Chapter 48
Research on Inventory Level Distribution for a Closed-loop Support System

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Abstract Performance-based logistics (PBL) is becoming a dominant support strategy for military equipment systems. This paper sets up a closed-loop support system which consisted of a spare parts warehouse and a repair workshop. Firstly, we set up the transition balance equations for the inventory level state. Then, the steady-state probability distribution of spare part inventory level is derived. Then, we conduct several experiments to investigate sensitivities of system's parameters. Our research will help to improving the support level for our army.

Keywords Repairable parts · Inventory level · Distribution · Closed-loop

48.1 Introduction

With equipment systems develop to large-scale and complexity, the repairable parts management of the weapon system is becoming more and more important, but the repair cost is also becoming higher than before. For example, the repair cost of the modern ship equipment is higher than the development cost and purchase price, it has become a major part of life cycle cost for ship equipment. Every year the most of military budget of Chinese Navy is the ship equipment repair cost. Every year United States Department of Defense spends 80% logistics budget on repairing weapon and combat support system [1]. Nowadays in the condition of the spare parts' rising price, traditional repair parts management make the contradictions between the limited budget and the shortage of spare parts more obviously, it may

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affect supplying repair parts timely. So the research of repair parts inventory is especially important.

In order to reduce military cost, through performance based logistics, America and other western developed countries want to solve the problem that weapon system keeps the sustainable fighting capability [2]. Based on life cycle management theory PBL is a US army's strategy which uses to keep the sustainable fighting capability of weapon system. The essence is the weapon system performance purchase, is not same with traditionally purchasing weapon products or repair parts, or purchasing repair service only. PBL make the strategies of Ministry of Defence turn traditionally single and scattered transaction patterns into performance based guarantees patterns, including weapon system availability, reliability, repair, logistics guarantee scale, logistics reaction speed, unit operation cost and so on.

Devries et al [3] studied the most common barriers and enablers at the system, subsystem and component level, as services go forward with PBL implementation. Nowicki et al [4] pointed out that suppliers compute cost not based on the number of repair and components inventory, but based on the effective time the costumers got. So performance based logistics management will become a popular pattern. But the studies above all focus on PBL contract price or the factors which impact on PBL contract, and they don't point out how to enhance the system reliability through improving system parameters. For this reason, coming up with a minimized cost program which based on a higher reliability has the great significant in both theory and practice. In fact, under the influence of random output level, failures, repair service, equipment maintenance and introduced new technology, some people raise some storage management strategy. For example, Braglia et al [7] studied that the supplier built central supply center and regional distribution center, and reduced total cost by the way that it moved repair parts from central supply center to regional distribution center. Ilgin and Tunali [8] pointed out that repair service efficiency exercise a great influence on the inventory level and joint optimization of preventive maintenance repairable parts supply. METRIC theory is used to study the problem of repairable parts inventory. Based on METRIC theory, Yang et al [9] explored the relationship between repair parts inventory level and equipment reliability, when the total cost was fixed; Sleptchenko et al [10] pointed out system reliability affected by repair service efficiency and the repairable parts inventory level; Basten et al [12] took the repair and inventory problem into consideration and built a inventory optimization model for repairable parts. Based on the queuing model; Xiao et al [13] analyzed the relationship between weapon system harm extent of failure and sending maintainers or rescue workers. But those researches above are all based on traditional spare part supply chain, they purpose on getting minimum part inventory or analyzing the factors that make great influence on system reliability. Obviously, they can't meet the system part management requirements that have become more and more complicated and expensive.

This paper considers a closed-loop support system for repairable components, in which both a component failure and the repair lead time of each service in the repair facility follow an independent Poisson distribution. Under the support performance, we build balance equations of repairable part inventory level based on the closes-loop queuing system. We derived the steady-state probability distribution of spare parts inventory level and reveal the properties of the steady-state probability distribution, which will help to improving the support level for our army.

