Chapter 8
A Third-party Logistics Network Design Model under Fuzzy Random Environment

Xiaoyang Zhou and Yan Tu

Abstract In the present paper, for the location problem of a third-party logistics company which is under the fuzzy random environment, we proposed an chance constraint model. In order to solve it, we transform it into an equivalent crisp model by some mathematical proofs. Finally, an illustrative examples are given in order to show the application of the proposed models.

Keywords 3PLs · Network design · Fuzzy random variable · Chance-constraint operator

8.1 Introduction

Today's competitive business environment has resulted in increasing cooperation among individual companies as members of a supply chain. In other words, the success of a companies will depend on their ability to achieve effective integration of worldwide organizational relationships within a supply chain [1]. Moreover, consumers now require high levels of customer services for a variety of products with a short life cycle. In such an environment, companies are under pressure with filling their customers' orders, keeping the deliveries of products up to speed, reducing inventory. Consequently, the individual companies of a supply chain are frequently faced with the challenges of restructuring their distribution network with respect to global need and volatile market changes. Faced with such a situation, 3PLs come into being to cooperate the manufacturing companies to improve the logistics efficiency.

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In the recent past, third-party logistics (3PLs), also referred to as logistics outsourcing [2–4], has received considerable attention from logistics scholars, resulting in a plethora of research and writing in this field. The interest of researchers in 3PLs should continue as several recent studies suggest that a steadily increasing number of companies across industry sectors use third-party providers for the management of all or part of their logistics operations [5, 6].

The main advantage of outsourcing services to 3PLs is that these 3PLs allow companies to get into a new business or a new market without interrupting forward flows; in addition, logistics costs can be greatly reduced. These 3PLs have evolved the logistics functions such as transportation management, warehouse management, inventory management etc. 3PLs are playing an increasing role in the management of supply chains.

In general, 3PLs operate clients' transportation and warehouses services. More specifically, through the use of these logistics centers, 3PLs provide inbound and outbound transportation, cross-docking, and distribution as well as holding inventory for their clients. So the design of 3PLs network is very important for a third party logistics enterprise.

Unfortunately, the 3PLs network design problem is subject to many sources of uncertainty besides random uncertainty and fuzzy uncertainty. In a practical decision-making process, we often face a hybrid uncertain environment. To deal with this twofold uncertainty, fuzzy random variable was proposed by Kwakernaak [13, 14] to depict the phenomena in which fuzziness and randomness appear simultaneously [11, 15]. Several research works have been published in recent years [16, 17]. However, in this paper, we consider the amount of demand on the products as normally distributed variable $N(\mu,\sigma^2)$ from the view point of probability theory, and the values of μ as a triangular fuzzy variable (a,b,c) because of scanty data to analyze. Therefore, probability 3PLs network with fuzzy parameters appears. In this case, random fuzzy variable which was presented by Liu [12] can be used to deal with this kind of combined uncertainty of randomness and fuzziness. How to model and solve the problem of 3PLs network design in random fuzzy environment is a new area of research interest. To the best of the author's knowledge, so far, there is little research in this area.

Our purpose in this paper is to make some contribution on 3PLs network design in an uncertain environment of combined fuzziness and randomness and obtain optimal solutions. We apply uncertain programming techniques to the real 3PLs network design problem, and provide optimal alternative solutions to the decision-maker.

The remainder of the paper is organized as follows: In Sect. 8.2, we introduce the third-party logistics problem, the fuzzy random chance constraint model and the details of modelling for 3PLS location problem. A crisp equivalent model is presented in Sect. 8.3. An application is presented in Sect. 8.4. Finally the conclusion has been drawn in Sect. 8.5.

8.2 Model for 3PLs Network Design

In this section, we will built up a model according to the third-party logistics network design problem under fuzzy random environment.

(1) Problem statement

The first important thing for a third-party logistics company is to decide the location of logistics centers, it's the core of a 3PLs company. The function of a third-party logistics company is to manage the products of the client plants, that is, to perform the outsourcing contract signed with client plants, which includes distributing the products to the customers to satisfy their demand, and manage the inventory for the client plants. So the client plants can just concentrate on the production, and let the 3PLS company to deal with the distribution and the inventory.

For a 3PLs company, first all of the products are transported to logistics centers, then according to the demand of customers the products are sent to the destination of the customers. For the superfluous amount, the 3PLs company will keep them as inventory, see Fig. 8.1.

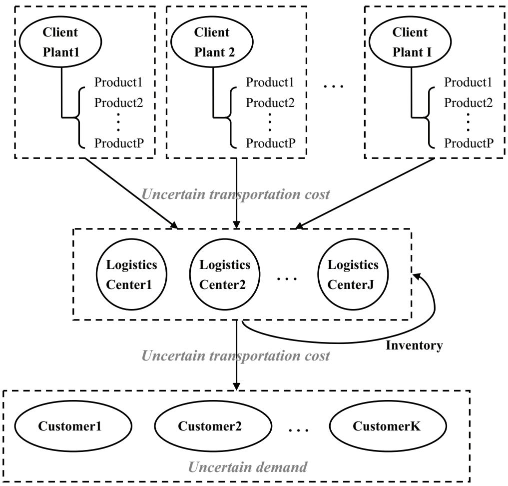

Fig. 8.1 Third-party logistics system

A third-party logistics company want to profit, they should minimize the total cost under the precondition that they should carry out the outsourcing contract.

(2) 3PLs model with fuzzy random coefficients

We try to present a 3PLS model under fuzzy random environment, here we first introduce some basic knowledge about fuzzy random, and then we give the details of establishing fuzzy random chance constraint 3PLS model.

