibilistic TBox $\mathcal{T}$ and the possibilistic ABox $\mathcal{A}$ will be used in the rest of the paper:

$$\begin{aligned}
&\mathcal{T} = \{\ \langle Teacher \sqsubseteq \neg Student, .8\rangle,\\
&\langle PhdStudent \sqsubseteq Student, .7\rangle,\\
&\langle PhdStudent \sqsubseteq Teacher, .9\rangle,\\
&\langle \exists teachesTo \sqsubseteq Teacher, .6\rangle,\\
&\langle \exists teachesTo^- \sqsubseteq Student, .5\rangle\}.\\
&\mathcal{A} = \{\langle Student\,(b)\,, .95\rangle\ ,\ \langle teachesTo\,(b, c)\,, 1\rangle\}.
\end{aligned}$$

In π-*DL-Lite* KB, the necessity degree attached to an axiom reflects its confidence and evaluates to what extent this axiom is considered as certain. For instance the axiom $\langle teachesTo\,(b, c)\,, 1\rangle$ states that we are absolutely certain that the "*the Teacher b teachesTo the Student c*". However the axiom $\langle PhdStudent \sqsubseteq Student, .7\rangle$ simply states that a *PhdStudent* may be a *Student* with a certainty degree equal or greater than .7.

4.2 From π-*DL-Lite* Knowledge Base to π-*DL-Lite* Possibility Distribution

The semantics of π-*DL-Lite* is given by a possibility distribution, denoted $\pi_{\mathcal{K}}$, defined over the set of all interpretations $I = \left(\Delta, .^I\right)$ of a *DL-Lite* language (see Section 3). As in standard possibilistic logic [7], given a π-*DL-Lite* knowledge base $\mathcal{K}$ the possibility distribution induced by $\mathcal{K}$ is defined as follow:

Definition 2. *For every $I \in \Omega$*

$$\pi_{\mathcal{K}}(I) = \begin{cases} 1 & if\, \forall \langle \phi_i, \alpha_i \rangle \in \mathcal{K}, I \models \phi_i \\ 1 - max\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{K} | I \not\models \phi_i\} & otherwise \end{cases}$$

where $\models$ is the satisfaction relation between *DL-Lite* formulas recalled in Section 2. $\langle \phi_i, \alpha_i \rangle \in \mathcal{K}$ means that $\langle \phi_i, \alpha_i \rangle \in \mathcal{K}$ either belongs to the TBox $\mathcal{T}$ or the ABox $\mathcal{A}$ of $\mathcal{K}$.

Example 2. (Example 1 continued) Using Definition 2, we compute the following possibility degree of three interpretations where $\Delta = \{b, c\}$:

I	$.^{I}$	$\pi_{\mathcal{K}}$
I_1	$(Student)^{I} = \{b, c\}$, $(PhdStudent)^{I} = \{b\}$, $(Teacher)^{I} = \{b\}$ $(teachesTo)^{I} = \{(b, c)\}$	.2
I_2	$(Student)^{I} = \{b, c\}$,$(PhdStudent)^{I} = \{\}$,$(Teacher)^{I} = \{\}$ $(teachesTo)^{I} = \{(b, c)\}$	.4
I_3	$(Student)^{I} = \{b\}$, $(PhdStudent)^{I} = \{\}$,$(Teacher)^{I} = \{c\}$ $(teachesTo)^{I} = \{(c, b)\}$	0

In this example, we can see that the interpretation I_1 does not satisfy $\langle Teacher \sqsubseteq \neg Student, .8 \rangle$, the interpretation I_2 does not satisfy $\langle \exists teachesTo \sqsubseteq Teacher, .6 \rangle$ and the interpretation I_3 does not satisfy $\langle teachesTo\,(b, c)\,, 1 \rangle$. Hence, no one of these interpretations is a model of $\mathcal{K}$.

