Chapter 9
The Optimization of the Bandpass Lengths in the Multi-Bandpass Problem

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Abstract The Bandpass problem has applications to provide a cost reduction in design and operating telecommunication network. Given a binary matrix $A_{m\times n}$ and a positive integer B called the Bandpass length, a set of B consecutive non-zero elements in any column is called a Bandpass. No two bandpasses in the same column can have common rows. The general Bandpass Problem consists of finding an optimal permutation of rows of the matrix A that produces the maximum total number of bandpasses having the same given bandpass length B in all columns. The Multi-Bandpass problem includes different bandpass lengths B_j in each column j of the matrix A, where $j = 1, 2, \cdots, n$. In this paper, we propose an extended formulation for the Multi-Bandpass problem. A given B_j may not be always efficient bandpass lengths for the communication network. Therefore, it is important to find an optimal values of the bandpass lengths in the Multi-Bandpass problem. In this approach, the lengths in each destination are defined as z_j and we present a model to find the optimal values of z_j. Then, we calculate the approximate solution of this model using genetic algorithm for the problem instances which are presented in an online library.

Keywords Combinatorial optimization · Bandpass problem · Telecommunication · Genetic algorithm

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9.1 Introduction

Since the beginning of the 21st century, usage of the internet, digital tv broadcasts and GSM networks have been dramatically increasing. Rapid growth in the number of users leads to the need for more effective communication techniques. Manufacturers research and develop to overcome these kinds of problems. Recent studies focus on data transmission speed and capacity increase in reasonable costs.

The Bandpass problem and the Multi-Bandpass problem (MBP) are problems which are aimed to reduce the costs of communication [2].

This work is organized as follows. In Sect. 9.2, we give the definition and a brief history of the bandpass problem. In Sect. 9.3, we present the mathematical model of the problem with a constant bandpass length *B*. In Sect. 9.4, we introduce another mathematical model for optimizing bandpass length *B* which is a decision variable in the model. In Sect. 9.5, we extend the bandpass problem to the MBP and give a mathematical model of it. In Sect. 9.6, we propose a new model consists of finding the optimal bandpass lengths. In Sect. 9.7, we improve genetic algorithms and present our experimental results. In Sect. 9.8, we give some concluding remarks.

9.2 The Bandpass Problem

Today, wavelengths multiplexing technology for fiber optic cables is a major milestone to increase transmission capacity and speed. This technology is called dense wavelength division multiplexing (DWDM) and provides a platform to exploit the huge capacity of optical fiber. DWDM increases the number of communication channels within a fiber-optic cable, thereby letting service providers obtain much more bandwidth without installing a new cable. The 101 Tb/s transmission, based on the wavelength division multiplexing (WDM) of 370 wavelengths each having a speed of 273 Gb/s, is the highest ever reported in the optical transmission field by NEC in 2011 [12].

An add-drop multiplexer (ADM) is one of the most important elements in a fiber optic network. An ADM is a device that can add, block, pass or redirect various wavelengths in a fiber optic network. Each ADM facilitates flows on some wavelengths to exit the cable according to their paths. In each ADM, special cards control each wavelength; they may either pass through the ADM or may be dropped at their destination. An ADM can be programmed to drop consecutive wavelengths by one special card. Thus, *n* consecutive wavelengths in the fiber optic cable can be dropped in a station using only one card instead of *n* cards.

Not only developing new technologies are important, but reducing the cost of these systems is crucial to ensure widespread use of them.

There are several wavelengths which are used by a source vertex to carry data to destinations in a communication network. In addition, some of these wavelengths are dropped at the intermediate stations. If the wavelengths are not ordered properly, we might encounter an inefficient network in terms of cost.

The bandpass problem is an optimization problem that finds an optimal permutation of wavelengths, and thus it provides an opportunity to reduce the number of cards to be used in the optical communication networks. This problem is first proposed by Babayev and Bell in 2004. Then it is proved to be NP-hard. In [3, 9, 10], Integer programming models of the bandpass problem are developed and some heuristic polynomial algorithms are presented. A library of the bandpass problem which includes the optimum and the best known solutions of 90 instances is published [1]. The MBP that includes several bandpass lengths is modeled and some approximation algorithms are given [6, 7]. In [4, 5], several genetic algorithms, in which initial population is generated randomly or using heuristic algorithms are improved for different models of the bandpass problem.