In order to establish a optimization network, we have the following assumptions:

- The 3PLs company sign a outsourcing contract with the client company for very long time, and we consider one period;
- The location and the number of client companies and customers are known;

- The demand of customers and the standard transaction costs are uncertain, we use random fuzzy variables to denote it.

Fuzzy random variable, which was introduced by Kwakemaak [7] in 1978, is a concept to depict the phenomena in which randomness and fuzziness appear simultaneously. Since then, its variants and extensions were presented by other researchers, e.g., Colubi et al. [8], Kruse and Meyer [9], Lǿpez-Diaz and Gil [10] , Puri and Ralescu [11] and Liu [12].

In this paper, the definitions about fuzzy random variable are cited from Liu [12].

Definition 8.1. [12] Let $(\Omega,\mathscr{A},Pr)$ be a probability space, $\mathscr{F}$ be a collection of fuzzy variables defined on the possibility space. A fuzzy random variable is a function $\xi:\Omega\rightarrow\mathscr{F}$ such that for any Borel set B of $\Re$, $\xi^*(B)=Pos\{\xi(\omega)\in B\}$ is a measurable function of ω.

Fuzzy random variable ξ is said to be triangular, if for each ω, $\xi(\omega)$ is a triangular fuzzy variable, denoted by $(X_1(\omega),X_2(\omega),X_3(\omega))$, with X_i are random variables defined on the probability space Ω. The randomness of ξ is said to be determined by random variables X_i, $i=1,2,3$.

In this problem, we assume each logistic center has enough storage space for the client plant. The notations for the proposed model are presented as follows:

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p : set of clients' product types, $p=\{1,\cdots,P\}$;
i : set of clients' plant locations, $i=\{1,\cdots,I\}$;
j : set of potential sites of logistics centers, $j=\{1,\cdots,J\}$;
k : set of fixed customers locations, $k=\{1,\cdots,K\}$.

Parameters

$\tilde{\bar{c}}^1_{pij}$: standard unit transportation cost from i to j by product p;
$\tilde{\bar{c}}^2_{pjk}$: standard unit transportation cost from j to k by product p;
f_j : fixed cost of potential logistics center j;
$\tilde{\bar{d}}_{pk}$: demand of customer k for product p;
s_{pi} : supply of product p of client company i;
t_{pj} : the unit storage cost of product p in logistics center j;
y_{pj} : the storage amount of product p in logistics center j.

Decision variables

x^1_{pij} : amount of product p from client plant i to logistics center j;
x^2_{pjk} : amount of product p from logistics center j to customer k;

$$z_j : \begin{cases} 1 \text{ if the logistics center } j \text{ is open,} \\ 0 \text{ otherwise.} \end{cases}$$

For a 3Pls company, the objective is minimizing the total cost which is composed of the transaction cost from client plant 1 to the logistics center j and from the logistics center j to customer k, the fixed cost of opening the logistics center j, the variable intermediary cost of logistics center j transfer the products, and the storage cost, ao we get the following objective function:

$$\min\ F = \sum_{p\in P}\sum_{i\in I}\sum_{j\in J} x^1_{pij}\tilde{\bar{c}}^1_{pij} + \sum_{p\in P}\sum_{j\in J}\sum_{k\in K} x^2_{pjk}\tilde{\bar{c}}^2_{pjk} + \sum_{j\in J} z_j f_j + \sum_{p\in P}\sum_{j\in J} t_{pj} y_{pj}. \quad (8.1)$$

The constraints include the following:
The total amount of products from logistics center to the customer should be just satisfied the demand of customer,

$$\sum_{j\in J} x^2_{pjk} \geq \tilde{\bar{d}}_{pk},\ \forall p,k. \quad (8.2)$$

The total amount of products from client plant to logistics center should be no more than the production of client plant,

$$\sum_{j\in J} x^1_{pij} \leq s_{pi},\ \forall p,i. \quad (8.3)$$

The total number of logistics centers that will be open should be not larger than a certain number,

$$\sum_{j\in J} z_j \leq n,\ \forall j. \quad (8.4)$$

For a logistics center and a product, the quantity difference between inbound and output is the products that should be stocked, that is the storage amount,

$$y_{pj} = \sum_{i\in I} x^1_{pij} - \sum_{k\in K} x^2_{pjk},\ \forall p,j. \quad (8.5)$$

The storage amount should be no less than 0,

$$y_{pj} \geq 0,\ \forall p,j. \quad (8.6)$$

In addition, there are some logical constrains,

$$\begin{aligned} &x^1_{pij} = x^1_{pij}\cdot z_j,\ \forall i,j, \quad x^2_{pjk} = x^2_{pjk}\cdot z_j,\ \forall j,k,\\ &x^1_{pij} \geq 0,\ \forall i,j, \quad x^2_{pjk} \geq 0,\ \forall j,k, \quad z_j = \{0,1\},\ \forall j. \end{aligned} \quad (8.7)$$

