Chapter 44
A Fusion Based Approach for Group Decision Making Based on Different Uncertain Preference Relations

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Abstract Many decision making problems such as personnel promotion and investment selection in organizations are often dealt by multiple experts in uncertain situation. The aim of this paper is to develop an approach to solve group decision making problems where the preference information provided by experts is in the form of interval multiplicative preference relations, interval fuzzy preference relations and interval linguistic preference relations. Firstly, taking interval linguistic preference relation as base representation element, non-homogenous information is unified by transformation functions. Then, an optimization model is established to obtain the maximum consensus level among the group by searching the weights of the experts. Next, the aggregation process and the selection process are carried out to find the best alternative(s). The entire procedure of the proposed approach is given. Finally, an example of the manager selection for a company is provided to show the effectiveness of the proposed approach.

Keywords Group decision making · Uncertain preference relation · Linguistic variable · Consensus · Alternative selection

44.1 Introduction

Decision making problems that address choosing the most appropriate option have been widely studied in the last decade [3, 6, 14]. For example, selecting a suitable advanced manufacturing technology is an important issue in operations managers when making capital investment decisions to improve their manufacturing performance [3]. In practice, because of the increasing complexity of the social-economic environment nowadays, many organizations have moved from a single

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J. Xu et al. (eds.), *Proceedings of the Seventh International Conference on Management Science and Engineering Management (Volume 1)*,
Lecture Notes in Electrical Engineering 241, DOI: 10.1007/978-3-642-40078-0_44,

decision maker or expert to a group of experts to accomplish the given tasks successfully.

In decision making problems with multiple experts as group decision making (GDM) problems, each expert expresses his/her preferences depending on the nature of the alternatives and on his/her own knowledge over them [10]. Preference relations are a popular and powerful tool used by experts to provide their preference information in the decision process. The use of preference relations facilitates experts when expressing their preferences. Alternative selection problems are mainly related to qualitative aspects which make it difficult to qualify them using precise values. To capture the uncertainty contained in these problems, various types of uncertain preference relations have been investigated in the literature, including interval multiplicative preference relations e.g. [17], interval fuzzy preference relations e.g. [20], and interval linguistic preference relations e.g. [5].

In GDM, it is quite natural that different experts who may have different background and knowledge will provide their preferences by different kinds of preference relations [7]. The use of non-homogenous information in decision problems is not an unusual situation [4, 10]. In this paper, we will assume a GDM model in which the preferences can be provided in any of the uncertain preference relations: interval multiplicative preference relations, interval fuzzy preference relations, and interval linguistic preference relations with multi-granularity. The main problem to deal with non-homogenous contexts is how to aggregate the information assessed in these contexts. Different methods have been proposed to unify the input information [4, 7, 8, 10, 13, 21].

Prior studies have made significant contributions to the GDM with diverse nature. However, there is no sufficient information to model the GDM problem with uncertain preference relations. The aim of this paper is to present a model for GDM with different uncertain preference relations. To make the information uniform, we extended the transformation functions into uncertain situations. To obtain the maximum degree of consensus in the aggregation process, an optimization model is constructed. The rest of this the paper is organized as follows. Sect. 44.2 deals with the preliminaries necessary to develop our model. Sect. 44.3 presents the conceptual framework of the proposed model. Sect. 44.4 introduces the information fusion methods. Sect. 44.5 presents the optimization method to compute the maximum consensus level of the group. Finally, an example of the manager selection for a company is provided in Sect. 44.6 and some concluding remarks are included in Sect. 44.7.

44.2 Basic Concepts and Definitions

In this section, we briefly introduce the basic concepts of linguistic approaches, and then for the convenience of analysis, we give definitions of different preference relations.

The linguistic approach considers the variables which participate in the problem assessed by means of linguistic terms, that is, variables whose values are not numbers but words or sentences in a nature or artificial language [23]. The basic notations and operational laws of linguistic variables can be found in [9] and [19]. Suppose that $S = \{s_\alpha | \alpha = -t, \cdots, -1, 0, 1, \cdots, t\}$ be a linguistic term set whose cardinality value is an odd one, where s_α represents a possible value for a linguistic variable. The semantics of the terms is given by fuzzy numbers defined in the $[0,1]$ interval, which are usually described by membership functions.

It is usually required that s_i and s_j satisfy the following additional characteristics:
(1) The set is ordered: $s_i > s_j$, if $i > j$;
(2) There is a negation operator: $\mathrm{neg}(s_i) = s_{-i}$, especially, $\mathrm{neg}(s_0) = s_0$;

In the process of information aggregation, however, some results may not exactly match any linguistic labels in S. To preserve all the information, Xu [19] extend the discrete linguistic label set S to a continuous linguistic label set $\bar{S} = \{s_\alpha | s_{-q} \leq s_\alpha \leq s_q, \alpha \in [0,q]\}$, where s_α meets all the characteristics above and $q(q \geq t)$ is a sufficiently large positive integer. If $s_\alpha \in S$, s_α is called the original term, otherwise, s_α is called the virtual term. In general, the original term is used to evaluate alternatives, while the virtual term can only appear in operations.