A π-*DL-Lite* KB is said to be consistent if the possibility distribution $\pi_{\mathcal{K}}$ is normalized, namely there exists an interpretation I such that $\pi_{\mathcal{K}}(I) = 1$. If not, $\mathcal{K}$ is said to be inconsistent and its inconsistency degree is defined semantically as follow:

Definition 3. *The inconsistency degree of a π-DL-Lite KB, denoted by $Inc\,(\mathcal{K})$, is semantically defined as follow: $Inc\,(\mathcal{K}) = 1 - \underset{I \in \Omega}{max}\{\pi_{\mathcal{K}}(I)\}$*

If $Inc\,(\mathcal{K}) = 1$ then $\mathcal{K}$ is fully inconsistent and if $Inc\,(\mathcal{K}) = 0$ then it is consistent.

Example 3. (Example 2 continued), in fact, one can check that the inconsistency degree of $\mathcal{K}$ according to $\pi_{\mathcal{K}}$ is : $Inc\,(\mathcal{K}) = 1 - \underset{I \in \Omega}{max}\{\pi_{\mathcal{K}}(I)\} = .6$, and hence $\mathcal{K}$ is inconsistent (in fact, there is no way to find an interpretation that satisfy $\mathcal{K}$ with a degree greater than .6).

Remark 1. In propositional possibilistic logic, each possibilistic KB induces a joint possibility distribution and conversely. Although each π-*DL-Lite* KB induces a unique joint possibility distribution, the converse does not hold. Consider again the example where we only have one concept A and two individuals a_1 and a_2. Consider four interpretations I_1, I_2, I_3 and I_4 having the same domain $\Delta = \{a_1, a_2\}$ and where $A^{I_1} = \{a_1\}$, $A^{I_2} = \{a_1\}$, $A^{I_2} = \{a_1, a_2\}$ and $A^{I_4} = \emptyset$.

Assume that $\pi(I_1) = \pi(I_2) = 1$ and $\pi(I_3) = \pi(I_4) = .5$. One can check that there is no π-*DL-Lite* KB such that $\pi_{\mathcal{K}} = \pi$.

5 Possibilistic Closure in π-*DL-Lite*

Let us first point out that one can easily add conjunctions in the right side of inclusion axioms. Proposition 1 shows that a complex inclusion axiom of the form $\langle B_1 \sqsubseteq B_2 \sqcap B_3, \alpha\rangle$ can be splitted into elementary inclusion axioms that can be added to $\mathcal{K}$ without modifying its possibility distribution. Proposition 1 can be derived from Proposition 5 in [14] for general DLs.

Proposition 1. Let $\mathcal{K} = \{IP \cup \{\langle B_1 \sqsubseteq B_2 \sqcap B_3, \alpha\rangle\}, \mathcal{A}\}$ *and* $\mathcal{K}' = \{IP \cup \{ \langle B_1 \sqsubseteq B_2, \alpha\rangle, \langle B_1 \sqsubseteq B_3, \alpha\rangle\}, \mathcal{A}\}$ then $\mathcal{K}$ and $\mathcal{K}'$ induces the same possibility distribution.

Hence, the language given in Section 2 is a simplification of the one based on conjunctions used in the right side (*resp.* disjunction in left side) of inclusion axioms.

The aim of this section is to define the so-called π-*negated* closure of a π-*DL-Lite* KB. This notion is crucial for defining the concepts of consistency and inference from a π-*DL-Lite* KB. A possibilistic TBox $\mathcal{T} = \{IP, IN\}$ can be viewed as composed of positive inclusions (*PI*) of the form $\langle B_1 \sqsubseteq B_2, \alpha\rangle$ or $\langle R_1 \sqsubseteq R_2, \alpha\rangle$ and negative inclusions (*NI*) of the form $\langle B_1 \sqsubseteq \neg B_2, \alpha\rangle$ or $\langle R_1 \sqsubseteq \neg R_2, \alpha\rangle$. Conceptually, the *PI* axioms (*resp. NI* axioms) represent subsomption (*resp.* disjunction) between concepts or roles. Roughly speaking, this closure denoted π-*neg*$(\mathcal{T})$, will contain possibilistic negated axioms of the form $\langle B_1 \sqsubseteq \neg B_2, \alpha\rangle$ or $\langle R_1 \sqsubseteq \neg R_2, \alpha\rangle$ that can be derived from $\mathcal{T}$. The set π-*neg*$(\mathcal{T})$ is obtained by applying a set of rules that extends the ones defined in standard *DL-Lite* when axioms are weighted with uncertainty degrees.