9.3 The Mathematical Model of the Bandpass Problem

Let A be a binary matrix of size $m \times n$ that represents the flow of data from the source to n destinations using m wavelengths. Such a matrix is shown in Table 9.1. The matrix is defined as follows:

$$A = [a_{ik}], a_{ik} = \begin{cases} 1, & \text{data in } \lambda_i \text{ is transmitted to destination } k\ , \\ 0, & \text{otherwise,} \end{cases}$$

where $i = 1, 2, \cdots, m$, $k = 1, 2, \cdots, n$.

We mentioned in the previous section that an ADM can drop consecutive wavelengths using one special card. Consecutive wavelengths form a bandpass. The number of wavelengths in a bandpass is called bandpass length (B). Every non-zero entry of the network flow matrix can be included in only one bandpass. This is because of that a wavelength can not be dropped in a station by two cards.

Table 9.1 Network flow matrix

	1^{st} station	2^{nd} station	$\cdots$	n^{th} station
λ_1	1/0	1/0	$\cdots$	1/0
λ_2	1/0	1/0	$\cdots$	1/0
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
λ_m	1/0	1/0	$\cdots$	1/0

In order to minimize the number of cards used in the communication network, we should maximize the number of bandpasses. Before giving the model of the problem, let us define the decision variables as follows:

$$x_{ik}, y_{kj} \in 0, 1,\ i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m,$$

$$x_{ik} = \begin{cases} 1, & \text{if row } \ i \ \text{ is relocated to position } k\ , \\ 0, & \text{otherwise,} \end{cases}$$

$$y_{kj} = \begin{cases} 1, & \text{if row } k \text{ is the first row of a bandpass in column } j\,, \\ 0, & \text{otherwise.} \end{cases}$$

The Bandpass problem can be modeled mathematically as follows:

$$\max \sum_{j=1}^{n} \sum_{k=1}^{M-B+1} y_{kj},$$

$$\text{s.t.} \begin{cases} \sum\limits_{k=1}^{m} x_{ik} = 1, \ \sum\limits_{i=1}^{m} x_{ik} = 1, \ i = 1, \cdots, m, \ k = 1, \cdots, m, \\ \sum\limits_{i=k}^{k+B-1} y_{ij} \leq 1, \ j = 1, \cdots, n, \ k = 1, \cdots, m-B+1, \\ B \cdot y_{kj} \leq \sum\limits_{i=k}^{M-B+1} \sum\limits_{r=1}^{m} a_{rj} x_{ri}, \ j = 1, 2, \cdots, n, \ 1 \leq k \leq M-B+1. \end{cases}$$

In this model, there are $2m + 2n(m - B + 1)$ constraints [11].

9.4 The Mathematical Model to Optimize Bandpass Length in the Bandpass Problem

In Sect. 9.3, B (bandpass length) is fixed and given as an input in the model of the Bandpass problem, but this constant length may not be so efficient for the communication network. Therefore, it is considered another model called "the mathematical model to optimize bandpass length in the bandpass problem" [8]. In this model, bandpass length is defined as a decision variable z and its optimal value is determined. We first introduce the variables for this model.

$$x_{ik}, y_{kj} \in 0,1, \ i = 1, 2, \cdots, m, \ j = 1, 2, \cdots, n, \ k = 1, 2, \cdots, m, \ z > 1, z \in Z,$$

$$x_{ik} = \begin{cases} 1, & \text{if row } i \text{ is relocated to position } k\,, \\ 0, & \text{otherwise,} \end{cases}$$

$$y_{kj} = \begin{cases} 1, & \text{if row } k \text{ is the first row of a bandpass in column } j\,, \\ 0, & \text{otherwise.} \end{cases}$$

The mathematical model to optimize bandpass length in the bandpass problem as follows:

$$\max \ z \cdot \sum_{j=1}^{n} \sum_{k=1}^{M-z+1} y_{kj},$$

$$\text{s.t.}\begin{cases}\sum_{k=1}^{m} x_{ik}=1,\ \sum_{i=1}^{m} x_{ik}=1,\ i=1,2,\cdots,m,\ k=1,2,\cdots,m,\\ \sum_{i=k}^{k+z-1} y_{ij}\leq 1,\ j=1,2,\cdots,n,\ k=1,2,\cdots,m-z+1,\\ z\cdot y_{kj}\leq \sum_{i=k}^{k+z-1}\sum_{r=1}^{m} a_{rj}x_{ri},\ j=1,2,\cdots,n,\ 1\leq k\leq m-z+1.\end{cases}$$