48.2 Model and Analysis

In this paper, we consider a closed-loop support system for maintaining repair parts of equipment systems. The closed-loop system is consisted of a spare parts warehouse and a repair workshop. The basic description for the model is as follows:

- There are N sets of equipment systems needed to be supported. These systems are independent on each other and with the same type of core components. Once a core component fails, the system will immediately stop working. If there are available spare parts in the warehouse, the failure component will be recovered and the system will regain to work. Let the failure distribution of a core component follows a Poisson distribution with consistent rate λ.
- The core components failures of equipment systems are independent on each other. If the warehouse holds spare parts to replace defective part, the equipment system failure will be immediately excluded and the system resumes normal operation. Otherwise, the failure equipment systems will wait maintenance services.
- The failure core component is immediately sent to the repair workshop for repairing. A recovered part is as good as the new one. We assume that the repair workshop is modeled as the single-server queuing system. The customers' arrival process is Poisson process with intensity λ and the repair time is an independent exponent random variable with mean μ^{-1}.
- The recovered part will be kept in the warehouse and the initial core parts inventory reserves is assumed to be s.
- In order to facilitate the analysis, we assume that the replacement time of failure parts is negligible and parts in the warehouse during the reserve period will not malfunction. The transit time for components in the closed-loop support system is also negligible.

Let x represent the component number in the warehouse. As there are$N$ independent, same type, system repairable components, x is from $-N$ to s. When x is negative, it means the shortage number of the warehouse. Suppose y is the number of components which is in the repair workshop. Let z be the number of running components in the independent system components. There are x repairable parts in the warehouse, y failure components in the repair station and z running components, and they satisfy the following relationships:

$$z = \begin{cases} N, & x > 0, \\ N + x, & -N \leq x \leq 0, \end{cases} \quad y = s - x, \ -N \leq x \leq s. \tag{48.1}$$

In the equilibrium, the inventory level state space $E = \{-N, \cdots, 0, 1, \cdots, s\}$ and the inventory level state process is a Markovian process, the rule of the inventory level state transition is expressed as follows:

- When $x \leq 0$, there is no ready-to-use spare parts in the warehouse, and the operational system is $N + x$ with the failure rate $(N + x)\lambda$. That is to say, at state x, the inventory will be transferred to $x - 1$ with intensity $(N + x)\lambda$;
- When $1 \leq x \leq s$, there is on shortage in the warehouse and the number of operational system is N with the failure rate $N\lambda$. At state x, the inventory will be transferred to $x - 1$ with intensity $N\lambda$;
- A repaired component arrives at the warehouse from the repair facility. At state x, the repair intensity is μ. That is to say, at state x, the inventory will be transferred to $x + 1$ with intensity with μ.

48.3 The Steady-state Probability Distribution

Let $P(x)(x = -N, -N + 1, \cdots, s)$ is the steady-state probability distribution of the ready-to-use parts in the warehouse, and $\sum_{x=-N}^{s} P(x) = 1$. Theorem 48.1 gives the steady-state probability distribution.

Theorem 48.1. *In the long-run equilibrium, the steady-state probability distribution of spare parts inventory level state can be expressed as follows:*

$$P_{-N} = \left[\sum_{x=-N}^{0} \frac{\rho^{N+x}}{(N+x)!} + \sum_{x=1}^{s} \frac{\rho^N}{N!} \left(\frac{\rho}{N} \right)^x \right]^{-1}, \tag{48.2}$$

$$P_x = \frac{\rho^{N+x}}{(N+x)!} P_{-N}, -N \leq x \leq 0, \tag{48.3}$$

$$P_x = \frac{\rho^N}{N!} \left(\frac{\rho}{N} \right)^x P_{-N}, 1 \leq x \leq s, \tag{48.4}$$

where, $\rho = \mu/\lambda$.

Proof. According to Markovian process of the spare parts inventory state level, we can set up the following balance equations.

$$\mu P_{-N} = \lambda P_{-N+1}, \tag{48.5}$$

$$[(x+N)\lambda + u] P_x = (x+N+1)\lambda P_{x+1} + uP_{x-1}, \; -N+1 \leq x \leq -1, \tag{48.6}$$

$$(N\lambda + u) P_x = N\lambda P_{x+1} + uP_{x-1}, 0 \leq x \leq s-1, \tag{48.7}$$

$$N\lambda P_s = uP_{s-1}. \tag{48.8}$$

In the long run equilibrium, the steady-state probability distributions of the inventory level $P(x)$ satisfy the above Equations (48.5) $\sim$ (48.8). The balance equations can be obtained by the fact that transition out of a state is equal to transition into a state for a Markov process. For example, we consider a type inventory level state x that lines in the range $-N+1 \leq x \leq -1$. The equation is presented in Equation (48.6). When x is in this range, there is no ready-to-use spare parts, the transition out this state can be only duce to either a failed part arrival or a repaired part arrival. This fact is presented on the left-hand side of Equation (48.6). Either a failed part state $x+1$ will reduce the inventory level by one unit, thus bring it to state x. State x can also be reached from state $x-1$ when a repaired part arrives.

