Practical Considerations about Error Analysis for Discrete Event Simulations Model

Giovanni Davoli[1], Peter Nielsen[2], Gabriele Pattarozzi[1], and Riccardo Melloni[1]

[1] Department of Mechanical and Civil Engineering (DIMeC), University of Modena and Reggio Emilia; via Vignolese 905, 41100, Modena, Italy
[2] Department of Production, Aalborg University, Fibigerstrede 16, DK 9220 Aalborg, Denmark
giovanni.davoli@unimore.it

Abstract. The purpose of making efficient and flexible manufacturing systems is often related to the possibility to analyze the system considering at the same time a wide number of parameters and their interactions. Simulation models are proved to be useful to support and drive company management in improving the performances of production and logistic systems. However, to achieve the expected results, a detailed model of the production and logistic system is needed as well as a structured error analysis to guarantee results reliability. The aim of this paper is to give some practical guide lines in order to drive the error analysis for discrete event stochastic simulation model that are widely used to study production and logistic system.

Keywords: discrete event, simulation model, error analysis, stochastic model.

1   Introduction

Stochastic, discrete events, simulation models are widely used to study production and logistic system. Apart from the development, one of the main problem of this approach is to perform the error analysis on the outputs of the simulation model. Simulation experiments are classified as either terminating or non-terminating as far as the goal of the simulation is concerned (Law and Kelton, 2000), (Fishman, 2000).

If we limit our interests on non-terminating simulation, the error analysis can be split into two different parts. The first part consists of individuating the initial transient period and the confidence interval of the outputs. The second part consists of estimating how the transient period and the outputs confidence interval varies when the initial model scenario is changed. The first part of the problem is widely studied, Kelton (1983-1989), Schruben (1982-1983), Welch (1982), Vassilacopoulos (1989) White (1997), and many methods are provided to determinate the transient period often related to output stability, that can be quantified in different ways. Between the proposed techniques Mean Squared Pure Error method, Mosca et al. (1985-1992), should be reminded as a practical method useful to determinate both transient period and confidence interval. On the other hand the second part of error analysis problem is not commonly addressed directly as reported in the recent work of Sandikc (2006) that tries to fill the gap for the initial transient period for simulation model addressing

production lines. The variance of outputs confidence interval between different scenario is often faced with the hypothesis that it is normally distributed around a central value used in the reference scenario according with the basic theory of statistics (Box et al. 2013). But in many practical cases there is no evidences that this hypothesis is correct and, moreover, the significance of central value, for the reference scenario, is lost. In fact in some recent simulation handbook (Chung 2004) the advice to quantify the confidence interval for all different simulated scenario is given.

2   Purpose

The aim of this paper is to give some practical guidelines in order to drive the error analysis for discrete event stochastic simulation models. The paper is focused on the study of confidence interval variance related to the variance of simulated scenario. Nowadays, in many practical applications, the calculation potential is large enough to perform "long" simulation run in order to assure to exceed the initial transient period. Much more important is to determinate the confidence interval for the outputs in different simulated scenario, because overestimate or underestimate these confidence intervals can drive analysts towards a wrong interpretation of the results.

3   Methodology

To address the aim of the paper a quite simple discrete event simulation model is considered and the MSPE (1) is used to estimate outputs confidence interval. Then the simulation are performed according to different scenario and the variance of confidence interval is studied for different outputs.

$$MSPE_i = \frac{\sum_{j=1}^{r}(y_{ji}-Y_i)^2}{r-1} *$$ (1)

$*$ i: day; y_{ji}: output value at day i, replication j; Y_i: output mean at day i, on r replications.

This paper is grounded on a discrete events simulation model reproducing a re-order point logistic system, in particular a single-item fixed order quantity system also known as: Economic Order Quantity (EOQ) model. The economic order quantity (EOQ), first introduced by Harris (1913), and developed by Brown (1963) and Bather (1966) with stochastic demand, is a well-known and commonly used inventory control techniques reported in a great variety of hand book, for example: Tersine (1988) and Ghiani (2004). The notation used in this paper is illustrated in table 1.

Table 1. Symbol and definitions

Symbol	Unit	Definition
N	Day	Number of days for simulation
D_i	Unit/day	Mean demand per day in units
Lt	Day	Mean lead time in day
C_o	Euro/order	Single order cost in euro
C_s	Euro/ unit*year	Stock cost in euro per unit per year
SS	Unit	Safety stocks in unit

3.1   Simulation Model

The simulation model was developed according with the standard EOQ model for single item. A set of stochastic functions, developed in SciLab environment, are used to generate the demand that activates the model. The simulation model was tested performing standard EOQ model with normal distributed demand (where σ_d is demand standard deviation) and normal distributed lead time (where σ_t is lead time standard deviation). The parameters set used in the reference scenario are illustrated in table 2.