9.5 The Mathematical Model of the Multi-Bandpass Problem

In the models given in Sect. 9.3 and Sect. 9.4, there is only one bandpass length for all destinations in the network. But ADMs which are placed in each destinations or stations may be programmed for different bandpass lengths. In this model, we define a bandpass length B_i for each destination point i. we first introduce some notation for this model.

$$x_{ik}, y_{kj} \in 0,1,\ i=1,2,\cdots,m,\ j=1,2,\cdots,n,\ k=1,2,\cdots,m,$$

B_j is the bandpass length for column j,

$$x_{ik}=\begin{cases}1, & \text{if row } i \text{ is relocated to position } k\,,\\ 0, & \text{otherwise,}\end{cases}$$

$$y_{kj}=\begin{cases}1, & \text{if row } k \text{ is the first row of a bandpass in column } j\,,\\ 0, & \text{otherwise.}\end{cases}$$

The mathematical model of the Multi-bandpass problem is as follows:

$$\max \sum_{j=1}^{n}\sum_{k=1}^{m-B_j+1} y_{kj},$$

$$\text{s.t.}\begin{cases}\sum_{k=1}^{m} x_{ik}=1,\ i=1,2,\cdots,m,\\ \sum_{i=1}^{m} x_{ik}=1,\ k=1,\cdots,m,\\ \sum_{i=k}^{k+B_j-1} y_{ij}\leq 1,\ j=1,2,\cdots,n,\ k=1,2,\cdots,m-B_j+1,\\ B_j\cdot y_{kj}\leq \sum_{i=k}^{M-B_j+1}\sum_{r=1}^{m} a_{rj}x_{ri},\ j=1,2,\cdots,n,\ 1\leq k\leq M-B_j+1.\end{cases}$$

9.6 A New Mathematical Model to Optimize Bandpass Lengths in the Multi-Bandpass Problem

In the previous model given in Sect. 9.5, bandpass lengths B_i are fixed and given as inputs. But this constant lengths may not be so efficient for the communication network. Therefore, we propose another model the so called the mathematical model to optimize bandpass lengths in the Multi-bandpass problem. We define the bandpass lengths in each destination (or station) by a decision variable z_i and we focus on to create a model finding their optimal values.

9.6.1 Determination the Objective Function

Let $A_{m\times n}$ be a given binary matrix defined as follows, where $i = 1,2,\cdots,m$ and $k = 1,2,\cdots,n$.

$$A = [a_{ik}], a_{ik} = \begin{cases} 1, & \text{data in } \lambda_i \text{ is transmitted to destination } k, \\ 0, & \text{otherwise.} \end{cases}$$

Let m_j be a sum of ones in column j such that:

$$m_j = \sum_{i=1}^{m} a_{ij}, \ j = 1,2,\cdots,n.$$

Bandpass length z_j in a column j must be chosen so that the sum of the total number of possible bandpasses and the total number of $1's$ which are not included in a bandpass is the minimum in the matrix. Then, we can write the objective function as follows:

$$\begin{aligned} \min F &= \sum_{j=1}^{n} \sum_{i=1}^{m-z_j+1} y_{ij} + \sum_{j=1}^{n} \left(m_j - \sum_{i=1}^{m-z_j+1} z_j y_{ij} \right) \\ &= \sum_{j=1}^{n} m_j + \sum_{j=1}^{n} \left(\sum_{i=1}^{m-z_j+1} y_{ij} - \sum_{i=1}^{m-z_j+1} z_j y_{ij} \right) \\ &= M + \sum_{j=1}^{n} \sum_{i=1}^{m-z_j+1} (y_{ij} - z_j y_{ij}) = M + \sum_{j=1}^{n} \sum_{i=1}^{m-z_j+1} (1 - z_j) y_{ij} \\ &= M + \sum_{j=1}^{n} (1 - z_j) \sum_{i=1}^{m-z_j+1} y_{ij} = M - \sum_{j=1}^{n} (z_j - 1) \sum_{i=1}^{m-z_j+1} y_{ij}, \end{aligned}$$

z_j is a decision variable for an optimal bandpass length in column j, $\sum_{j=1}^{n} \sum_{i=1}^{m-z_j+1} y_{ij}$ is the number of total bandpasses in the matrix, $\sum_{j=1}^{n}(m_j - \sum_{i=1}^{m-z_j+1} z_j y_{ij})$ is the number of all remaining $1's$ which are not included in a bandpass in the matrix. We can easily see if $z_j = 1$ for all $j = 1,\cdots,n$ then $\min F = M$. Therefore, we suppose that $z_j > 1$ for all $j = 1,2,\cdots,n$. As it can be seen in the last form of the objective function F, M is a constant value. Hence, if we want to find the minimum value of the function F, we need to calculate the maximum value of the function $F' = \sum_{j=1}^{n}(z_j - 1)\sum_{i=1}^{m-z_j+1} y_{ij}$. Then, we can find the value of the function F in the following formula: $F = M - F'$.