Definition 44.1. Let $\tilde{s} = [s_\alpha, s_\beta]$, where $s_\alpha, s_\beta \in \overline{S}$, s_α and s_β are the lower and upper limits, respectively, then $\tilde{s}$ is called an uncertain linguistic variable.

Let $\tilde{S}$ be the set of all uncertain linguistic variables. Consider any three uncertain linguistic variables $\tilde{s} = [s_\alpha, s_\beta]$, $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}]$, then their operational laws are defined as:
(1) $\tilde{s}_1 \oplus \tilde{s}_2 = [s_{\alpha_1}, s_{\beta_1}] \oplus [s_{\alpha_2}, s_{\beta_2}] = [s_{\alpha_1} \oplus s_{\alpha_2}, s_{\beta_1} \oplus s_{\beta_2}] = [s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}]$;
(2) $\tilde{s}_1 \oplus \tilde{s}_2 = \tilde{s}_2 \oplus \tilde{s}_1$;
(3) $\mu\tilde{s} = \mu[s_\alpha, s_\beta] = [\mu s_\alpha, \mu s_\beta] = [s_{\mu\alpha}, s_{\mu\beta}]$, where $\mu \in [0,1]$.

From the above descriptions, we can see that the operation on two linguistic terms can be converted to the operation on lower indices of the corresponding terms. Thus we denote $I(s)$ as the positive index of s in $\bar{S}$. For example, $I(s_\alpha) = \alpha$. The function I translates a linguistic term to a numerical one and has an inverse function noted as I^{-1} which translates a numerical value into a linguistic type.

In many practical cases, crisp values are inadequate to model real-life decision problems because of the inherent subjective nature of the human judgments. The experts may have vague knowledge about the preference degrees of one alternative over another, and can not estimate their preferences with exact values. It is suitable for the experts expressing their opinions with uncertain formats of preference relation. Different uncertain preference relations are stated as follows.

Definition 44.2. [15] An interval multiplicative preference relation on a set of alternatives X is represented by an interval matrix, $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, $\tilde{a}_{ij} = [a_{ij}^L, a_{ij}^U]$, $0 < a_{ij}^L \leq a_{ij}^U$, being $\tilde{a}_{ij}$ belonged to the Satty's $[1/9,\ 9]$ scale. The reciprocal property by assumption holds, i.e., $a_{ij}^L = 1/a_{ji}^U$, $a_{ij}^U = 1/a_{ji}^L$, $a_{ii}^L = a_{ii}^U = 1$, for all $i, j \in N$.

Definition 44.3. [21] An interval fuzzy preference relation on a set of alternatives X is represented by an interval matrix, $\tilde{P} = (\tilde{p}_{ij})_{n\times n}$, $\tilde{p}_{ij} = [p_{ij}^L, p_{ij}^U]$, $0 \leq p_{ij}^L \leq p_{ij}^U$, being $\tilde{p}_{ij}$ belonged to the $[0,\ 1]$ scale. The reciprocal property by assumption holds, i.e., $p_{ij}^L + p_{ji}^U = p_{ij}^U + p_{ji}^L = 1$, $p_{ii}^L = p_{ii}^U = 0.5$, for all $i, j \in N$.

Definition 44.4. [16] Let S_T be a linguistic terms set with granularity T. An interval linguistic preference relation on a set of alternatives X is represented by an interval matrix, $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$, $\tilde{l}_{ij} = [l_{ij}^L, l_{ij}^U] \in \tilde{S}$, where the reciprocal property by assumption holds, i.e., $l_{ij}^L \oplus l_{ji}^U = l_{ij}^U \oplus l_{ji}^L = s_0$, $l_{ii}^L = l_{ii}^U = s_0$, for all $i, j \in N$.

44.3 A Conceptual Model Based on Uncertain Preference Relations

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite set of alternatives. The alternatives will be classified from best to worst (ordinal ranking), using the information already known according to a finite set of purposes. Without loss of generality, suppose there are m experts $E = \{e_1, e_2, \cdots, e_m\}$ who provide their evaluations on alternatives with different preference relations. Let $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$ be the weight vector of experts which is to be determined. Suppose m_1 experts give uncertain multiplicative preference relations $\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_{m_1}\}$, m_2 experts give uncertain fuzzy preference relations $\{\tilde{P}_1, \tilde{P}_2, \cdots, \tilde{P}_{m_2}\}$, and m_3 experts give uncertain linguistic preference relations $\{\tilde{L}_1, \tilde{L}_2, \cdots, \tilde{L}_{m_3}\}$, such that $m = m_1 + m_2 + m_3$. The problem addressed in this paper is how to rank alternatives or select desired alternatives in a rational way.

