

# Phase Average Visualization of a Finite Cylinder Wake as Predicted by Large Eddy Simulation

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**Abstract** The current paper reports a large eddy simulation (LES) of turbulent flow over a finite-height square cylinder mounted normal to a ground plane. The cylinder aspect ratio is  $AR = 3$  and the Reynolds number based on the cylinder width and inlet velocity is  $Re = 500$ . The flow field is complex, since the flow over the top of the cylinder interacts with the flow along the ground plane to create a complicated wake structure. The wake is characterized by a velocity field that changes rapidly in both direction and magnitude. Phase averaging based on the Strouhal number reveals a wake structure with quasi-periodic features that is much different from the structure suggested by the mean vorticity field.

**Keywords** Wake · Vortex structure · Finite square cylinder · Phase average · Large eddy simulation

## 1 Introduction

Turbulent flow over external bodies which is characterized by shear-layer separation is a complex phenomenon, e.g., it involves both the large-scale periodic motions associated with vortex shedding and smaller-scale motions associated with the fluid turbulence. In order to predict the aerodynamic forces on the body, as well as the potential for sound generation within the flow, the instantaneous and mean structure of the wake need to be understood. In this paper, we focus on the special case of a square cylinder, which is finite in length and mounted perpendicular to a

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ground plane. The flow approaching the cylinder is characterized by a boundary layer of thickness,  $\delta$ . An important characteristic of the flow is the aspect ratio  $AR = H/D$  (where  $H$  is the height and  $D$  is the width of the cylinder), which can be regarded as a measure of the slenderness of the cylinder. For a finite cylinder, the structure of the wake is influenced by both the flow over the top of the free end of the cylinder, as well as the boundary layer developing on the ground plane. Recent studies of finite (circular) cylinder flows (Sumner et al. 2004; Palau-Salvador et al. 2010) have indicated that at small aspect ratios, the wake structure becomes distinct from that at larger aspect ratios. The measurements of Wang and Zhou (2009) indicate both symmetric and staggered shedding, and they have also postulated a vortical structure for the cylinder wake. In contrast, Bourgeois et al. (2011) conclude that the wake structure should be understood in terms of the phase-average behavior. For the case of a square prism of  $AR = 4$ , they have identified a so-called half-loop structure. The present paper reports a large eddy simulation (LES) of a low Reynolds number ( $Re = UD/\nu = 500$ ) flow over a square cylinder of aspect ratio  $AR = 3$ . The approaching boundary layer was thin and laminar. Two- and three-dimensional visualization of the phase-average structures was used to investigate the wake. It confirms the presence of quasi-periodic vortical structures, which is surprising at this low aspect ratio.

## 2 Numerical Methods

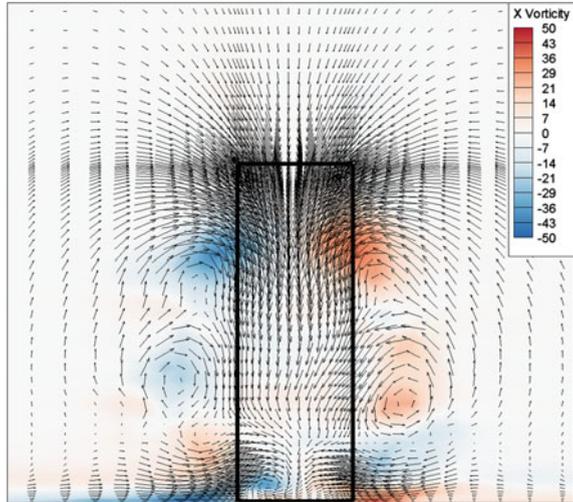
The mathematical model consisting of the filtered Navier–Stokes equations was discretized using the finite-volume method. A localized dynamic Smagorinsky model was used for the subgrid-scale stress terms. The resultant equation set was solved using a fractional step method, where the convective and diffusive terms were advanced in time using the Crank–Nicolson method, and a multi-grid (MG) method was used to efficiently solve the linear algebraic equation set. The initial velocity field was first developed in time to obtain a realistic turbulent flow. After this, velocity data were collected and processed to obtain the time-averaged or mean velocity field, the resolved-scale fluctuations, and the phase-average fields.