From the discussions above, we can formulate a fuzzy random mixed-integer non-linear programming model as follows:

$$\min\ F = \sum_{p\in P}\sum_{i\in I}\sum_{j\in J} x^1_{pij}\tilde{\bar{c}}^1_{pij} + \sum_{p\in P}\sum_{j\in J}\sum_{k\in K} x^2_{pjk}\tilde{\bar{c}}^2_{pjk} + \sum_{j\in J} z_j f_j + \sum_{p\in P}\sum_{j\in J} t_{pj} y_{pj},$$

$$
\text{s.t.}\begin{cases}
\sum_{j\in J} x^2_{pjk} \geq \tilde{\bar{d}}_{pk},\ \forall p,k,\\
\sum_{j\in J} x^1_{pij} \leq s_{pi},\ \forall p,i,\\
y_{pj} = \sum_{i\in I} x^1_{pij} - \sum_{k\in K} x^2_{pjk},\ \forall p,j,\\
y_{pj} \geq 0,\ \forall p,j,\\
x^1_{pij} = x^1_{pij} z_j,\ \forall i,j,\\
x^2_{pjk} = x^2_{pjk} z_j,\ \forall j,k,\\
x^1_{pij} \geq 0,\ \forall i,j,\\
x^2_{pjk} \geq 0,\ \forall j,k,\\
z_j = \{0,1\},\ \forall j.
\end{cases} \tag{8.8}
$$

Generally, in order to solve the model above, we have to transform these fuzzy random variables into crisp parameters. In this paper we use the chance operator to transform the fuzzy random programming to a chance-constraint programming model.

Before the transformation, we give the following three useful definitions.

Definition 8.2. [17] (Probability measure) Let Ω be a nonempty set, and $\mathscr{A}$ a σ-algebra over Ω, A is an event in $\mathscr{A}$. The set function Pr is called a probability measure if it satisfies the following three axioms.
Axiom 1. $Pr\{\Omega\} = 1$.
Axiom 2. $Pr\{A\} \geq 0$ for any $A \in \mathscr{A}$.
Axiom 3. For every countable sequence of mutually disjoint events $\{A_i\}_{i=1}^{\infty}$, we have:

$$Pr\left\{\bigcup_{i=1}^{\infty} A_i\right\} = \sum_{i=1}^{\infty} Pr\{A_i\}.$$

Definition 8.3. [17](Possibility measure) Given a universe Γ, $\mathscr{P}(\Gamma)$ is the power set of Γ and Pos is a set function defined on $\mathscr{P}(\Gamma)$. Pos is said to be a possibility measure, if Pos satisfies the following conditions:
(1) $Pos\{\Phi\} = 0$;
(2) $Pos(\Gamma) = 1$, and
(3) $Pos\left(\bigcup_{i\in I} A_i\right) = \sup_{i\in I} Pos(A_i)$ for any subclass $\{A_i | i \in I\}$ of $\mathscr{P}(\Gamma)$.

Definition 8.4. [17] (Chance measure) Let ξ be a fuzzy random variable, and B a Borel set of $\Re$. Then the chance of fuzzy random event $\xi \in B$ is a function from $(0,\ 1]$ to $[0,\ 1]$, define as:

$$Ch\{\xi \in B\}(\alpha) = \sup_{Pr\{A\}\geq\alpha} \inf_{\omega\in A} Pos\{\xi(\omega) \in B\}.$$

Therefore, $Ch\{f_i(\xi) \leq 0\}(\alpha_i),\ i = 1,2,\cdots,m$ means the possibility of the fuzzy random event $f_i(\xi) \leq 0$ standing under the probability level α_i. $Ch\{f_i(\xi) \leq 0\}(\alpha_i) \geq \beta_i,\ i = 1,2,\cdots,m$ means the possibility of the fuzzy random event $f_i(\xi) \leq 0$ standing under the probability level α_i is no less than β_i.

For the purpose of minimizing the optimistic value F' of the fuzzy random objective, we turn the objective function to Equation (8.9),

$$\min\ F', \tag{8.9}$$

and we add constraint (8.9),

$$Ch\Bigg\{\sum_{p\in P}\sum_{i\in I}\sum_{j\in J}x^1_{pij}\tilde{\bar{c}}^1_{pij}+\sum_{p\in P}\sum_{j\in J}\sum_{k\in K}x^2_{pjk}\tilde{\bar{c}}^2_{pjk}+\sum_{j\in J}z_jf_j$$
$$+\sum_{p\in P}\sum_{j\in J}v_{pj}x^1_{pij}+\sum_{p\in P}\sum_{j\in J}t_{pj}y_{pj}\le F'\Bigg\}(\varphi)\ge\delta, \tag{8.10}$$

where $\delta,\varphi\in[0,1]$ are confidence levels, and $\min F'$ is the (δ,φ)-optimistic return, $Ch\{\cdot\}$ denotes the chance of the event in $\{\cdot\}$.

According to the definition of chance, constraint (8.10) could be written as Equation (8.11),

$$Pr\Bigg\{\omega\Bigg|Pos\Bigg\{\sum_{p\in P}\sum_{i\in I}\sum_{j\in J}x^1_{pij}\tilde{\bar{c}}^1_{pij}(\omega)+\sum_{p\in P}\sum_{j\in J}\sum_{k\in K}x^2_{pjk}\tilde{\bar{c}}^2_{pjk}(\omega))$$
$$+\sum_{j\in J}z_jf_j+\sum_{p\in P}\sum_{j\in J}v_{pj}x^1_{pij}+\sum_{p\in P}\sum_{j\in J}t_{pj}y_{pj}\le F'\Bigg\}\ge\delta\Bigg\}\ge\varphi, \tag{8.11}$$

where $Pos\{\cdot\}$ denotes the possibility of the event in $\{\cdot\}$, and $Pr\{\cdot\}$ denotes the probability of the event in $\{\cdot\}$.