The proposed method for solving the above GDM problem with different kinds of uncertain preference relations is presented graphically in Fig. 44.1.

Although any one kind of uncertain preference relation can be used to manage the non-homogenous information, the linguistic type of preference relation is selected as the representation model for the convenience of computation. First, we use the transformation functions proposed in the next section to make the different uncertain preference relations uniform. Then, the decision model develops two steps to accomplish the selection process. The aggregation phase utilizes the uncertain linguistic weighted average operator $ULWA$ to aggregate information which guarantees that the collective preference relation is a reciprocal interval linguistic preference relation as well. A maximizing consensus method is introduced to determine the weights of the experts. Such weight vector is also used in the aggregation process. The exploitation phase consists of choosing the alternatives "best" acceptable to the group of individuals as a whole. To do so, the uncertain linguistic ordered weighted average operator $ULOWA$ acts over the collective linguistic preference relation to quantify the dominance of one alternative over all the others in a fuzzy majority sense. Finally, a ranking method could be used to obtain the rank order of the alternatives and the best option.

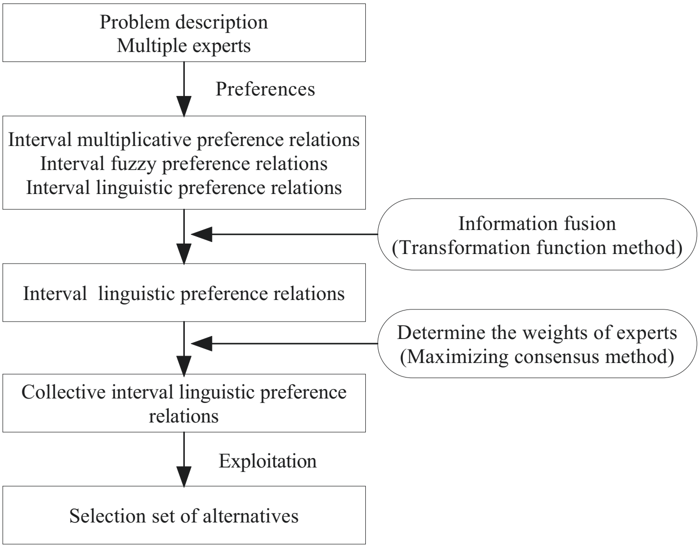

Fig. 44.1 The resolution process of the GDM problem

44.4 Fusions of Different Uncertain Preference Relations

In this section, we give different transformation functions to make the different scales uniform. Based on the work of [4] and [7], we get the following propositions.

Proposition 44.1. *Let S_{T_1} and S_{T_2} be two linguistic scales predefined as in Sect. 44.2. Their granularities are T_1 and T_2, respectively. Let, $l_1 \in S_{T_1}, l_2 \in S_{T_2}$. Then the transformation function from S_{T_1} to S_{T_2} is given as follows:*

$$l_2 = \frac{T_2 - 1}{T_1 - 1} l_1. \tag{44.1}$$

Proposition 44.2. *Let P be the $[0,1]$ fuzzy scale, and S_T be a linguistic scale with granularity T. Let $p \in P$ and $l \in S_T$. Then the corresponding transformation function from P to S_T is given as follows:*

$$l = I^{-1}((T-1)(p-0.5)). \tag{44.2}$$

Proposition 44.3. *Let A be the $[1/9, 9]$ scale, and S_T be a linguistic scale with granularity T. Let $a \in A$ and $l \in S_T$, then the corresponding transformation function from A to S_T is given as follows:*

$$l = I^{-1}\left(\frac{T-1}{2} \log_9 a\right). \tag{44.3}$$

In a situation, where different experts give different uncertain preference relations, to make the information uniform, we have the following transformation functions.