9.6.2 Boolean Integer Programming Model

Now, we propose the mathematical model to optimize bandpass lengths in the Multi-bandpass problem. We first introduce the variables for this model.

$$x_{ik}, y_{kj} \in 0,1,\ i=1,2,\cdots,m,\ j=1,2,\cdots,n,\ k=1,2,\cdots,m,$$
$$z_j \in Z \text{ is the bandpass length for column } j,\ \text{where } z_j > 1,$$
$$x_{ik} = \begin{cases} 1, & \text{if row } i \text{ is relocated to position } k\ , \\ 0, & \text{otherwise,} \end{cases}$$
$$y_{kj} = \begin{cases} 1, & \text{if row } k \text{ is the first row of a bandpass in column } j\ , \\ 0, & \text{otherwise.} \end{cases}$$

Boolean integer programming model is as follows:

$$\max \sum_{j=1}^{n} (z_j - 1) \sum_{i=1}^{m-z_j+1} y_{ij},$$
$$\text{s.t.} \begin{cases} \sum\limits_{k=1}^{m} x_{ik} = 1, \sum\limits_{i=1}^{m} x_{ik} = 1,\ i=1,2,\cdots,m,\ k=1,2,\cdots,m, \\ \sum\limits_{i=k}^{k+z_j-1} y_{ij} \le 1,\ j=1,2,\cdots,n,\ k=1,2,\cdots,m-z_j+1, \\ z_j \cdot y_{kj} \le \sum\limits_{i=k}^{k+z_j-1} \sum\limits_{r=1}^{m} a_{rj} x_{ri},\ j=1,2,\cdots,n,\ 1 \le k \le m-z_j+1. \end{cases}$$

9.7 Genetic Algorithm and Computational Tests

The solution of the MBP is a permutation of the input matrix of size mxn and it is easy to adapt for using genetic algorithms. Therefore, we improve a genetic algorithm (GA) having three crossover and four mutation operators to solve the MBP. In the GA, chromosomes of initial set of population are created randomly and then the best bandpass lengths in each column are determined for each chromosome. Then, the chromosomes are sorted in increasing order by the fitting values which mean the value of objective function of the mathematical model. In the beginning of the GA, initial population size (*ps*), crossover rate (*cr*), mutation rate (*mr*), crossover number (*cn*) and mutation number (*mn*) are determined as below:
m : the number of rows in the matrix;
n : the number of columns in the matrix;
d : the density of the matrix;
$ps = (n \cdot m)/2$;
$cr = 0.9$;
$mr = 1 - cr$;

$cn = cr \cdot ps$;
$mn = mr \cdot ps$.

Two main crossover methods named as C1 and C2 have been used. The first crossover method C1 selects two parents using roulette wheel selection. This operation is repeated cn times in the population set and the new solutions (offsprings) are formed. C2 crosses the best cn parents with randomly selected parents and then the new offsprings are formed. The last crossover operator C3 uses previous crossovers sequentially.

The mutation operators M1 and M2 use 2-opt and they are differ from each other by selection. The third mutation operator M3 uses 3-opt method. In this method, a chromosome is chosen using roulette wheel selection and 5 new chromosomes are obtained by permuting of these 3 genes. The chromosome which has the maximum fitness value continues to live. The last mutation operator M4 is an exchange of a determined random row length.