Remark 8.1. When the fuzzy random variable $\tilde{\bar{c}}_p$ degenerates to random variable $\bar{c}_p$, the constraint (8.11) is equivalent to Equation (8.12),

$$Pr\Bigg\{\sum_{p\in P}\sum_{i\in I}\sum_{j\in J}x^1_{pij}\bar{c}_{pij}(\omega)+\sum_{p\in P}\sum_{j\in J}\sum_{k\in K}x^2_{pjk}\bar{c}_{pjk}(\omega)$$
$$+\sum_{j\in J}z_jf_j+\sum_{p\in P}\sum_{j\in J}v_{pj}x^1_{pij}+\sum_{p\in P}\sum_{j\in J}t_{pj}y_{pj}\le F'\Bigg\}\ge\varphi. \tag{8.12}$$

And similarly, when the fuzzy random variable $\tilde{\bar{c}}_p$ degenerates to fuzzy variable $\tilde{c}_p$, the constraint (8.11) is equivalent to Equation (8.13),

$$Pos\Bigg\{\sum_{p\in P}\sum_{i\in I}\sum_{j\in J}x^1_{pij}\tilde{c}_{pij}(\omega)+\sum_{p\in P}\sum_{j\in J}\sum_{k\in K}x^2_{pjk}\tilde{c}_{pjk}(\omega)$$
$$+\sum_{j\in J}z_jf_j+\sum_{p\in P}\sum_{j\in J}v_{pj}x^1_{pij}+\sum_{p\in P}\sum_{j\in J}t_{pj}y_{pj}\le F'\Bigg\}\ge\delta. \tag{8.13}$$

For given confidence level (α_{pk},β_{pk}), constraint (8.2) can be transformed to Equation (8.14),

$$Ch\left\{\sum_{j\in J} x^2_{pjk} \geq \tilde{\bar{d}}_{kp}\right\}(\alpha_{pk}) \geq \beta_{pk}, \tag{8.14}$$

and it also can be written as Equation (8.15),

$$Pr\left\{\omega \middle| Pos\left\{\sum_{j\in J} x^2_{pjk} \geq \tilde{\bar{d}}_{pk}(\omega)\right\} \geq \beta_{pk}\right\} \geq (\alpha_{pk}), \tag{8.15}$$

where α_{pk} and β_{pk} are predetermined confidence levels.

Remark 8.2. When the random fuzzy variable $\tilde{\bar{d}}_{pk}$ degenerates to random variable $\bar{d}_{kp}$, the constraint (8.14) is equivalent to Eqaution (8.16),

$$Pr\left\{\sum_{j\in J} x^2_{pjk} \geq \bar{d}_{pk}(\omega)\right\} \geq (\alpha_{pk}). \tag{8.16}$$

And similarly, when the fuzzy random variable $\tilde{\bar{d}}_{pk}$ degenerates to fuzzy variable $\tilde{d}_{kp}$, the constraint (8.15) is equivalent to Equation (8.17),

$$Pos\left\{\sum_{j\in J} x^2_{pjk} \geq \tilde{d}_{pk}(\omega)\right\} \geq \beta_{pk}. \tag{8.17}$$

We propose the chance-constraint programming model under random fuzzy environment as follows,

$$\begin{array}{l}
\min\ F', \\
\text{s.t.}\left\{\begin{array}{l}
Ch\left\{\sum_{p\in P}\sum_{i\in I}\sum_{j\in J} x^1_{pij}\tilde{\bar{c}}^1_{pij} + \sum_{p\in P}\sum_{j\in J}\sum_{k\in K} x^2_{pjk}\tilde{\bar{c}}^2_{pjk} + \sum_{j\in J} z_j f_j \right. \\
\qquad \left. + \sum_{p\in P}\sum_{j\in J} t_{pj}y_{pj} \leq F'\right\}(\varphi) \geq \delta, \\
Ch\left\{\sum_{j\in J} x^2_{pjk} \geq \tilde{\bar{d}}_{pk}\right\}(\alpha_{pk}) \geq \beta_{pk},\ \forall p,k, \\
\sum_{j\in J} x^1_{pij} \leq s_{pi},\ \forall p,i, \\
y_{pj} = \sum_{i\in I} x^1_{pij} - \sum_{k\in K} x^2_{pjk},\ \forall p,j, \\
y_{pj} \geq 0,\ \forall p,j, \\
x^1_{pij} = x^1_{pij}\cdot z_j,\ \forall i,j, \\
x^2_{pjk} = x^2_{pjk}\cdot z_j,\ \forall j,k, \\
x^1_{pij} \geq 0,\ \forall i,j, \\
x^2_{pjk} \geq 0,\ \forall j,k, \\
z_j = \{0,1\},\ \forall j.
\end{array}\right.
\end{array} \tag{8.18}$$

8.3 Model Analysis

One way of solving a chance constraint programming model is to convert the constraints of problem (8.18) into their respective crisp equivalents. As we know, this process is usually a hard work and only successful for some special cases. Next, we will consider a special case and present the result in this section.

Lemma 8.1. *Let $\tilde{m}$ and $\tilde{n}$ be two independently fuzzy numbers with continuous membership functions. For given confidence level $\alpha \in [0,1]$,*

$$Pos\{\tilde{m} \geq \tilde{n}\} \geq \alpha = \sup\{\mu_{\tilde{m}}(u) \wedge \mu_{\tilde{n}}(v) | u > v\}.$$

By using the α-level sets of the fuzzy variables, the above Lemma 8.1 can also be rewritten as:

$$Pos\{\tilde{m} \geq \tilde{n}\} \geq \alpha \Leftrightarrow m_\alpha^R \geq n_\alpha^L, \tag{8.19}$$

where m_α^L, m_α^R and n_α^L, n_α^R are the left and right side extreme points of the α-level sets $[m_\alpha^L, m_\alpha^R]$ and $[n_\alpha^L, n_\alpha^R]$ of $\tilde{m}$ and $\tilde{n}$, respectively, and $Pos\{\tilde{m} \geq \tilde{n}\}$ means the degree of possibility that $\tilde{m}$ is greater than or equal to $\tilde{n}$.