Proposition 44.4. *Let S_{T_1} and S_{T_2} are two linguistic scales predefined as in Sect. 44.2. Their granularities are T_1 and T_2, respectively. Let $\tilde{L}_1 = (\tilde{l}_{ij,1})_{n\times n}$ and $\tilde{L}_2 = (\tilde{l}_{ij,2})_{n\times n}$ be two interval linguistic preference relations, where $\tilde{l}_{ij,1} = [l^L_{ij,1}, l^U_{ij,1}]$, $l^L_{ij,1}$, $l^U_{ij,1} \in S_{T_1}$, $\tilde{l}_{ij,2} = [l^L_{ij,2}, l^U_{ij,2}]$, $l^L_{ij,2}, l^U_{ij,2} \in S_{T_2}$. Then the transformation function from $\tilde{L}^1$ to $\tilde{L}^2$ is given as follows:*

$$\tilde{l}_{ij,2} = \frac{T_2 - 1}{T_1 - 1}\tilde{l}_{ij,1}. \tag{44.4}$$

Proof. This proposition is a generalization of Proposition 44.1. We only need to verify that the preference relation after the transformation function (44.4) is in deed a reciprocal interval linguistic preference relation. Firstly, we have:

$$l^L_{ij,2} \oplus l^U_{ji,2} = \frac{T_2 - 1}{T_1 - 1} l^L_{ij,1} \oplus \frac{T_2 - 1}{T_1 - 1} l^U_{ji,1} = \frac{T_2 - 1}{T_1 - 1}(l^L_{ij,1} \oplus l^U_{ji,1}) = s_0. \tag{44.5}$$

In a similar way, we have $l^U_{ij,2} \oplus l^L_{ji,2} = s_0$. This completes the proof of Proposition 44.4. □

Proposition 44.5. *Let $\tilde{P} = (\tilde{p}_{ij})_{n\times n}$ be an interval fuzzy preference relation, and $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ be an interval linguistic preference relation with granularity T. Then the corresponding transformation function from $\tilde{P}$ to $\tilde{L}$ is given as follows:*

$$\tilde{l}_{ij} = I^{-1}((T-1)(\tilde{p}_{ij} - 0.5)). \tag{44.6}$$

Proof. This proposition is a generalization of Proposition 44.2. We only need to verify the reciprocal property of $\tilde{L}$. According to the reciprocity of $\tilde{P} = (\tilde{p}_{ij})_{n\times n}$, we have

$$I(l^L_{ij}) + I(l^U_{ji}) = (T-1)(p^L_{ij} - 0.5)) + (T-1)(p^U_{ji} - 0.5)) = (T-1)((p^L_{ij} + p^U_{ji}) - 1) = 0. \tag{44.7}$$

From Equation (44.7), it follows that:

$$l^L_{ij} \oplus l^U_{ji} = s_0. \tag{44.8}$$

Similarly, we get $l^U_{ij} \oplus l^L_{ji} = s_0$. Thus $\tilde{L}$ is reciprocal, This completes the proof of Proposition 44.5. □

Proposition 44.6. *Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ be an interval multiplicative preference relation, and $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ be an interval linguistic preference relation with granularity T. Then the corresponding transformation function from $\tilde{A}$ to $\tilde{L}$ is given as follows:*

$$\tilde{l}_{ij} = I^{-1}\left(\frac{T-1}{2}\log_9 \tilde{a}_{ij}\right). \tag{44.9}$$

Proof. This proposition is a generalization of Proposition 44.3. Again, we verify the reciprocal property of $\tilde{L}$. According to the reciprocity of $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, we have:

$$I(l_{ij}^L) + I(l_{ji}^U) = \frac{T-1}{2}\log_9 a_{ij}^L + \frac{T-1}{2}\log_9 a_{ji}^U = \frac{T-1}{2}\log_9 a_{ij}^L a_{ji}^U = 0. \quad (44.10)$$

From Equation (44.10), it follows that:

$$l_{ij}^L \oplus l_{ji}^U = s_0. \quad (44.11)$$

Similarly, we get $l_{ij}^U \oplus l_{ji}^L = s_0$. Thus $\tilde{L}$ is reciprocal, This completes the proof of Proposition 44.6. □

In this section, we give propositions to make different uncertain preference relations uniform, given the interval linguistic preference relation as the basic representation structure. The transformations between different preference relations are the generalizations of the conversions between different judgement scales in essence.

44.5 Maximizing Consensus

The experts's judgements are somewhat subjective. It is possible that conflicts and contradicts may exist among the experts. Therefore, before the selection process, a consensus process is carried out to obtain a solution of maximum degree of agreement between the set of group members. Some researches have presented interesting results on consensus models based linguistic information e.g. [1, 2, 11, 19] or other information formats e.g. [12, 20]. Generally speaking, research progress in GDM with one kind of preference relation can benefit research in other kind preference relation. In the following, we introduce an optimization model to obtain the maximal consensus level among the experts for GDM with interval linguistic preference relations.