From Equation (48.5), we get:

$$P_{-N+1} = \rho P_{-N}. \tag{48.9}$$

By the iterative method, we can get Equation (48.10) from (48.6).

$$P_x = \frac{\rho^{N+x}}{(N+x)!} P_{-N}, -N+1 \leq x \leq 0. \tag{48.10}$$

With the same manipulation, we get Equation (48.4). Then, we insert Equation (48.3) and Equation (48.4) into $\sum_{x=-N}^{s} P_x = 1$, and get Equation (48.2).

The steady-state probability distribution of the parts inventory level state is the basis of computing various support performance measures. □

48.4 Numerical Experiments

In this section, we present will reveal properties of the steady-state probability distribution of the parts inventory level state. We conducted several experiments because each of them investigated a case and the main parameters which should be considered. In the following, we investigate the sensitivities of system parameters. The numerical results are summed up as follows.

Case 48.1. The effects of failure rate λ on the steady-state probability distribution of the parts inventory. We set other parameters as $\mu = 5,\ N = 5,\ s = 3$.

We sum all the probability distributions at the right column which is denoted $\mathrm{Sum}(P)$. Our experiments show $\mathrm{Sum}(P) = 1$, which is should to be 1 in theory. From Table 48.1, we can get that the failure rate has dramatic effect on the steady-state probability distribution of the parts inventory.

Case 48.2. The effects of repair rate μ on the steady-state probability distribution of the parts inventory. The other parameters are set to be $\lambda = 0.5,\ N = 5,\ s = 3$.

Table 48.2 shows that the repair rate has effect on the steady-state probability distribution of the parts inventory dramatically. For example, P3 increases from 0.0023 to 0.7220 with repair rate increasing from 1 to 9. While P-5 decreases from 0.1345 to 0 with repair rate increasing from 1 to 9. We also can find some other features of the probability distribution for the Fig. 48.1.

Table 48.1 The sensitivities of failure rate

λ	P_{-5}	P_{-4}	P_{-3}	P_{-2}	P_{-1}	P_0	P_1	P_2	P_3	Sum(P)
0.1	0.0000	0.0000	0.0000	0.0000	0.0001	0.0009	0.0090	0.0900	0.9000	1
0.2	0.0000	0.0000	0.0000	0.0002	0.0013	0.0064	0.0320	0.1600	0.8001	1
0.3	0.0000	0.0000	0.0002	0.0014	0.0057	0.0189	0.0631	0.2102	0.7006	1
0.4	0.0000	0.0002	0.0012	0.0049	0.0154	0.0386	0.0964	0.2409	0.6024	1
0.5	0.0001	0.0008	0.0038	0.0127	0.0317	0.0634	0.1268	0.2536	0.5072	1
0.6	0.0003	0.0022	0.0093	0.0260	0.0541	0.0901	0.1502	0.2504	0.4173	1
0.7	0.0007	0.0053	0.0189	0.0451	0.0805	0.1150	0.1643	0.2347	0.3353	1
0.8	0.0017	0.0106	0.0331	0.0691	0.1079	0.1349	0.1686	0.2107	0.2634	1
0.9	0.0034	0.0186	0.0517	0.0958	0.1331	0.1478	0.1643	0.1825	0.2028	1