Theorem 8.1. *Let $\tilde{\bar{d}}_{pk}$ be a fuzzy random variable which is characterized by the following membership function,*

$$\mu_{\tilde{\bar{d}}_{pk}(\omega)}(t) = \begin{cases} L\left(\frac{d_{pk}(\omega)-t}{a_{pk}^d}\right), & t \leq d_{pk}(\omega),\ a_{pk}^d > 0, \\ R\left(\frac{t-d_{pk}(\omega)}{b_{pk}^d}\right), & t \geq d_{pk}(\omega),\ b_{pk}^d > 0, \end{cases} \quad \omega \in \Omega, \tag{8.20}$$

where random vector $d_{pk}(\omega)$ is normally distributed with mean vector u^d and variance ${\sigma_{pk}^d}^2$, written as $d_{pk}(\omega) \sim \mathscr{N}(u_{pk}^d, {\sigma_{pk}^d}^2)$, a_{pk}^d, b_{pk}^d are positive numbers expressing the left and right spreads of $\tilde{\bar{d}}_{pk}(\omega)$, and reference functions $L, R : [0,1] \to [0,1]$ with $L(1) = R(1) = 0$ and $L(0) = R(0) = 1$ are non-increasing, continuous functions. Then, we have:

$$Pr\left\{\omega \middle| Pos\left\{\sum_{j\in J} x_{pjk}^2 \geq \tilde{\bar{d}}_{pk}(\omega)\right\} \geq \alpha_{pk}\right\} \geq \beta_{pk},$$

if and only if

$$\sum_{j\in J} x_{pjk}^2 \geq u_{pk}^d - L^{-1}(\alpha_{pk}) a_{pk}^d + \Phi^{-1}(\beta_{pk}) \sigma_{pk}^d,$$

where Φ is the standardized normal distribution and $\alpha_{pk}, \beta_{pk} \in (0,1)$ are predetermined confidence levels.

Proof. From assumption we know that $\tilde{\bar{d}}_{pk}$ is a fuzzy number with membership function $\mu_{\tilde{\bar{d}}_{pk}}(t)$ for given $\omega \in \Omega$. For convenience, we denote $\tilde{\bar{d}}_{pk}(\omega) = (d_{pk}(\omega), a_{pk}^d, b_{pk}^d)_{LR}$. By Lemma (8.1), we have that:

$$Pos\left\{\sum_{j\in J} x_{pjk}^2 \geq \tilde{\bar{d}}_{pk}(\omega)\right\} \geq \alpha_{pk} \;\Leftrightarrow\; \sum_{j\in J} x_{pjk}^2 \geq d_{pk}(\omega) - L^{-1}(\alpha_{pk})a_{pk}^d.$$

Since $d_{pk}(\omega) \sim \mathscr{N}(u_{pk}^d, {\sigma_{pk}^d}^2)$, for given confidence levels $\alpha_{pk}, \beta_{pk} \in (0,1)$, we have:

$$\begin{aligned}
& Pr\left\{\omega \middle| Pos\left\{\sum_{j\in J} x_{pjk}^2 \geq \tilde{\bar{d}}_{pk}(\omega)\right\} \geq \alpha_{pk}\right\} \geq \beta_{pk} \\
& \Leftrightarrow Pr\left\{\omega \middle| \sum_{j\in J} x_{pjk}^2 \geq d_{pk}(\omega) - L^{-1}(\alpha_{pk})a_{pk}^d\right\} \geq \beta_{pk} \\
& \Leftrightarrow Pr\left\{\omega \middle| \frac{d_{pk}(\omega) - u_{pk}^d}{\sigma_{pk}^d} \leq \frac{\sum_{j\in J} x_{pjk}^2 + L^{-1}(\alpha_{pk})a_{pk}^d - u_{pk}^d}{\sigma_{pk}^d}\right\} \geq \beta_{pk} \\
& \Leftrightarrow \Phi\left(\frac{\sum_{j\in J} x_{pjk}^2 + L^{-1}(\alpha_{pk})a_{pk}^d - u_{pk}^d}{\sigma_{pk}^d}\right) \geq \beta_{pk} \\
& \Leftrightarrow \sum_{j\in J} x_{pjk}^2 \geq u_{pk}^d - L^{-1}(\alpha_{pk})a_{pk}^d + \Phi^{-1}(\beta_{pk})\sigma_{pk}^d.
\end{aligned}$$

This completes the proof. □

Theorem 8.2. *Let $\tilde{\bar{c}}$ be a fuzzy random variable which is characterized by the following membership function,*

$$\mu_{\tilde{\bar{c}}(\omega)}(t) = \begin{cases} L\left(\frac{c(\omega)-t}{a^c}\right), \; t \leq c(\omega), a^c > 0, \\ R\left(\frac{t-c(\omega)}{b^c}\right), \; t \geq c(\omega), b^c > 0, \end{cases} \quad \omega \in \Omega, \tag{8.21}$$

where random vector $(C(\omega))$ is normally distributed with mean vector u^c and positive define covariance matrix V^c, written as $(c(\omega)) \sim \mathscr{N}(u^c, V^c)$, a^c, b^c are positive numbers expressing the left and right spreads of $\tilde{\bar{c}}(\omega)$, and reference functions $L, R : [0,1] \to [0,1]$ with $L(1) = R(1) = 0$ and $L(0) = R(0) = 1$ are non-increasing, continuous functions. Then, we have:

$$Pr\{\omega | Pos\{\tilde{\bar{c}}(\omega)^T x \leq \bar{F}\} \geq \delta\} \geq \varphi,$$

if and only if

$$\bar{F} \geq u^{cT} x - L^{-1}(\delta)a^{cT} x + \Phi^{-1}(\varphi)\sqrt{x^T V^c x},$$

where Φ is the standardized normal distribution and $\delta, \varphi \in [0,1]$ are predetermined confidence levels.