Definition 44.5. [5] Let $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$, $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}]$ be two uncertain linguistic variables, the distance between $\tilde{s}_1$ and $\tilde{s}_2$ is defined as:

$$d(\tilde{s}_1, \tilde{s}_2) = \frac{1}{2t}(|I(s_{\alpha_1}) - I(s_{\alpha_2})| + |I(s_{\beta_1}) - I(s_{\beta_2})|). \quad (44.12)$$

Definition 44.6. Let $\tilde{L}_1 = (\tilde{l}_{ij,1})_{n\times n}, \tilde{L}_2 = (\tilde{l}_{ij,2})_{n\times n}, \cdots, \tilde{L}_m = (\tilde{l}_{ij,m})_{n\times n}$ be m interval linguistic preference relations. Then their weighted combination $\tilde{L} = \lambda_1\tilde{L}_1 \oplus \lambda_2\tilde{L}_2 \oplus \cdots \oplus \lambda_m\tilde{L}_m$ by *ULWA* operator is the group interval linguistic preference relation, $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$, where, $\tilde{l}_{ij} = \lambda_1\tilde{l}_{ij,1} \oplus \lambda_2\tilde{l}_{ij,2} \oplus \cdots \oplus \lambda_m\tilde{l}_{ij,m}$.

Definition 44.7. Let $\tilde{L}_1 = (\tilde{l}_{ij,1})_{n\times n}, \tilde{L}_2 = (\tilde{l}_{ij,2})_{n\times n}, \cdots, \tilde{L}_m = (\tilde{l}_{ij,m})_{n\times n}$ and $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ be m interval linguistic preference relations and the group interval linguistic preference relation, respectively. Then based on the distance function d, the group consensus index of $\tilde{L}_k$ is defined by:

$$\mathrm{GCI}(\tilde{L}_k) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d(\tilde{l}_{ij,k}, \tilde{l}_{ij}). \tag{44.13}$$

Let $e = (1, 1, \cdots, 1)^T$ be a m dimensional vector. Let $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$ be the weight vector of experts such that $e^T\lambda = 1$, $\lambda_i \geq 0$. It is most desirable that the consensus indexes of every expert should be kept as small as possible, which leads to the following optimization model to be constructed.

$$\begin{aligned} &\min\ \mathrm{GCI}(\tilde{L}_k),\ k = 1, 2, \cdots, m, \\ &\text{s.t. } e^T\lambda = 1,\ \lambda \geq 0. \end{aligned}$$

Considering every single objective is of equal importance, the above model can be transformed into the following concrete programming:

$$\begin{aligned} &\min\ J = \tfrac{1}{t \times n \times (n-1)} \sum_{k=1}^{m} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d(\tilde{l}_{ij,k}, \tilde{l}_{ij}), \\ &\text{s.t. } \sum_{k=1}^{m} \lambda_k = 1,\ \lambda_k \geq 0,\ k = 1, 2, \cdots, m. \end{aligned}$$

Letting

$$\varepsilon_{ij,k}^{+} = \frac{1}{2}(\varepsilon_{ij,k} + |\varepsilon_{ij,k}|) \quad \text{and} \quad \varepsilon_{ij,k}^{-} = \frac{1}{2}(-\varepsilon_{ij,k} + |\varepsilon_{ij,k}|), \tag{44.14}$$

$$\eta_{ij,k}^{+} = \frac{1}{2}(\eta_{ij,k} + |\eta_{ij,k}|) \quad \text{and} \quad \eta_{ij,k}^{-} = \frac{1}{2}(-\eta_{ij,k} + |\eta_{ij,k}|), \tag{44.15}$$

where

$$\varepsilon_{ij,k} = I(l_{ij,k}^{L}) - \sum_{h=1}^{m} \lambda_h I(l_{ij,h}^{L}) \quad \text{and} \quad \eta_{ij,k} = I(l_{ij,k}^{U}) - \sum_{h=1}^{m} \lambda_h I(l_{ij,h}^{U}). \tag{44.16}$$

We have:

$$|\varepsilon_{ij,k}| = \varepsilon_{ij,k}^{+} + \varepsilon_{ij,k}^{-} \quad \text{and} \quad |\eta_{ij,k}| = \eta_{ij,k}^{+} + \eta_{ij,k}^{-}. \tag{44.17}$$

Accordingly, the optimization model can be rewritten as the following linear programming model:

$$\min J=\frac{1}{t\times n\times(n-1)}\sum_{k=1}^{m}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\varepsilon_{ij,k}^{+}+\varepsilon_{ij,k}^{-}+\eta_{ij,k}^{+}+\eta_{ij,k}^{-}),$$
$$\text{s.t.}\begin{cases} I(l_{ij,k}^{L})-\sum\limits_{h=1}^{m}\lambda_h I(l_{ij,h}^{L})-\varepsilon_{ij,k}^{+}+\varepsilon_{ij,k}^{-}=0,\\ \quad i=1,2,\cdots,n-1,\ j=i+1,\cdots,n,\ k=1,2,\cdots,m,\\ I(l_{ij,k}^{U})-\sum\limits_{h=1}^{m}\lambda_h I(l_{ij,h}^{U})-\eta_{ij,k}^{+}+\eta_{ij,k}^{-}=0,\\ \quad i=1,2,\cdots,n-1,\ j=i+1,\cdots,n,\ k=1,2,\cdots,m,\\ \sum\limits_{k=1}^{m}\lambda_k=1,\quad \lambda_k\geq 0,\quad k=1,2,\cdots,m. \end{cases} \tag{44.18}$$

The solution to the above problem could be easily found by Matlab software or Lingo software package. Thus, we obtain the weights of the experts by considering maximizing the consensus among the group.

