Towards the Numerical Modelling of Floating Offshore Renewables

A. Viré, J. Xiang, M. D. Piggott, C. J. Cotter, J-P. Latham and C. C. Pain

Abstract Two finite element models are coupled, with the aim of computing the mutual interactions between fluids and floating solids. The fluid and solid domains are discretised differently in space and time, and at every time step, the solid mesh is mapped onto the fluid mesh. The effect of the solid on the fluid dynamics, and vice versa, is modelled through a volumetric penalty force added to the momentum balances of the fluids and solids. A novel algorithm ensures that the action-reaction principle is satisfied at the discrete level. The coupled models are used to simulate uniform flow past a wind turbine, which is represented as a fixed actuator disc. Preliminary results on a floating pile also demonstrate the applicability of the models to fully coupled simulation of a floating spar. This work is a first-step towards the fully coupled modelling of floating wind turbines.

Keywords Fluid-structure interactions · Immersed-body approach · Finite element method · Actuator disc · Floating pile · Floating wind turbines

1 Introduction

The numerical modelling of fluid–structure interactions is important in the context of offshore renewables. This work targets floating wind turbines, where the turbine is supported by a floating pile moored to the seabed. Such devices are attractive in

A. Viré (✉) · J. Xiang · M. D. Piggott · J-P. Latham · C. C. Pain
Department of Earth Science and Engineering, Imperial College London,
SW7 2AZ London, UK
e-mail: avire@imperial.ac.uk

M. D. Piggott
Grantham Institute for Climate Change, Imperial College London, SW7 2AZ London, UK

C. J. Cotter
Department of Aeronautics, Imperial College London, SW7 2AZ London, UK

deep seas (typically deeper than 50 m), where bottom-mounted foundations are too expensive. The numerical modelling of floating wind turbines is very challenging because of the dynamic interaction between the moving air–water and fluid–solid interfaces, and the presence of both rigid (i.e. the wind turbine) and deformable (i.e. the moorings) solids. Computer models are, however, attractive in studying such coupled fluid–solid problems, because they can analyse different configurations while limiting expensive laboratory or onsite testing. In this work, two finite element models are coupled in order to represent the mutual interactions between fluids and floating solids. The use of separate models, based on different spatial and temporal discretisations, allows for the mesh resolution to be tailored to the specific needs of the fluids and solids, respectively.

2 Numerical Models

Consider a solid V_s immersed in a fluid domain V_f. In this work, the interactions between fluids and solids are modelled by filling the regions covered by solids with the surrounding fluid and relaxing the flow to the behaviour of the solids in those regions. The Navier–Stokes equations describing the fluid dynamics are thus modified in two ways. First, they are expressed in terms of monolithic fields, which are defined over the extended domain $V = V_f \cup V_s$. In particular, the monolithic velocity field is given by $u = \alpha_f u_f + \alpha_s u_s$, where $\alpha_f = V_f/V$ and $\alpha_s = V_s/V$ are the volume fractions of the fluid and solid, respectively. Second, the monolithic velocity is relaxed to the solid velocity in V_s. This is achieved by adding a volumetric penalty force to the right-hand side of the momentum balance. Considering incompressible fluids, the fluid-dynamics equations are given by:

$$\nabla \cdot u = 0, \tag{1}$$

and

$$\rho_f \frac{\partial u}{\partial t} + \rho_f (u \cdot \nabla) u - \nabla p + \nabla \cdot \left(2\mu \overline{\overline{S}}^* \right) + F_f + B, \tag{2}$$

where p is the pressure field, μ is the dynamic viscosity of the fluid, $\overline{\overline{S}}^*$ is the deviatoric part of the strain-rate tensor $S_{ij} = \partial_j u_i + \partial_i u_j$, B represents external body forces (e.g. buoyancy), and F_f is the penalty force. The latter is non-zero close to the fluid–solid interface only, and is expressed as

$$F_f = \beta \alpha_f \alpha_s \left(u_s - u_f \right) = \beta(\widehat{u}_s - \alpha_s u). \tag{3}$$

where $\widehat{u}_s = \alpha_s u_s$, $\beta = \rho_f/\Delta t$ is a factor which dictates how fast the fluid and solid velocities relax to one another at the interface, and Δt is the time step. The magnitude of the relaxation factor is driven solely by inertial effects. The extension to viscous effects is the subject of further work.

Equations 1–2 are solved using the open-source model 'Fluidity-ICOM' (Pain et al. 2001). This fluid/ocean dynamics model uses unstructured finite elements and has the capability of adapting the mesh dynamically in time. The fluid mesh is therefore modified, either to track accurately the fluid–solid interfaces when a solid moves, or to refine the resolution around specific flow features which develop in the wake of a fixed solid.

The finite-element model 'Y3D' (Xiang et al. 2009) solves the solid-dynamics equations on a separate solid mesh, which covers the regions Vs. The solid-dynamics equations are expressed as

$$\mathcal{L}d_s + F_{\text{int}} = F_{\text{ext}} + F_c + F_s, \tag{4}$$

where d_s denotes the solid displacement, $\mathcal{L}$ is an operator dependent on the velocity gradient, F_{int} stands for the internal forces, F_c is the contact force when multiple solids impact on each other, and F_{ext} is the external force including the surface traction force and all the body forces other than the action–reaction force exerted by the fluid. A novel algorithm was implemented in order to ensure that the action–reaction principle is satisfied at a discrete level, when coupling 'Fluidity-ICOM' and 'Y3D'(Viré et al. 2012). The models are applied to two different cases, which are relevant for the modelling of floating wind turbines: (1) a fixed wind turbine modelled as a thin actuator-disc, and (2) a pile floating at the interface between air and water.