Proof. From assumption we know that $\tilde{\bar{c}}$ is a fuzzy number with membership function $\mu_{\tilde{\bar{c}}}(t)$ for given $\omega \in \Omega$. It follows from extension principle [12] that fuzzy number $\tilde{\bar{c}}^T x$ is characterized by the following membership function

$$\mu_{\tilde{\bar{c}}(\omega)^T x}(r) = \begin{cases} L\left(\frac{c(\omega)^T x - r}{a^{cT} x}\right), & r \leq c(\omega)^T x, \\ R\left(\frac{r - c(\omega)^T x}{b^{cT} x}\right), & r \geq c(\omega)^T x. \end{cases} \tag{8.22}$$

For convenience, we denote $\tilde{\bar{c}}(\omega) = (c(\omega), a^c, b^c)_{LR}$ and $\tilde{\bar{c}}(\omega)^T x = (c(\omega)^T x, a^{cT} x, b^{cT} x)_{LR}$ respectively. By Equation (8.1), we have:

$$Pos\{\tilde{\bar{c}}(\omega)^T \leq \bar{F}\} \leq \delta \Leftrightarrow c(\omega)^T x - L^{-1}(\delta) a^{cT} x \leq \bar{F}.$$

Since $c(\omega) \sim \mathscr{N}(u^c, V^c)$, it follows that $c(\omega)^T x \sim \mathscr{N}(u^{cT} x, x^T V^c x)$. So, for given confidence levels $\delta, \varphi \in (0,1)$, we have:

$$\begin{aligned} & Pr\{\omega | Pos\{c(\omega)^T x \geq \bar{F}\} \geq \delta\} \geq \varphi \\ & \Leftrightarrow Pr\{\omega | c(\omega)^T x \leq \bar{F} + L^{-1}(\delta) a^{cT} x\} \geq \varphi \\ & \Leftrightarrow Pr\left\{\omega \left| \frac{c(\omega)^T x - u^{cT} x}{\sqrt{x^T V^c C x}} \leq \frac{\bar{F} + L^{-1}(\delta) a^{cT} x - u^{cT} x}{\sqrt{x^T V^c x}} \right.\right\} \geq \varphi \\ & \Leftrightarrow \Phi\left(\frac{\bar{F} + L^{-1}(\delta) a^{cT} x - u^{cT} x}{\sqrt{x^T V^c x}}\right) \geq \varphi \\ & \Leftrightarrow \bar{F} \geq u^{cT} x - L^{-1}(\delta) a^{cT} x + \Phi^{-1}(\varphi) \sqrt{x^T V^c x}. \end{aligned}$$

This completes the proof. □

8.4 Numerical Example

Suppose there is a 3PLs company, the first important decision for this company is to design the logistics network, which including choose the logistics centers, limited by the capital, only $n = 2$ logistics centers could be established. This 3Pls company will be in charge of the transportation from two client plants to six customers via two logistic centers.

Table 8.1 Parameters about logistics center

Logistics center	Fixed cost(f_j) (RMB)	Unit inventory cost(t_j) (RMB)
1	1800	5
2	2200	6
3	1900	5
4	2000	6

Here we just consider one type of product, that is $P = 1$. So we can apply Model (8.18) to this numerical problem, and obtain the following specific Model (8.23).

$$\min F',$$
$$\text{s.t.}\begin{cases} Pr\left\{\omega|Pos\left\{\sum_{i=1}^{2}\sum_{j=1}^{4} x_{ij}^1\tilde{\bar{c}}_{ij}^1+\sum_{j=1}^{4}\sum_{j=1}^{6} x_{jk}^2\tilde{\bar{c}}_{jk}^2 \le F' - \sum_{j=1}^{4} z_j f_j - \sum_{j=1}^{4} t_j y_j\right\}\ge\delta\right\}\ge\varphi, \\ Pr\left\{\omega|Pos\left\{\sum_{j=1}^{4} x_{jk}^2 \ge \tilde{\bar{d}}_k\right\}\ge\alpha_k\right\}\ge\beta_k,\ k=1,\cdots,6, \\ \sum_{j=1}^{4} x_{ij}^1 \le s_i,\ i=1,2, \\ \sum_{j=1}^{4} z_j \le 2,\ j=1,\cdots,4, \\ y_j = \sum_{i=1}^{2} x_{ij}^1 - \sum_{k=1}^{6} x_{jk}^2,\ j=1,\cdots,4, \\ y_j \ge 0, j=1,\cdots,4, \\ x_{ij}^1 = x_{ij}^1 z_j,\ i=1,2, j=1,\cdots,4, \\ x_{jk}^2 = x_{jk}^2 z_j,\ j=1,\cdots,4,\ k=1,\cdots,6, \\ x_{ij}^1 \ge 0,\ i=1,2,\ j=1,\cdots,4, \\ x_{jk}^2 \ge 0,\ j=1,\cdots,4,\ k=1,\cdots,6, \\ z_j=\{0,1\},\ j=1,\cdots,4. \end{cases} \tag{8.23}$$

It's known that there are two client plants which produce one kind of product. Client plant 1 will produce 1800 products, and client plant 1 will produce 1500 products. After doing some surveys, there are 4 potential logistics centers. Some important data are in Table 8.1. The standard unit transportation costs of this product from client plants to logistic centers and from the logistic centers to customers are shown in Tables 8.2 and 8.3. Table 8.4 reveals the demand of each customers.