At the beginning π-*neg*$(\mathcal{T})$ is set to an empty set.

Rule 1. *Let* $\mathcal{T} = \{IP, IN\}$ *then* $IN \subseteq \pi - neg(\mathcal{T})$.

This rule simply means that negated axioms explicitly stated in $\mathcal{T}$ can be trivially derived from $\mathcal{T}$.

Example 4. (Example 1 continued): Using Rule 1, we add $\langle Teacher \sqsubseteq \neg Student, .8\rangle$ as NI to $\pi - neg(\mathcal{T})$.

Rule 2. *If* $\langle B_1 \sqsubseteq B_2, \alpha_1\rangle \in \mathcal{T}$ *and* $\langle B_2 \sqsubseteq \neg B_3, \alpha_2\rangle \in \pi - neg(\mathcal{T})$ *or* $\langle B_3 \sqsubseteq \neg B_2, \alpha_2\rangle \in \pi - neg(\mathcal{T})$ *then add* $\langle B_1 \sqsubseteq \neg B_3, min(\alpha_1, \alpha_2)\rangle$ *to* $\pi - neg(\mathcal{T})$.

Rule 3. *If* $\langle R_1 \sqsubseteq R_2, \alpha_1\rangle \in \mathcal{T}$ *and* $\langle R_2 \sqsubseteq \neg R_3, \alpha_2\rangle \in \pi - neg(\mathcal{T})$ *or* $\langle R_3 \sqsubseteq \neg R_2, \alpha_2\rangle \in \pi - neg(\mathcal{T})$ *then add* $\langle R_1 \sqsubseteq \neg R_3, min(\alpha_1, \alpha_2)\rangle$ *to* $\pi - neg(\mathcal{T})$.

Rules 2 and 3 simply state that transitivity holds with a weight equal to the least weight of premises axioms.

Rule 4. *if* $\langle R_1 \sqsubseteq R_2, \alpha_1\rangle \in \mathcal{T}$ *and* $\langle \exists R_2 \sqsubseteq \neg B, \alpha_2\rangle \in \pi - neg\,(\mathcal{T})$ *or* $\langle B \sqsubseteq \neg\exists R_2, \alpha_2\rangle \in \pi - neg\,(\mathcal{T})$ *then add* $\langle \exists R_1 \sqsubseteq \neg B, min\,(\alpha_1, \alpha_2)\rangle$ *to* $\pi - neg\,(\mathcal{T})$.

Rule 5. *If* $\langle R_1 \sqsubseteq R_2, \alpha_1\rangle \in \mathcal{T}$ *and* $\left\langle \exists R_2^- \sqsubseteq \neg B, \alpha_2\right\rangle \in \pi - neg\,(\mathcal{T})$ *or* $\left\langle B \sqsubseteq \neg\exists R_2^-, \alpha_2\right\rangle \in \pi - neg\,(\mathcal{T})$ *then add* $\left\langle \exists R_1^- \sqsubseteq \neg B, min\,(\alpha_1, \alpha_2)\right\rangle$ *to* $\pi - neg\,(\mathcal{T})$.

Rule 6. *If* $\langle R \sqsubseteq \neg R, \alpha\rangle \in \pi - neg\,(\mathcal{T})$ *or* $\langle \exists R \sqsubseteq \neg\exists R, \alpha\rangle \in \pi - neg\,(\mathcal{T})$ *or* $\langle \exists R^- \sqsubseteq \neg\exists R^-, \alpha\rangle \in \pi - neg\,(\mathcal{T})$ *then add* $\langle R \sqsubseteq \neg R, \alpha\rangle$ *and* $\langle \exists R \sqsubseteq \neg\exists R, \alpha\rangle$ *and* $\langle \exists R^- \sqsubseteq \neg\exists R^-, \alpha\rangle$ *to* $\pi - neg\,(\mathcal{T})$.

Proposition 2. *Let* $\mathcal{T} = \{IP, IN\}$ *and* $\pi - neg\,(\mathcal{T})$ *be the closure of* $\mathcal{T}$ *obtained using Rules (1-6). Then* $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ *and* $\mathcal{K}^{'} = \{\mathcal{T} \cup \pi - neg\,(\mathcal{T}), \mathcal{A}\}$ *induce the same possibility distribution, namely* $\forall I$, $\pi_{\mathcal{K}}\,(I) = \pi_{\mathcal{K}'}\,(I)$.