3 Results

3.1 Uniform Flow Past a Fixed Actuator Disc

Wind turbines can be parameterised by thin discs, which exert a constant thrust force on the flow. The force is derived from actuator disc theory (Conway 1995), which assumes that the flow is homogeneous, incompressible, steady, frictionless and that no external force acts on the fluid upstream or downstream of the rotor. This formalism is used to model a fixed wind turbine subjected to a uniform flow. As described in the previous section, two meshes are used: the fluid mesh covers the whole computational domain, while the solid mesh only discretises the disc. Because the disc is fixed, the solid model does not solve the solid-dynamics equations. A solid-concentration field is defined on the fluid mesh, in order to identify the region occupied by the disc. This field is obtained by projecting a unitary field from the solid to fluid mesh. A thrust force acting on the fluid is spread uniformly across the disc, and is given by:

$$T = \frac{1}{2}\rho C_T A_d u_0^2, \tag{5}$$

where ρ is the fluid density, C_T is the thrust coefficient of the wind turbine, A_d is the frontal area of the disc, and u_0 is the far-upstream velocity. The size of the computational domain is $25D \times 10D \times 10D$, where D is the disc diameter. The disc centre is placed at a distance $5D$ from the domain inlet and sides. The disc normal is aligned with the flow. The Reynolds number is set to $Re_D = u_0D/v = 10^3$, where v is the kinematic viscosity of the fluid. Three values of the thrust coefficients are considered ($C_T = 0.2; 0.45; 0.7$). The fluid mesh adapts every $T = 1$ s, in order to refine the regions of high curvatures in the velocity and pressure fields. The total number of mesh nodes at steady state is of the order of 10^6. The time step is such that the Courant number is fixed at 0.3. The streamwise evolution of the streamwise velocity at the disc centreline is shown in Fig. 1 where the numerical results are indicated by dashed lines and the continuous lines represent the semi-analytical solution (Conway 1995). The velocity deficit induced by the disc is well predicted by the present algorithm. The discrepancy between numerical and theoretical results in the far-wake is explained by the role of the fluid viscosity in the numerical simulation. No wake recovery is observed in the theoretical results, which are derived in the inviscid approximation. The effect of the resolution on the numerical results is under further investigation.

4 Dynamics of a Floating Pile

The dynamic response of a pile is analysed when placed at the interface between air ($\rho_1 = 1.2\,\text{kg} \cdot \text{m}^{-3}, \mu_1 = 18.27\mu\text{Pa} \cdot \text{s}$) and water ($\rho_2 = 1000\,\text{kg} \cdot \text{m}^{-3}, \mu_2 = 1.002\,m\text{Pa} \cdot \text{s}$). The pile density is uniform and equals half the water density. It is centred in a box of edge length $20D$, where D is the pile diameter and $4D$ is its length. Initially, the fluids are at rest and the pile lies vertically across the fluid interface (Fig. 2 left). As time evolves, the pile moves in heave before falling into water. The pile eventually reaches an equilibrium position, that is horizontal and half submerged in water, in accordance with Archimedes' principle.

Fig. 1 Streamwise evolution of the streamwise velocity (non-dimensionalised by the far-upstream velocity) for an actuator disc of diameter D, centred at $x = 0$, and three values of thrust coefficient C_T. The lines represent the semi-analytical solution (Conway 1995), while the dashed lines show the numerical results
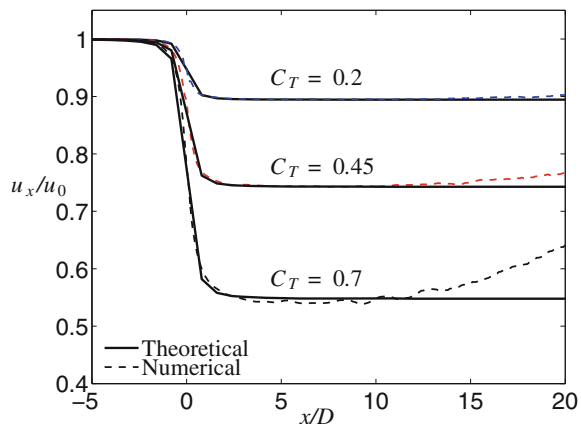
Fig. 2 Snapshots of the dynamics of a floating pile, at three different instants in time, when placed at the interface between air and water

5 Conclusions

This work presents a novel algorithm to couple two finite element models and demonstrates their ability to simulate: (1) the flow past a fixed wind turbine represented as an actuator disc, and (2) the dynamics of a cylindrical pile floating at the interface between air and water. Results in both cases are shown to be in good agreement with theoretical predictions, and the effect of different parameters will be the subject of further work. This study is a first step towards the fully coupled numerical simulation of floating solids, applicable to the next generation of offshore renewables.

References

Conway J (1995) Analytical solutions for the actuator disk with variable radial distribution of load. J Fluid Mech 297:327–355

Pain C, Umpleby A, de Oliveira C, Goddard A (2001) Tetrahedral mesh optimisation and adaptivity for steady-state and transient finite element calculations. Comput Methods Appl Mech Engrg 190:3771–3796

Viré A, Xiang J, Milthaler F, Farrell P, Piggott M, Latham JP, Pavlidis D, Pain C (2012) Modelling of fluid-solid interactions using an adaptive-mesh fluid model coupled with a combined finite-discrete element model. Ocean Dyn 62:1487–1501

Xiang J, Munjiza A, Latham JP (2009) Modelling of fluid-solid interactions using an adaptive-mesh fluid model coupled with a combined finite-discrete element model. Int J Numer Meth Engng 79:946–978