Table 8.2 Transportation cost from client plants to logistic centers

$\tilde{\bar{c}}_{11}^1 = [c_{11}^1, 0.5, 0.5]_{LR}$ with $c_{11}^1 \sim N(5, 0.5)$	$\tilde{\bar{c}}_{12}^1 = [c_{12}^1, 0.5, 0.5]_{LR}$ with $c_{12}^1 \sim N(6, 0.5)$
$\tilde{\bar{c}}_{13}^1 = [c_{13}^1, 0.3, 0.3]_{LR}$ with $c_{13}^1 \sim N(3, 0.2)$	$\tilde{\bar{c}}_{14}^1 = [c_{14}^1, 0.5, 0.4]_{LR}$ with $c_{12}^1 \sim N(5, 0.5)$
$\tilde{\bar{c}}_{21}^1 = [c_{21}^1, 0.5, 0.5]_{LR}$ with $c_{21}^1 \sim N(4, 0.4)$	$\tilde{\bar{c}}_{22}^1 = [c_{22}^1, 0.5, 0.5]_{LR}$ with $c_{22}^1 \sim N(4, 0.5)$
$\tilde{\bar{c}}_{13}^1 = [c_{23}^1, 0.4, 0.4]_{LR}$ with $c_{23}^1 \sim N(5, 0.2)$	$\tilde{\bar{c}}_{24}^1 = [c_{24}^1, 0.5, 0.5]_{LR}$ with $c_{22}^1 \sim N(3, 0.3)$

Because the fuzzy random variables in this model are normally distributed, so we can use Theorems 8.1 and 8.2 which in Sect. 8.3 to transform the above model (8.23) to an equivalent crisp model (8.24) as follows, here we suppose $\delta = \varphi = \alpha_k = \beta_k = 0.9$.

$$
\min F', \\
\text{s. t.} \begin{cases}
F' - [1800z_1 + 2200z_2 + 1900z_3 + 2000z_4 + 5y_1 + 6y_2 + 5y_3 + 6y_4] \\
\quad \geq 5x^1_{11} + \cdots + 3x^1_{24} + 7x^2_{11} + \cdots + 12x^2_{46} - L^{-1}(0.9) \\
\quad (0.5x^1_{11} + \cdots + 0.5x^1_{24} + x^2_{11} + \cdots + 2x^2_{46}) \\
\quad + \Phi(0.9)\sqrt{0.5{x^1_{11}}^2 + \cdots + 0.3{x^1_{24}}^2 + 0.5{x^2_{11}}^2 + \cdots + 2{x^2_{46}}^2}, \\
\sum_{j=1}^{4} x_{j1} \geq 800 - 5L^{-1}(0.9) + \sqrt{16}\Phi^{-1}(0.9), \\
\vdots \\
\sum_{j=1}^{4} x_{j6} \geq 400 - 10L^{-1}(0.9) + \sqrt{9}\Phi^{-1}(0.9), \\
x^1_{11} + x^1_{12} + x^1_{13} + x^1_{14} + x^1_{15} + x^1_{16} \leq 1800, \\
x^1_{21} + x^1_{22} + x^1_{23} + x^1_{24} + x^1_{25} + x^1_{26} \leq 1500, \\
\sum_{j=1}^{4} z_j \leq 2, \\
y_1 = x^1_{11} + x^1_{21} - (x^2_{11} + x^2_{12} + x^2_{13} + x^2_{14} + x^2_{15} + x^2_{16}), \\
\vdots \\
y_4 = x^1_{14} + x^1_{24} - (x^2_{41} + x^2_{42} + x^2_{43} + x^2_{44} + x^2_{45} + x^2_{46}), \\
x^1_{ij} = x^1_{ij} \cdot z_j \ i = 1,2; j = 1,\cdots,4, \\
x^2_{jk} = x^2_{jk} \cdot z_j \ j = 1,\cdots,4; k = 1,\cdots,6, \\
x^1_{ij} \geq 0 \ i = 1,2; j = 1,\cdots,4, \\
x^2_{jk} \geq 0 \ j = 1,\cdots,4; k = 1,\cdots,6, \\
z_j = \{0,1\} \ j = 1,\cdots,4.
\end{cases}
\tag{8.24}
$$