Table 2. Used parameters set

Parameter	Set value
D_i	1.000,00
σ_d	300,00
Lt	7,00
σ_t	2,00
C_o	1.000,00
C_s	1,00
Imposed SL	0,95

To evaluate model performances, in terms of achieved service level, a set of 4 Key Performance Indicators (KPI) is defined. The used KPI are illustrated in table 3.

Table 3. Used KPI

KPI	Unit	Definition
SL1	%	1-Number of stock-out in days per day
SL2	%	1-Number of stock-out in units per day
SL3	%	1-Number of stock-out in units per day during lead time
SL4	%	1-Number of stock-out event during lead time period

3.2   Design of the Experiments

To investigate the influence of different parameters on confidence intervals four factors are considered. These four factors are:

- Demand distribution;
- Lead time distribution;
- Ratio C_o/C_s;
- SS, safety stocks.

A full factorial experiment with three levels is used in this paper. Four factors and three levels give $3^4 = 81$ combinations. For each combination a number of 5 replications were conducted for a number of 405 simulations. The three settings for the four factors are shown in table 4.

Table 4. Factors setting

	Low (-1)	Mean (0)	High (+1)
D	Normal distribution, mean = 1,000 units/day, standard deviation = 300 units/day	Uniform distribution, minimum = 500 units/day, maximum = 1,500 units/day	Exponential distribution, mean = 1,000 units/day
Lt	Normal distribution, mean = 7 day, standard deviation = 2 day	Uniform distribution, minimum = 1day, maximum = 13 day	Exponential distribution, mean = 7 day
C_o/C_s	100	1.000	1.900
SS	0 units	1.000 units	2.000 units

4   Findings

The presented experiments are evaluated in terms of stability of the results and confidence interval width for all considered KPI. The simulations are conducted for a length of 1.000 days and this guarantee the stability of outputs for all KPI. Initial transient period length varies according with different parameters set and the variance is more significant for certain KPI, as shown in figure 1.
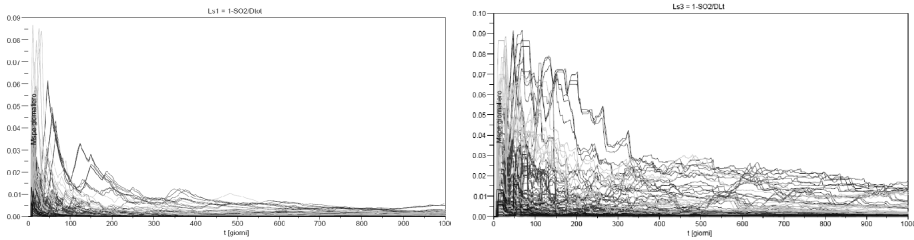

Fig. 1. MSPE for KPI SL1 and SL3

To evaluate the significance of confidence intervals the results are presented for each KPI as the ratio between half interval and the mean for each KPI. Confidence half intervals are calculated for a 95% level of significance according with (2).

$$I_{\frac{1}{2}} = Z_{\frac{\alpha}{2}} \frac{\sigma_{KPIn}}{\sqrt{r}} \quad {}^{*} \tag{2}$$

$* Z_{\alpha/2}$ $normalized$ $standard$ $varible, \sigma_{KPIn}$ $standard$ $deviation$ for $KPIn$ and r is $replication$ $number.$

Table 5. ANOVA test P-value results, codes: 0 '***' 0.001 '**' 0.01 '*' 0.05

Factors	LS1		LS2		LS3		LS4	
Demand distribution	3,15E-14	***	4,99E-14	***	4,85E-15	***	0,0002638	***
Lead time distribution	< 2.2e-16	***	< 2.2e-16	***	< 2.2e-16	***	0,3792613	
Safety Stocks	0,004364	**	0,013121	*	0,0001933	***	8,62E-09	***
Ratio Co/Cs	0,132156		0,126539		< 2.2e-16	***	3,49E-07	***
Demand dist.: Lead time dist.	< 2.2e-16	***	< 2.2e-16	***	4,03E-16	***	0,1436708	
Demand dist.: Safety Stocks	0,639589		0,952472		0,8427638		0,9926262	
Demand dist.: Ratio Co/Cs	0,016213	*	0,031195	*	0,0091386	**	0,0912483	
Lead time dist.: Safety Stocks	0,570304		0,801933		0,0424879	*	0,2119648	
Lead time dist.: Ratio Co/Cs	0,025063	*	0,009775	**	< 2.2e-16	***	0,1682513	
Safety Stocks: Ratio Co/Cs	0,92423		0,918936		0,9432251		0,8817591	