Example 5. From the $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ of Example 1, one can check that applying Rule 1 - Rule 6 gives the following $\pi - neg\,(\mathcal{T})$ where :

$$
\begin{aligned}
&\pi - neg\,(\mathcal{T}) = \{\langle Teacher \sqsubseteq \neg Student, .8\rangle,\\
&\langle PhdStudent \sqsubseteq \neg Teacher, .7\rangle,\\
&\langle PhdStudent \sqsubseteq \neg Student, .8\rangle,\\
&\langle \exists teachesTo \sqsubseteq \neg Student, .6\rangle,\\
&\langle \exists teachesTo^- \sqsubseteq \neg Teacher, .5\rangle\}\\
&\mathcal{A} = \{\langle Student\,(b)\,, .95\rangle\ ,\ \langle teachesTo\,(b,c)\,, 1\rangle\}
\end{aligned}
$$

6 Checking Inconsistency

From now on, $\pi - neg\,(\mathcal{T})$ denotes the result of applying Rules 1-6 until reaching the closure (namely, no negated axioms can be added using Rules 1-6). An important result is that computing inconsistency of $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ comes down to compute inconsistency degree of $\mathcal{K}^{'} = \{\pi - neg\,(\mathcal{T}), \mathcal{A}\}$.

Proposition 3. *Let* $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ *and let* $\mathcal{K}^{'} = \{\pi - neg\,(\mathcal{T}), \mathcal{A}\}$ *then* $Inc\,(\mathcal{K}) = Inc\,(\mathcal{K}')$.

Proposition 3 is important since it provides a way to compute the inconsistency degree of a π-*DL-Lite* KB. Indeed, verifying inconsistency of $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ is reduced to verifying the inconsistency of $\mathcal{K}^{'} = \{\pi - neg\,(\mathcal{T}), \mathcal{A}\}$. A contradiction is presented when a same individual (*resp.* two individuals) belongs to two negated concepts (*resp.* negated roles) (*i.e. NI* in $\pi - neg\,(\mathcal{T})$). Then, checking inconsistency is done by means a set of weighted queries issued from $\pi - neg\,(\mathcal{T})$. Subsection 6.1 formalizes this concept of weighted queries while subsection 6.2 provides the algorithm to compute inconsistency degrees using a set of weighted queries.

6.1 Weighted Queries

The idea is to evaluate over $\mathcal{A}$ suitable weighted queries expressed from $\pi - neg\,(\mathcal{T})$ to exhibit whether the ABox $\mathcal{A}$ contains or not contradictions and to compute the inconsistency degree. To obtain the set of weighted queries q_c from $\pi - neg(\mathcal{T})$, we propose a translation function ψ. ψ has an argument a possibilistic *NI* $\langle B_1 \sqsubseteq \neg B_2, \alpha\rangle$ or $\langle R_1 \sqsubseteq \neg R_2, \alpha\rangle$ and produces a weighted first order formula.

Definition 4. *ψ is a function that transforms all axioms in $\pi - neg(\mathcal{T})$ to weighted query q_c:*

- $\psi(\langle B_1 \sqsubseteq \neg B_2, \alpha\rangle) = \langle (x, \gamma_1, \gamma_2).\lambda_1(x, \gamma_1) \wedge \lambda_2(x, \gamma_2), \alpha\rangle$ *with*
  - $\lambda_i(x, \gamma_i) = A_i(x, \gamma_i)$ *if* $B_i = A_i$
  - $\lambda_i(x, \gamma_i) = \exists y_i.P_i(x, y_i, \gamma_i)$ *if* $B_i = \exists P_i$
  - $\lambda_i(x, \gamma_i) = \exists y_i.P_i(y_i, x, \gamma_i)$ *if* $B_i = \exists P_i^-$
- $\psi(\langle R_1 \sqsubseteq \neg R_2, \alpha\rangle) = \langle (x, y, \gamma_1, \gamma_2).\nu_1(x, y, \gamma_1) \wedge \nu_2(x, y, \gamma_2), \alpha\rangle$ *with*
  - $\nu_i(x, y, \gamma_i) = P_i(x, y, \gamma_i)$ *if* $R_i = P_i$
  - $\nu_i(x, y, \gamma_i) = P_i(y, x, \gamma_i)$ *if* $R_i = P_i^-$