Table 8.3 Transportation cost from logistic centers to customers

$\tilde{\bar{c}}^2_{11} = [c^2_{11}, 1, 1]_{LR}$ with $c^2_{11} \sim N(7, 0.5)$	$\tilde{\bar{c}}^2_{12} = [c^1_{12}, 1, 1]_{LR}$ with $c^2_{12} \sim N(6, 0.5)$
$\tilde{\bar{c}}^2_{13} = [c^1_{13}, 1, 1]_{LR}$ with $c^2_{13} \sim N(8, 1)$	$\tilde{\bar{c}}^2_{14} = [c^1_{14}, 1, 1]_{LR}$ with $c^2_{14} \sim N(9, 0.5)$
$\tilde{\bar{c}}^2_{15} = [c^2_{15}, 1, 1]_{LR}$ with $c^2_{15} \sim N(5, 0.2)$	$\tilde{\bar{c}}^2_{16} = [c^1_{16}, 1, 2]_{LR}$ with $c^2_{16} \sim N(6, 1)$
$\tilde{\bar{c}}^2_{21} = [c^2_{21}, 2, 1]_{LR}$ with $c^2_{21} \sim N(14, 1)$	$\tilde{\bar{c}}^2_{22} = [c^1_{22}, 1, 1]_{LR}$ with $c^2_{22} \sim N(9, 0.5)$
$\tilde{\bar{c}}^2_{23} = [c^1_{23}, 1, 1]_{LR}$ with $c^2_{23} \sim N(13, 1)$	$\tilde{\bar{c}}^2_{24} = [c^1_{24}, 2, 2]_{LR}$ with $c^2_{24} \sim N(14, 2)$
$\tilde{\bar{c}}^2_{25} = [c^2_{25}, 1, 2]_{LR}$ with $c^2_{25} \sim N(12, 2)$	$\tilde{\bar{c}}^2_{26} = [c^1_{26}, 2, 1]_{LR}$ with $c^2_{26} \sim N(13, 2)$
$\tilde{\bar{c}}^2_{31} = [c^2_{31}, 1, 1]_{LR}$ with $c^2_{31} \sim N(1, 0.8)$	$\tilde{\bar{c}}^2_{32} = [c^1_{32}, 0.6, 0.6]_{LR}$ with $c^2_{32} \sim N(7, 0.5)$
$\tilde{\bar{c}}^2_{33} = [c^1_{33}, 1, 1]_{LR}$ with $c^2_{33} \sim N(9, 1)$	$\tilde{\bar{c}}^2_{34} = [c^1_{34}, 0.6, 0.6]_{LR}$ with $c^2_{34} \sim N(7, 0.5)$
$\tilde{\bar{c}}^2_{35} = [c^2_{35}, 0.6, 0.8]_{LR}$ with $c^2_{35} \sim N(8, 1)$	$\tilde{\bar{c}}^2_{36} = [c^1_{36}, 2, 2]_{LR}$ with $c^2_{36} \sim N(10, 2)$
$\tilde{\bar{c}}^2_{41} = [c^2_{41}, 1, 1]_{LR}$ with $c^2_{41} \sim N(10, 2)$	$\tilde{\bar{c}}^2_{42} = [c^1_{42}, 2, 3]_{LR}$ with $c^2_{42} \sim N(15, 3)$
$\tilde{\bar{c}}^2_{43} = [c^1_{43}, 1, 2]_{LR}$ with $c^2_{43} \sim N(13, 2)$	$\tilde{\bar{c}}^2_{44} = [c^1_{44}, 1, 1]_{LR}$ with $c^2_{44} \sim N(10, 1)$
$\tilde{\bar{c}}^2_{45} = [c^2_{45}, 1, 2]_{LR}$ with $c^2_{45} \sim N(13, 2)$	$\tilde{\bar{c}}^2_{46} = [c^1_{46}, 2, 1]_{LR}$ with $c^2_{46} \sim N(12, 2)$

Note that the crisp model (8.24) is transformed from (8.23), and Model (8.24) is a crisp mixed integer programming model and it can be solved by some existing methods. Here we employ the software Lingo 9.0 to solve it, and we can obtain

Table 8.4 Demand of customer

$\tilde{d}_1 = [\rho_1, 5, 5]_{LR}$ with $\rho_1 \sim N(800, 16)$	$\tilde{d}_2 = [\rho_2, 4, 4]_{LR}$ with $\rho_2 \sim N(500, 25)$
$\tilde{d}_3 = [\rho_3, 5, 5]_{LR}$ with $\rho_3 \sim N(700, 9)$	$\tilde{d}_4 = [\rho_4, 4, 4]_{LR}$ with $\rho_2 \sim N(500, 25)$
$\tilde{d}_5 = [\rho_5, 5, 5]_{LR}$ with $\rho_5 \sim N(600, 30)$	$\tilde{d}_6 = [\rho_6, 10, 10]_{LR}$ with $\rho_2 \sim N(400, 9)$

the following results: this 3Pls company will choose the first and the third place to establish the logistics centers, and the cost for one period will be 38700RMB, and the transportation scheme is shown in Fig. 8.2.

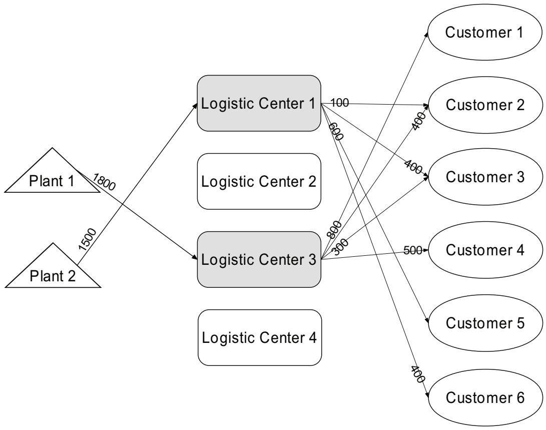

Fig. 8.2 The result of the transportation scheme

8.5 Conclusion

In this paper, for the first time, we have formulated a fuzzy random model about 3PLs network design problems in fuzzy random environments. Till now, no 3PLs network design model has been formulated in such environments. Besides, we creatively introduced the economic factors of scale which were important factors in real-life transportation problem into the proposed model, and make the model more effective. We transform the fuzzy random model into a chance-constraint model which utilize the chance operator of the fuzzy random variables, and for a special type of fuzzy random variables, a crisp equivalent model is proposed for the chance constraint programming model. At the end of this paper, we use an example problem to show the efficiency of the model.

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