In sum, a procedure for alternative selection in GDM with different formats of uncertain preference relations is given in the following.

Algorithm 1

Step 1. Form a committee of experts and identify the set of alternatives.
Step 2. Each expert provides preferences over the alternatives in the form of different uncertain preference relations.
Step 3. Making the information uniform. Utilize Proposition 44.6 to transform different uncertain preference relations into the same kind of interval linguistic preference relations.
Step 4. Aggregation. Utilize optimization model (44.18) to obtain the weight vector of the experts. Compute the collective interval linguistic preference relation by uncertain linguistic weighted average (*ULWA*) operator.
Step 5. Exploitation. Calculate the overall assessment value of each alternative by the uncertain linguistic ordered weighted average(*ULOWA*) operator.
Step 6. Rank the overall assessment values. Choose the best alternative with the maximal ranking value.
Step 7. Return the result to the experts and the decision maker. If the result is convincing and the decision maker is satisfied with it, then terminate the decision procedure, otherwise start a new round of decision.

Remark 44.1. If the consensus level reached does not meet predefined requirements, a consensus reaching algorithm can be introduced to achieve the goal [18]. We only consider one preference relation given by each expert for the alternative selection problem in this paper. However, similar to the analytical hierarchy process (AHP), we may consider constructing a hierarchy structure based on uncertain preference relations and extended the proposed approach into this general case.

44.6 Application Example

In this section, an example is provided to demonstrate how the proposed approach works in practice. Suppose a company wants to employ a new manger to manage its new business. Four outstanding candidate $\{x_1, x_2, x_3, x_4\}$ entered the final round of competition. There are four experts, $\{e_1, e_2, e_3, e_4\}$ provide their evaluations over the candidates according to the general performance of each candidate. Concretely, expert e_1 constructs an interval multiplicative preference relation, $\tilde{A}$, using Saaty's $[1/9, 9]$ scale. Expert e_2 an interval fuzzy preference relation, $\tilde{P}$, using the $[0, 1]$ scale. Experts e_3 and e_4 provide interval linguistic preference relations $\tilde{L}_3, \tilde{L}_4$ with granularity 7 and 9, respectively. These data are shown as follows.

$$\tilde{A} = \begin{pmatrix} [1,1] & [1,2] & [6,8] & [3,5] \\ [1/2,1] & [1,1] & [5,7] & [3,4] \\ [1/8,1/6] & [1/7,1/5] & [1,1] & [1/3,1/2] \\ [1/5,1/3] & [1/4,1/3] & [2,3] & [1,1] \end{pmatrix},$$

$$\tilde{P} = \begin{pmatrix} [0.5,0.5] & [0.5,0.6] & [0.8,0.9] & [0.6,0.8] \\ [0.4,0.5] & [0.5,0.5] & [0.7,0.8] & [0.5,0.7] \\ [0.1,0.2] & [0.2,0.3] & [0.5,0.5] & [0.3,0.4] \\ [0.2,0.4] & [0.3,0.5] & [0.6,0.7] & [0.5,0.5] \end{pmatrix},$$

$$\tilde{L}_3 = \begin{pmatrix} [s_0,s_0] & [s_0,s_1] & [s_2,s_3] & [s_1,s_2] \\ [s_{-1},s_0] & [s_0,s_0] & [s_2,s_3] & [s_0,s_1] \\ [s_{-3},s_{-2}] & [s_{-3},s_{-2}] & [s_0,s_0] & [s_{-2},s_0] \\ [s_{-2},s_{-1}] & [s_{-1},s_0] & [s_0,s_2] & [s_0,s_0] \end{pmatrix},$$

$$\tilde{L}_4 = \begin{pmatrix} [s_0,s_0] & [s_1,s_2] & [s_3,s_4] & [s_1,s_2] \\ [s_{-2},s_{-1}] & [s_0,s_0] & [s_2,s_3] & [s_0,s_0] \\ [s_{-4},s_{-3}] & [s_{-3},s_{-2}] & [s_0,s_0] & [s_{-2},s_{-1}] \\ [s_{-2},s_{-1}] & [s_0,s_0] & [s_1,s_2] & [s_0,s_0] \end{pmatrix}.$$

On the basis of Algorithm 1 described in the last section, the resolution process for this problem is divided into the following stages.