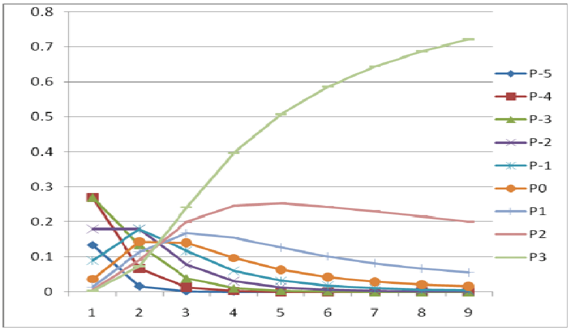

Fig. 48.1 The sensitivities of the probability distribution

Table 48.2 The sensitivities of repair rate

μ	P_{-5}	P_{-4}	P_{-3}	P_{-2}	P_{-1}	P_0	P_1	P_2	P_3	Sum(P)
1	0.1345	0.2691	0.2691	0.1794	0.0897	0.0359	0.0144	0.0057	0.0023	1
2	0.0168	0.0672	0.1344	0.1792	0.1792	0.1434	0.1147	0.0918	0.0734	1
3	0.0022	0.0130	0.0389	0.0778	0.1167	0.1400	0.1680	0.2016	0.2419	1
4	0.0004	0.0028	0.0113	0.0302	0.0604	0.0967	0.1547	0.2475	0.3960	1
5	0.0001	0.0008	0.0038	0.0127	0.0317	0.0634	0.1268	0.2536	0.5072	1
6	0.0000	0.0002	0.0015	0.0059	0.0177	0.0424	0.1018	0.2443	0.5862	1
7	0.0000	0.0001	0.0006	0.0030	0.0105	0.0293	0.0822	0.2301	0.6442	1
8	0.0000	0.0000	0.0003	0.0016	0.0066	0.0210	0.0672	0.2151	0.6882	1
9	0.0000	0.0000	0.0002	0.0010	0.0043	0.0155	0.0558	0.2007	0.7220	1

Case 48.3. The effects of initial inventory level s on the steady-state probability distribution of the parts inventory. The other parameters are set to be $\lambda = 0.5,\ \mu = 5,\ N = 5$.

Table 48.3 shows that the probability distribution $P_x\ (x = -N, -N+1, \cdots 0, \cdots, s)$ is decreasing with initial inventory level s increasing from 0 to 7.

Table 48.3 The sensitivities of initial inventory level

s	0	1	2	3	4	5	6	7
P_{-5}	0.0007	0.0003	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
P_{-4}	0.0068	0.0032	0.0015	0.0008	0.0004	0.0002	0.0001	0.0000
P_{-3}	0.0338	0.0159	0.0077	0.0038	0.0019	0.0009	0.0005	0.0002
P_{-2}	0.1128	0.053	0.0257	0.0127	0.0063	0.0031	0.0016	0.0008
P_{-1}	0.2820	0.1325	0.0643	0.0317	0.0157	0.0078	0.0039	0.002
P_0	0.5640	0.265	0.1286	0.0634	0.0315	0.0157	0.0078	0.0039
P_1		0.5301	0.2573	0.1268	0.0629	0.0314	0.0157	0.0078
P_2			0.5146	0.2536	0.1259	0.0627	0.0313	0.0156
P_3				0.5072	0.2518	0.1254	0.0626	0.0313
P_4					0.5036	0.2509	0.1252	0.0626
P_5						0.5018	0.2504	0.1251
P_6							0.5009	0.2502
P_7								0.5004
Sum(P)	1	1	1	1	1	1	1	1

Case 48.4. The effects of system sets number N on the steady-state probability distribution of the parts inventory. The other parameters are set to be $\lambda = 0.5$, $\mu = 5$, $s = 3$.

Table 48.4 shows that the probability distribution P_x $(x = -N, -N+1, \cdots, 0, \cdots, s - 1)$ is increasing with the N increasing from 1 to 5. But P_s $(s = 3)$ is decreasing with the N increasing from 1 to 5. For fixed N, the probability P_x is increasing with x increasing from $-N$ to s.

Table 48.4 The sensitivities of number N

N	P_{-5}	P_{-4}	P_{-3}	P_{-2}	P_{-1}	P_0	P_1	P_2	P_3	Sum(P)
1					0.0001	0.0009	0.0090	0.0900	0.9000	1
2				0.0001	0.0013	0.0064	0.0320	0.1600	0.8002	1
3			0.0001	0.0011	0.0057	0.0189	0.0631	0.2102	0.7008	1
4		0.0001	0.0009	0.0046	0.0154	0.0386	0.0964	0.2411	0.6028	1
5	0.0001	0.0008	0.0038	0.0127	0.0317	0.0634	0.1268	0.2536	0.5072	1

48.5 Conclusions

In traditional repairable system, the most of studies focus on how to decide the most suitable repairable part inventory and to reduce the total cost of the support system. But, they do not consider the support performance in conjunction with support cost.

This paper, we set up a closed-loop support system for repairable parts of N sets of independent and identical equipment systems under performance-based contracting. The support system is consisted of a spare parts warehouse and a repairable workshop. We established the inventory levels state as Markov process and derived the steady-state probability of inventory levels, which can be use to calculating various support performance measures. We also investigate the sensitivities of the main parameters. The steady-state probability distribution of the inventory level is affected by the main parameters obviously. Our results can be applied to optimize the closed-loop support system.

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