Intuitively, if $\langle B_1 \sqsubseteq \neg B_2, \alpha\rangle$ belongs in $\pi - neg(\mathcal{T})$, then a query associated to $B_1 \sqsubseteq \neg B_2$ simply means return all $\{B_1(x, \gamma_1), B_2(x, \gamma_2)\}$ that are present in the ABox.

Example 6. From Example 5, we obtain the following weighted queries using Definition 4:

$$q_c = \langle (x, \gamma_1, \gamma_2).Teacher(x, \gamma_1) \wedge Student(x, \gamma_2), .8\rangle$$
$$q_c = \langle (x, \gamma_1, \gamma_2).PhdStudent(x, \gamma_1) \wedge Teacher(x, \gamma_2), .7\rangle$$
$$q_c = \langle (x, \gamma_1, \gamma_2).PhdStudent(x, \gamma_1) \wedge Student(x, \gamma_2), .8\rangle$$
$$q_c = \langle (x, \gamma_1, \gamma_2).(\exists y.teachesTo(x, y, \gamma_1)) \wedge Student(x, \gamma_2), .6\rangle$$
$$q_c = \langle (x, \gamma_1, \gamma_2).(\exists y.teachesTo(y, x, \gamma_1)) \wedge Teacher(x, \gamma_2), .5\rangle$$

6.2 An Algorithm for Computing Inconsistency Degrees

Now, we provide the algorithm *Inconsistency*, which takes as input a $\mathcal{K}' = \{\pi - neg(\mathcal{T}), \mathcal{A}\}$ and computes $Inc(\mathcal{K})$, the inconsistency degree of $\mathcal{K}$.

Algorithmus 1. $Inconsistency(\mathcal{K})$

Input: $\mathcal{K}' = \{\pi - neg(\mathcal{T}), \mathcal{A}\}$
Output: $Inc(\mathcal{K})$

```
cont := {0}
for all (φ_i, α_i) ∈ π − neg(T); i = 1..|π − neg(T)| do
    (q_c, α_q) := (ψ(φ_i, α_i))
    if Eval(q_c, A) ≠ ∅ then
        β := max(Eval(q_c, A))
        if β > α_q then
            cont := cont ∪ {α_q}
        else
            cont := cont ∪ {β}
return max(cont)
```

In this algorithm, the set *cont* stores the inconsistency degrees founded during the algorithm. $Eval\,(q_c, \mathcal{A})$ denotes the evaluation of a weighted query q_c over $\mathcal{A}$ obtained by transforming $\pi - neg\,(\mathcal{T})$ with the function given in Definition 4. For (a, α_i) and (a, α_j) presented in a query result, we only consider one individual $(a, min\,(\alpha_j, \alpha_j))$. $\beta = max\,(Eval\,(q_c, \mathcal{A}))$ represents the maximum weight of all tuples in $Eval\,(q_c, \mathcal{A})$. At this point, if the weight of the query is less than β (*i.e.* $\alpha_q < \beta$) then the contradiction is issued from the query and implicitly form the TBox corresponding axioms. Otherwise (*i.e.* $\alpha_q \geq \beta$) then the contradiction is issued from the result of the query evaluation and implicitly from ABox assertions. Finally, the inconsistency degree of $\mathcal{K}$ ($Inc\,(\mathcal{K})$) is the maximum of all contradiction degrees of the *cont*. In case of consistency, the "if part" of the algorithm (lines 4-9) is never used, and the algorithm returns the value 0 (namely, $Inc\,(\mathcal{K}) = 0$). This explain why *cont* is initialized to $\{0\}$(line 1).

Example 7. From Example 6, only the query $\langle (x, \gamma_1, \gamma_2).\,(\exists y.teachesTo\,(x, y, \gamma_1)) \wedge Student\,(x, \gamma_2)\,, .6\rangle$ presents a contradiction: $\langle q_c, .6\rangle = \{(b, .95, 1)\}$. Thus, the inconsistency degree of the KB is $Inc\,(\mathcal{K}) = .6$.