The ANOVA test reveals that the considered factors have different impact on confidence interval. Demand and lead time distribution have a very strong effects in comparison with the other parameters and even their interaction is important, as shown in in figure 2 for SL1.
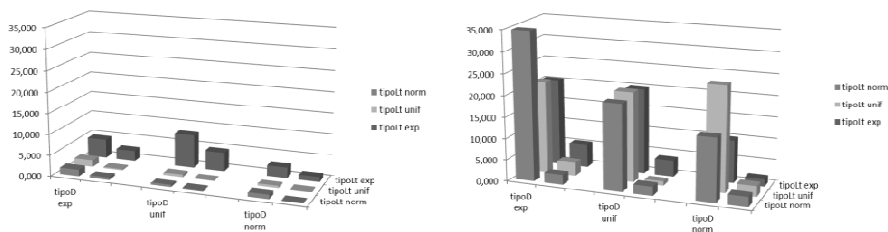

Fig. 2. Confidence half interval (min – max), in terms of %, for SL1and SL4

5   Conclusions

The case study presented here can be used to make some practical considerations to support error analysis for discrete event simulation models. First, a "long" simulation period, in order to pass the initial transient period, is relatively easy to set, even if different behavior have been observed for different KPI. Second, the initial transient period and the related confidence interval depend in a very different way by the considered parameters. In particular, for numeric parameters, the hypothesis that confidence interval variance is normal distributed around a central value calculated in

the reference scenario is almost verified. On the other hand, when the studied parameters are not numerical, for example distribution type as in the considered case study, the confidence interval must be re-calculated in each scenario because the variance could be high and the interaction are almost unpredictable. So, in practice, the effort to check the confidence interval related to discrete event simulation should be done when the modified parameters are not simply numeric. This kind of analysis, thanks to the actual computational resource, is not prohibitive in terms of time when we manage a rather simple model.

6   Limitation and Further Work

The number of replications for each scenario provided in the DOE is fixed, a deeper study about this aspect should be investigate.

References

1. Bather, J.A.: A continuous time inventory model. J. Appl. Prob. 3, 538–549 (1966)
2. Box, G.E.P., Hunter, W.E., Hunter, J.S.: Statistics for experimenters. John Wiley & Sons (1978)
3. Brown, R.G.: Smoothing, Forecasting and Prediction of Discrete Time Series. Prentice-Hall, Englewood Cliffs (1963)
4. Chung, A.C.: Simulation Modeling Handbook – A Pratical Approach. Industrial and Manufacturing Engineering Series, Series Editor. CRC Press (2004)
5. Fishman, G.S.: Discrete Event Simulation: Modeling, Programming, and Analysis. Springer (2001)
6. Ghiani, G., Laporte, G., Musmanno, R.: Introduction to logistics systems planning and control. John Wiley & Sons Ltd., West Sussex (2004)
7. Harris, F.W.: Howmany parts to make at once. Factory, The Magazine of Management 10(2), 135–136 (1913); 152, reprinted in Operations Research 38(6) (November-December 1990)
8. Kelton, W.D.: Random initialization methods in simulation. IIE Transactions 21(4), 355–367 (1989)
9. Kelton, W.D., Law, A.M.: A new approach for dealing with the startup problem in discrete event simulation. Naval Research Logistics Quarterly 30, 641–658 (1983)
10. Law, A.M., Kelton, W.D.: Simulation Modeling and Analysis, 3rd edn. McGraw-Hill (2000)
11. Mosca, R., Giribone, P.: Teoria degli esperimenti e simulazione. Quaderni di gestione degli impianti industriali, Università di Genova (1985)
12. Mosca, R., Giribone, P., Schenone, M.: Integrated management of a bishuttle FMS using discrete/stochastic simulator. Computer-Integrated Manufacturing Systems 5(2) (1992)
13. Sandıkc, B., Sabuncuoglu, I.: Analysis of the behavior of the transient period in non-terminating simulations. European Journal of Operational Research 173, 252–267 (2006)
14. Schruben, L.W.: Detecting initialization bias in simulation output. Operations Research 30, 569–590 (1982)
15. Schruben, L.W., Singh, H., Tierney, L.: Optimal test for initialization bias in simulation output. Operations Research 31, 1167–1178 (1983)

16. Tersine, R.J.: Principles of inventory and materials management. Elsevier Science Publishing Co.,Inc., North-Holland (1988)
17. Vassilacopoulus, G.: Testing for initialization bias in simulation output. Simulation 52, 151–153 (1989)
18. Welch, P.D. A graphical approach to the initial transient problem in steady-state simulations. In: Proceedings of the 10th IMACS World Congress on Systems, Simulation, and Scientific Computation, Montreal, pp. 219–221 (1982)
19. White Jr., K.P.: An effective truncation heuristic for bias reduction in simulation output. Simulation 69(6), 323–334 (1997)