Table 9.2 Computational experiments of the library problems

Problem	m	n	Min-cost	# of bpasses	Problem	m	n	Min-cost	# of bpasses
MBP-P1	64	8	107	71	MBP-P24	96	16	243	149
MBP-P2	64	8	61	39	MBP-P25	96	16	205	115
MBP-P3	64	8	96	62	MBP-P26	96	16	184	108
MBP-P4	64	8	60	27	MBP-P27	96	25	499	262
MBP-P5	64	8	69	40	MBP-P28	96	25	405	221
MBP-P6	64	8	40	24	MBP-P29	96	25	319	183
MBP-P7	64	12	159	104	MBP-P30	96	25	352	204
MBP-P8	64	12	151	88	MBP-P31	64	8	37	25
MBP-P9	64	12	129	71	MBP-P32	64	12	46	28
MBP-P10	64	12	97	58	MBP-P33	64	16	159	96
MBP-P11	64	12	115	60	MBP-P34	64	25	225	127
MBP-P12	64	12	100	70	MBP-P35	64	25	176	84
MBP-P13	64	16	240	146	MBP-P36	64	25	169	87
MBP-P14	64	16	196	132	MBP-P37	96	8	70	47
MBP-P15	64	16	199	121	MBP-P38	96	25	276	158
MBP-P16	64	16	145	90	MBP-P39	96	8	106	61
MBP-P17	64	16	152	80	MBP-P40	96	8	78	48
MBP-P18	64	16	168	75	MBP-P41	96	16	149	72
MBP-P19	96	8	128	63	MBP-P42	96	16	208	129
MBP-P20	96	8	104	59	MBP-P43	96	16	263	154
MBP-P21	96	8	106	64	MBP-P44	96	25	294	168
MBP-P22	96	8	96	68	MBP-P45	96	25	260	148
MBP-P23	96	16	299	176					

Using above 3 crossover and 4 mutation operators, 12 GA implementations have been created and these GA implementations have been tested on MBP problem instances which are published on http://fen.ege.edu.tr/ arifgursoy/mbpopt/ and the best solutions of the GA implementations are listed in Table 9.2. In this table, there are 45 instances having different number of rows and columns. Further details about the problems can be found at the web page.

9.8 Conclusion

In this paper, a new extended mathematical model are presented for optimization of the Multi-Bandpass problem. 12 genetic algorithm implementations are created using combinations of 2 crossover operators and 4 mutation operators, and an online problem library is created including 45 problem instances in the Web page http://fen.ege.edu.tr/ arifgursoy/mbpopt/. These problems are tested using the genetic algorithm implementations and the results are presented.

References

1. Babayev DA, Nuriyev UG, Bell GI et al (2007) Library of bandpass problems, Available at http://sci.ege.edu.tr/ math/ BandpassProblemsLibrary/
2. Babayev DA, Nuriyev UG, Bell GI et al (2008) Mathematical modelling of a telecommunication packing problem and a heuristic approach for finding a solution. In: International Conference on Control and Optimization with Industrial Applications, Baku, Azerbaijan, June 2-4:44
3. Babayev DA, Bell GI, Nuriyev UG (2009) The bandpass problem: Combinatorial optimization and library of problems. Journal of Combinatorial Optimization 18:151–172
4. Berberler ME, Gürsoy A (2010) Genetic algorithm approach for bandpass problem. In: 24th Mini EURO Conference on Continuous Optimization and Information-Based Technologies in the Financial Sector (MEC EurOPT 2010), İzmir 201–206
5. Gen M (2006) Genetic algorithms and their applications, in Springer Handbook of Engineering Statistics. In: Hoang P (ed.), Springer, London 749–773
6. Gürsoy A, Nuriyev UG, (2012) Genetic algorithm for multi bandpass problem and library of problems. In: Proceeding of the Fourth International Conference “Problems of Cybernetics and Informatics”, Section I, “Information and Communication Technologies”, Baku, Azerbaijan, September 12-14:134–138
7. Gürsoy A, Kurt M, Berberler ME et al (2012) On the solution approaches of the combinatorial bandpass problem in telecommunication. In: 25th Conference of European Chapter on Combinatorial Optimization, Antalya, Turkey, April 26-28
8. Gürsoy A, Kurt M, Kutucu H (2012) On the optimization of the bandpass length for the bandpass problem. In: The 2nd World Conference on Soft Computing, Baku, Azerbaijan, December 3-5:540–546
9. Li Z, Lin G (2011) The three column bandpass problem is solvable in linear time. Theoretical Computer Science 412(4-5):281–299
10. Lin G (2011) On the bandpass problem. Journal of Combinatorial Optimization 22(1):71–77
11. Nuriyev UG, Kutucu H, Kurt M (2011) Mathematical models of the bandpass problem and ordermatic computer game. Mathematical and Computer Modelling 53(5-6):1282–1288
12. Ultrafast fibre optics set new speed record, NTT. 2011-04-29. Retrieved 2012-02-26