Stage 1: Make the information uniform

According to the methods in Sect. 44.4, we transform $\tilde{A}, \tilde{P}, \tilde{L}_3$ into interval linguistic preference relations with granularity 9. The transformed linguistic preference relations for e_1 and e_2 are denoted as $\tilde{L}_1$, $\tilde{L}_2$, and for e_3, it is still denoted as $\tilde{L}_3$ for simplicity.

$$\tilde{L}_1 = \begin{pmatrix} [s_0,s_0] & [s_0,s_{1.3}] & [s_{3.3},s_{3.8}] & [s_1,s_2] \\ [s_{-1.3},s_0] & [s_0,s_0] & [s_{2.9},s_{3.5}] & [s_2,s_{2.5}] \\ [s_{-3.8},s_{-3.3}] & [s_{-3.5},s_{-2.9}] & [s_0,s_0] & [s_{-2},s_{-1.3}] \\ [s_{-2.9},s_{-2}] & [s_0,s_0] & [s_1,s_2] & [s_0,s_0] \end{pmatrix},$$

$$\tilde{L}_2 = \begin{pmatrix} [s_0, s_0] & [s_0, s_{0.8}] & [s_3, s_4] & [s_{0.8}, s_{2.4}] \\ [s_{-0.8}, s_0] & [s_0, s_0] & [s_{1.6}, s_{2.4}] & [s_0, s_{1.6}] \\ [s_{-3.2}, s_{-2.4}] & [s_{-2.4}, s_{-1.6}] & [s_0, s_0] & [s_{-1.6}, s_{-0.8}] \\ [s_{-2.4}, s_{-0.8}] & [s_{-1.6}, s_0] & [s_{0.8}, s_{1.6}] & [s_0, s_0] \end{pmatrix},$$

$$\tilde{L}_3 = \begin{pmatrix} [s_0, s_0] & [s_0, s_{4/3}] & [s_{8/3}, s_4] & [s_{4/3}, s_{8/3}] \\ [s_{-4/3}, s_0] & [s_0, s_0] & [s_{8/3}, s_4] & [s_0, s_{4/3}] \\ [s_{-4}, s_{-8/3}] & [s_{-4}, s_{-8/3}] & [s_0, s_0] & [s_{-8/3}, s_0] \\ [s_{-8/3}, s_{-4/3}] & [s_{-1.6}, s_0] & [s_0, s_{8/3}] & [s_0, s_0] \end{pmatrix}.$$

Stage 2: Aggregation

To obtain the collective interval linguistic preference relation, we have to determine the weights of expert in the aggregation process. Using the maximizing consensus model (44.18), we construct the corresponding optimization model.

The optimal value for the above linear programming by Lingo software is $J = 20.36667$. At the same time, we get the weight vector of the expert $\lambda = (0.098,\ 0.453,\ 0.273,\ 0.176)^T$. From Definition 44.6, the group interval linguistic preference relation is computed as:

$$\tilde{L}_3 = \begin{pmatrix} [s_0, s_0] & [s_{0.18}, s_{1.20}] & [s_{2.66}, s_{3.62}] & [s_{1.10}, s_{2.45}] \\ [s_{-1.20}, s_{-0.18}] & [s_0, s_0] & [s_{2.09}, s_{3.06}] & [s_{0.20}, s_{1.34}] \\ [s_{-3.62}, s_{-2.66}] & [s_{-3.06}, s_{-2.09}] & [s_0, s_0] & [s_{-2.00}, s_{-0.66}] \\ [s_{-2.45}, s_{-1.10}] & [s_{-1.34}, s_{-0.20}] & [s_{0.66}, s_{2.00}] & [s_0, s_0] \end{pmatrix}.$$

Stage 3: Exploitation

We use the *ULOWA* operator with a quantifier *at least half*, which implies $(0, 0.4, 0.5, 0.1)^T$ is the weighting vector, to compute the overall assessment value for each alternative. We obtain the interval priority vector:

$$([0.5275, 1.5831],\ [-0.0417, 0.5170],\ [-2.6898, -1.5775],\ [-0.9138, -0.2081])^T$$

corresponding to each alternative. As the interval data are not intersecting, we immediately have $x_1 \succ x_2 \succ x_4 \succ x_3$. The best choice is the first candidate, x_1.