## 3 Results and Discussion

Sampling of the velocity field in the wake of the cylinder indicated a Strouhal number of  $St = 0.128$ . Based on the period inferred from the Strouhal number, the velocity field behind the cylinder was processed in terms of five distinct phases ( $N = 1$  to 5) to give a picture of the periodicity of the flow. The results were averaged over five consecutive periods.

Figure 1 shows the mean streamwise vorticity and in-plane velocity vectors in a vertical plane section located at  $X/D = 2$  (where  $X$  is the streamwise coordinate).

**Fig. 1** Mean streamwise vorticity and velocity vectors for a vertical section located at  $X/D = 2$



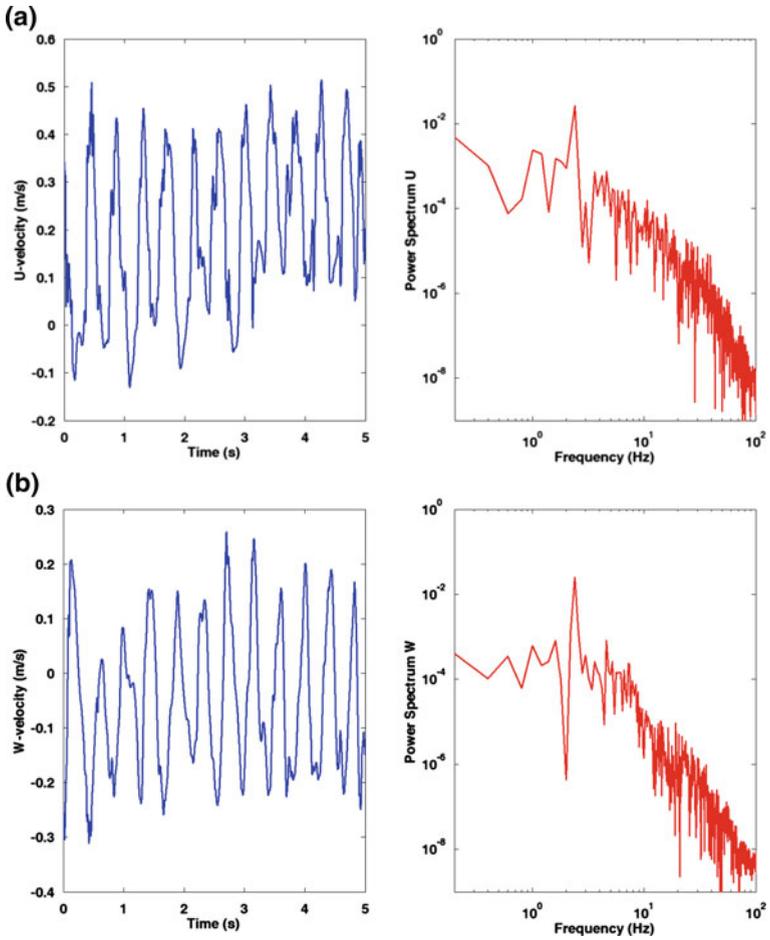
Two pairs of tip vortices and much weaker base vortices are evident, and the flow field is almost symmetric about the center-plane.

Figure 2 shows a temporal trace of the instantaneous streamwise and crosswise velocity components and the associated power spectra at a reference point located in the cylinder wake. The velocity exhibits a strong periodic component with a frequency of  $f = 2.4$  Hz.

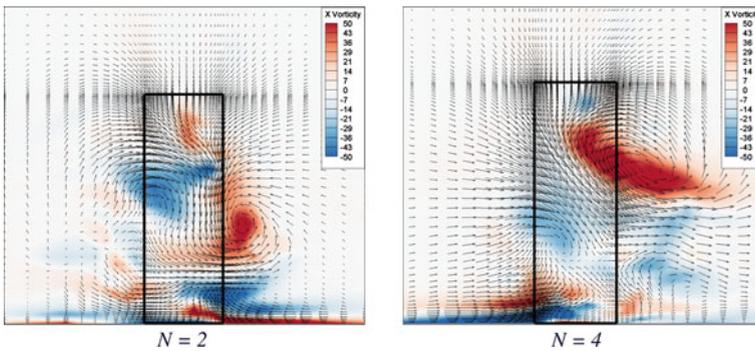
Figure 3 shows the phase-averaged streamwise vorticity and velocity vectors in the same vertical plane as in Fig. 1. Two phases are shown,  $N = 2, 4$ . The velocity and vorticity fields are observed to sweep across the wake creating strongly asymmetric flow patterns. Similar to the mean velocity field shown in Fig. 1, Fig. 3 indicates that a strong downwash flow exists between the counter-rotating streamwise structures for each phase.