We now provide two propositions that show on one hand that our π-*DL-Lite* extends standard *DL-Lite* and on other hand that the computational complexity of Algorithm 1 is the same as the one in standard *DL-Lite.*

Proposition 4. *Let* $\mathcal{K}_s = \{\mathcal{T}_s, \mathcal{A}_s\}$ *be a standard DL-Lite. Let* $\mathcal{K}_\pi = \{\mathcal{T}_\pi, \mathcal{A}_\pi\}$ *where* $\mathcal{T}_\pi$ *(resp.* $\mathcal{A}_\pi$*) is defined from* $\mathcal{T}_s$ *(resp.* $\mathcal{A}_s$*) by assigning a degree 1 to each axiom of* $\mathcal{T}_s$ *(resp.* $\mathcal{A}_s$*), namely :* $\mathcal{T}_\pi = \{\langle \phi_i, 1\rangle : \phi_i \in \mathcal{T}_s\}$ *and* $\mathcal{A}_\pi = \{\langle \phi_i, 1\rangle : \phi_i \in \mathcal{A}_s\}$*. Then* $\mathcal{K}_s$ *is consistent (in the sense of standard DL-Lite) iff* $Inc\,(\mathcal{K}_\pi) = 0$ *and* $\mathcal{K}_s$ *is inconsistent iff* $Inc\,(\mathcal{K}_\pi) = 1$.

Proposition 5. *The complexity of Algorithm 1 is the same as the one used in standard DL-Lite ([5], section 3.3 , Theorem 26)*

To see why proposition 5 holds it is enough to see the differences between Algorithm 1 and the one used in ([5], section 3.1.3) for standard *DL-Lite.* The first remarks, concerns the returned result. On our algorithm, results of queries are weighted while in standard DL-Lite, they are not. This does not change the complexity. The difference concerns lines 4-9, where in standard *DL-Lite* algorithm they are replaced by:

```
1: if Eval(q_c, A) ≠ ∅ then
2:     return True
3: else
4:     return False
```

It is easy first to see that in case of consistency both algorithms perform same steps, because the " if part of the algorithm" is never considered. Now in case of inconsistency, the worst case appears when the whole "loop" is used, namely inconsistency appears with the last element of $\pi - neg\,(\mathcal{T})$. In both cases let $\mathcal{A}$ be the result of the evaluation of $Eval\,(q_c)$. This needs at least $O\,(|\mathcal{A}|)$ steps. Algorithm 1 (contrary to the algorithm in standard *DL-Lite* [5]) computes also

$max\{\alpha_i : \langle\phi_i, \alpha_i\rangle \in \mathcal{A}\}$ which needs again $O(|\mathcal{A}|)$. Since trivially, $O(2|\mathcal{A}|) = O(|\mathcal{A}|)$, our algorithm has the same complexity as in standard *DL-Lite.* Hence we increase the expressive power of *DL-Lite* while keeping the complexity as low as the one of standard *DL-Lite.*

7 Inference in Possibilistic DL-Lite

In this section, we first present classical inference problem (*i.e.* subsumption and instance checking). First, we define an $\alpha - cut$ of $\mathcal{T}$ (*resp.* $\mathcal{A}$ and $\mathcal{K}$), denoted $\mathcal{T}_{>\alpha}$ (resp. $\mathcal{A}_{>\alpha}$, $\mathcal{K}_{>\alpha}$), a sub base of $\mathcal{T}$ (*resp.* $\mathcal{A}$ and $\mathcal{K}$) composed of formulas having a weight greater than alpha (α). In possibilistic *DL* inference problems such as subsumption and instance checking can be reduced to the task of computing the inconsistency degree of the KB [14]. We present in the following inference services in π-*DL-Lite*:

- Flat subsumption: Let $\mathcal{T}$ be a possibilistic TBox, B_1 and B_2 be two general concepts, A be an atomic concept not appearing in $\mathcal{T}$, and a be a constant. Then, $\mathcal{K} \models_\pi B_1 \sqsubseteq B_2$ iff the KB $\mathcal{K}_1 = \{\mathcal{T}_1, \mathcal{A}_1\}$ where $\mathcal{T}_1 = \mathcal{T}_{>Inc(\mathcal{K})} \cup \{\langle A \sqsubseteq B_1, 1\rangle, \langle A \sqsubseteq \neg B_2, 1\rangle\}$ and $\mathcal{A}_1 = \{\langle A(a), 1\rangle\}$ is inconsistent whatever is the degree ($\exists\alpha > 0$ such that $Inc(\mathcal{K}_1) = \alpha$).
- Subsumption with a necessity degree: Let $\mathcal{T}$ be a possibilistic TBox, B_1 and B_2 be two general concepts, A be an atomic concept not appearing in $\mathcal{T}$, and be a a constant. Then, $\mathcal{K} \models_\pi \langle B_1 \sqsubseteq B_2, \alpha\rangle$ iff the KB $\mathcal{K}_1 = \{\mathcal{T}_1, \mathcal{A}_1\}$ where $\mathcal{T}_1 = \mathcal{T}_{\geq\alpha} \cup \{\langle A \sqsubseteq B_1, 1\rangle, \langle A \sqsubseteq \neg B_2, 1\rangle\}$ and $\mathcal{A}_1 = \{\langle A(a), 1\rangle\}$ is inconsistent where $Inc(\mathcal{K}_1) = \alpha$ and $\alpha > Inc(\mathcal{K})$.
- Flat instance checking: Let $\mathcal{K}$ be a π-*DL-Lite* KB, B be a concept, A be an atomic concept not appearing in $\mathcal{T}$, and a be a constant. Then, $\mathcal{K} \models_\pi B(a)$ iff the KB $\mathcal{K}_1 = \{\mathcal{T}_1, \mathcal{A}_1\}$ where $\mathcal{T}_1 = \mathcal{T}_{>Inc(\mathcal{K})} \cup \{\langle A \sqsubseteq \neg B, 1\rangle\}$ and $\mathcal{A}_1 = \{\langle A(a), 1\rangle\}$ is inconsistent (whatever is the degree).
- Instance checking with a necessity degree: Let $\mathcal{K}$ be a π-*DL-Lite* KB, B be a concept, A be an atomic concept not appearing in $\mathcal{T}$, and a be a constant. Then, $\mathcal{K} \models_\pi \langle B(a), \alpha\rangle$ iff the KB $\mathcal{K}_1 = \{\mathcal{T}_1, \mathcal{A}_1\}$ where $\mathcal{T}_1 = \mathcal{T}_{>\alpha} \cup \{\langle A \sqsubseteq \neg B, 1\rangle\}$ and $\mathcal{A}_1 = \{\langle A(a), 1\rangle\}$ is inconsistent where $Inc(\mathcal{K}_1) = \alpha$ and $\alpha > Inc(\mathcal{K})$.

KB consistency is verified by Algorithm *Inconsistency*, presented above, where $Inc(\mathcal{K}) = 0$. Hence, all these basic inferences can be obtained using Algorithm 1. Note the difference between flat subsumption (*resp.* instance checking) and subsumption with a necessity degree (*resp.* instance checking with a necessity degree) is that in the first case we only check whether the subsumption holds whatever is the degree, while is the second case, subsumption should be satisfied to some degree.

8 Conclusions and Future Works

In this paper, we investigated a possibilistic extension of *DL-Lite.* We first introduced the syntax and the semantics of such extensions. We provided properties of π-*DL-Lite* and show how to compute the inconsistency degree of π-*DL-Lite* KB having a complexity identical to the one used in standard *DL-Lite*. This is done by defining π-*DL-Lite* negative closure that extends the one of standard *DL-Lite*. Then, we gave a method to check consistency for π-*DL-Lite*. Finally, we discussed inference problems. In particular we distinguish different inference tasks depending whether we use flat inferences or weighted inferences. Results of this paper are important since they extended *DL-Lite* languages to deal with priority (between TBox axioms or ABox axioms) or uncertainty without changing the computational complexity. Future works concern the revision of π-*DL-Lite* KB in presence of new pieces of information.

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