44.7 Concluding Remarks

In management decision making problem, because of the internal subjectivity and imprecision of human judgments, the information available from the multiple experts often appears as different uncertain formats. In this paper, we have developed an information fusion and maximizing consensus integrated approach to deal with such problems. An example of selecting the optimal manager for a company is illustrated to show the effective of the proposed model. The main characteristics of the proposed model are: 1) It allows experts to express their opinions with much flexibility; 2) It incorporates consensus concept into the aggregation process which makes the final solution more acceptable by the experts as well as the decision maker. Although we develop our model initially for alternative selection, it can be applied to other management decision problems. The proposed methods can also be extended into group multiple criteria decision making problems, which allow various evaluation scales to express the attribute values of the alternatives.

Acknowledgements This work is supported by Sichuan University Young Teacher Scientific Research Starting Funds under Grant No. 2012SCU11086 and 2012SCU11016.

References

1. Ben-Arieh D, Chen ZF (2006) Linguistic-labels aggregation and consensus measure for autocratic decision making using group recommendations. IEEE Transactions on Systems, Man, and Cybernetics. Part A: Systems and Humans 36:558–568
2. Cabrerizo FJ, Moreno JM, Pérez IJ et al (2010) Analyzing consensus approaches in fuzzy group decision making: advantages and drawbacks. Soft Computing 14:451–463
3. Chang PL, Chen YC (1994) A fuzzy multi-criteria decision making method for technology transfer strategy selection in biotechnology. Fuzzy Sets and Systems 63:131–139
4. Chiclana F, Herrera F, Herrera-Viedma E (1998) Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations. Fuzzy Sets and Systems 97:33–48
5. Chen HY, Zhou LG, Han B (2011) On compatibility of uncertain additive linguistic preference relations and its application in the group decision making. Knowledge-Based Systems 24:816–823
6. Chuu SJ (2009) Selecting the advanced manufacturing technology using fuzzy multiple attributes group decision making with multiple fuzzy information. Computers & Industrial Engineering 57:1033–1042
7. Dong YC, Xu, YF, Yu S (2009) Linguistic multiperson decision making based on the use of multiple preference relations. Fuzzy Sets and Systems 160:603–623
8. Fan ZP, Liu Y (2010) A method for group decision-making based on multi-granularity uncertain linguistic information. Expert Systems with Applications 37:4000–4008
9. Herrera F, Herrera-Viedma E (2000) Linguistic decision analysis: steps for solving decision problems under linguistic information. Fuzzy Sets and Systems 115:67–82
10. Herrera F, Martínez L, Sánchez PJ (2005) Managing non-homogeneous information in group decision making. European Journal of Operational Research 166:115–132
11. Herrera-Viedma E, Martínez L, Mata F, Chiclana F (2005) A consensus support systems model for group decision making problems with multigranular linguistic preference relations. IEEE Transactions on Fuzzy Systems 13:644–658
12. Kacprzyk J, Zadrożny S (2010) Soft computing and Web intelligence for supporting consensus reaching. Soft Computing 14:833–846
13. Ma J, Fan ZP, Jiang YP, Mao JY (2006) An optimization approach to multiperson decision making based on different formats of preference information. IEEE Transactions on Systems, Man, and Cybernetics. Part A: Systems and Humans 36:876–889
14. Parreiras RO, Ekel PYa, Morais DC (2012) Fuzzy set based consensus schemes for multicriteria group decision making applied to strategic planning. Group Decision and Negotiation 21:153–183
15. Saaty TL, Vargas L (1987) Uncertainty and rank order in the analytic hierarchy process. European Journal of Operational Research 32: 107–117.
16. Tapia García JM, del Moral MJ, Martínez MA, Herrera-Viedma E. (2012) A consensus model for group decision making problems with linguistic interval fuzzy preference relations. Expert Systems with Applications 39: 10022–10030.

17. Wang YM, Elhag TMS (2007) A goal programming method for obtaining interval weights from an interval comparison matrix. European Journal of Operational Research 177:458–471
18. Xu JP, Wu ZB (2013) A maximizing consensus approach for alternative selection based on uncertain linguistic preference relations. Computers & Industrial Engineering 64:999–1008
19. Xu ZS (2005) Deviation measures of linguistic preference relations in group decision making. Omega 33:249–254
20. Xu ZS (2011) Consistency of interval fuzzy preference relations in group decision making. Applied Soft Computing 11:3898–3909.
21. Xu ZS, Chen J (2008) MAGDM linear-programming models with distinct uncertain preference structures. IEEE Transactions on Systems, Man, and Cybernetics. Part B: Cybernetics 38:1356–1370
22. Yager RR (1988) On ordered weighted averaging aggregation operators in multicriteria decision making. IEEE Transactions on Systems, Man, and Cybernetics 18:183–190
23. Zadeh LA (1975) The concept of a linguistic variable and its applications to approximate reasoning, Part I. Information Sciences 8:199–249