Figure 4 shows the velocity vectors and associated in-plane spanwise vorticity at a height of  $Y/H = 1/4$  for phases  $N = 2, 4$ . The velocity vectors indicate patterns which are typical of vortex shedding, e.g., the dominant vortex immediately behind the cylinder is located at a different transverse location for each phase.

Figure 5 shows the three-dimensional structure of the wake (also for  $N = 2, 4$ ) visualized by the contours of the second invariant of the velocity gradient tensor. The flow structure is clearly asymmetric for each phase, with the initially vertical vortex tubes being reoriented in the streamwise direction. There is some evidence of the half-loop structure as described by Bourgeois et al. (2011). The vortex patterns observed in Fig. 3 are seen to coincide with the streamwise structures shown in Fig. 5.



**Fig. 2** Trace and power spectrum of **a** streamwise and **b** cross-stream velocity at a point in the near-wake of the cylinder



**Fig. 3** Phase-averaged streamwise vorticity and velocity vectors at  $X/D = 2$  for phases  $N = 2, 4$

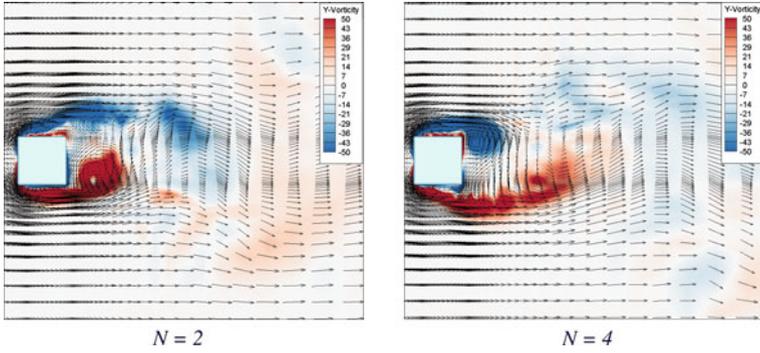


Fig. 4 Phase-averaged vertical vorticity and velocity vectors at  $Y/H = 0.25$  for phases  $N = 2, 4$

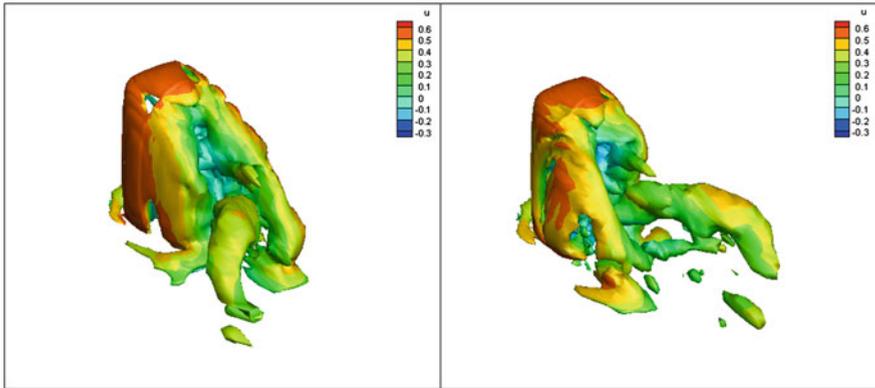


Fig. 5 Visualization of the phase-averaged vorticity field using the second invariant ( $Q = 200$ ), flooded by streamwise velocity for phases  $N = 2, 4$

## 4 Conclusions

LES predictions of low Reynolds number flow over a finite-height square cylinder mounted on a ground plane are reported in this paper. The flow field predicted for this sub-critical aspect ratio of  $AR = 3$  reveals a complex wake structure. Phase-average analysis of the cylinder wake reveals patterns of quasi-periodic vortical structures. The originally vertical-oriented vortex elements formed by the roll-up of the shear layers shed from the lateral sides of the cylinder evolve to create streamwise structures that connect back to the structures on the opposite side of the wake. The phase-averaged velocity field in transverse plane sections through the wake indicates strong downwash flow between pairs of dominant vortical streamwise structures that change their location over the course of a single phase-average cycle.

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