Chapter 12
Simplification of Large Scale Network in Time-cost Tradeoff Problem

Zhixiong Su, Jianxun Qi and Zhinan Kan

Abstract For the time-cost tradeoff problem, if the involved super large-scale CPM network is simplified, then any correlative algorithm which used to solve the problem is simplified too. According to the idea, firstly, property of free float and relation of free float and path length is analyzed, and some new conceptions and free float theorem are deduced; secondly, an algorithm of simplifying the super large-scale network in time-cost tradeoff problem is designed by using these conceptions and the theorem, and validity of the algorithm is proved; finally, application of the algorithm is discussed by illustration. The theoretic proof and illustration show that if the algorithm is used to simplify the time-cost tradeoff problem, any correlative algorithm which used to solve the problem could be greatly simplified.

Keywords CPM network planning · Time-cost trade off problem · Free float theorem · Simplification

12.1 Introduction

Through developing quickly more than ten years, modern project management not only becomes a new knowledge, but also has become a profession. According to the Project Management Body of Knowledge (PMBOK for short) which written by America Project Management Body of Knowledge (PMI for short), project management has been separated into nine domains. Thereinto, "project time management" and "project cost management" are two core domains [1]. The time-cost tradeoff problem [2–4] represents crossover of the two core domains, and it is applied very widely in practice.

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There are mainly two aspects about time-cost tradeoff problem: firstly, which activities' durations need to be shortened; secondly, how many quantities of these activities' durations need to be shortened. Current studies [5–14] have proved that using CPM network planning technology especially theory of float to analyze time-cost tradeoff problem could solve above two aspects problems more intuitionisticly. At present, there are five conceptions of float, which named total float, free float, safety float, node float and interference float respectively in international. But now common algorithms [15, 16] are difficult to solve the problem, and have biggish computation, especially when face super large scale project. One very important reason is that the whole project needs to be considered object when using these algorithms. Generally speaking, people are mainly interested to simplification of algorithm, by improving and designing algorithm to try to decrease difficult of solving problem. Although the approach is feasible, it is hard to avoid biggish difficulty, and limited algorithms could be accomplished by using the approach.

Now we could try to consider from other angle. Under many conditions, all involved objects need not to be considered, therefore, if object which might be considered in problem could be simplified, and some parts which need not to be considered are deleted, then the problem could be solved more simple by using any algorithms, and difficulty could be decreased. For example, for above time-cost tradeoff problem, if we want to shorten total duration of project by 5 days, we only need to decrease lengths of longer paths to 95 days which are bigger than 95 days in correlative network. If we could compose a sub-network with these paths whose lengths are bigger than 95 days, then it is equivalent to shorten total duration of the sub-network and original complicated network. Shorten total duration of the sub-network is simpler assuredly than shorten total float of original complicated network by using any algorithms. But find out path with certain length is very difficult in complicated network. Domestic and overseas scholars have designed many algorithms to simplify complicated network equivalently, but nearly all these algorithms have biggish complexity [17], or lack theory evidence [17, 18], and so on.

In this paper, according to the idea that simplify object of problem is equivalent to simplify any algorithms to solve the problem, through studying inherent rule of CPM network planning, properties of activity's free float and relation between free float and path's length, we deduce free float theorem. On the basis of the theory, for realizing purposes of simplifying object of problem and all correlative algorithms, we design algorithm to simplify super large-scale network equivalently in time-cost tradeoff problem, and don't affect the final result.

12.2 Conception and Theorem

12.2.1 Correlative Conception

(1) Total float

The total float of activity (i,j) which marked as TF_{ij} is defined as: $TF_{ij} = LS_{ij} - ES_{ij} = LF_{ij} - EF_{ij} = LT_j - ET_i - T_{ij}$. The total float denotes the time an activity can be delayed without causing a delay in the project.

(2) Free float

The free float of activity (i,j) which marked as FF_{ij} is computed as:

$$FF_{ij} = ES_{jr} - EF_{ij} = ET_j - ET_i - T_{ij}. \tag{12.1}$$

The free float denotes the time an activity can be delayed without affecting its immediate successor activities.

Similarly, the free float of any path μ which marked as FF_μ is computed as: $FF_\mu = \sum_{(i,j)\in\mu} FF_{ij}$. The path safety float FF_μ represents the sum of free float of all activities which are on any path μ in activity-on -arc representation network.

(3) Critical path

The critical path is the longest path in CPM network which marked as μ^∇. Activity and node on the path are named critical activity and activity node respectively.

(4) Fore main chain

The fore main chain of node (i) or activity (i,j) which marked as μ_i^* or μ_{ij}^* represents part of a path which starts from start node (i) composed of activities whose free floats are zero.

(5) Host activity, assistant activity and fundus activity

If $FF_{ij} > 0$, activity (μ,ν) on fore main chain of the activity (i,j) or node (j) is named host activity of the activity (i,j) or node (j). Immediate predecessor activity $(t,\mu) \notin \mu_i^*$ of node (μ) is named assistant activity of activity (i,j) or node (μ). And activity (i,j) is named fundus activity of its host activity and assistant activity. If $FF_{ij} > 0$, and (j) is critical node, then activity (i,j) is named assistant activity of network's terminal node (n).

(6) Eigenvalue of activity

The conception mainly contains three aspect:

- Eigenvalue which marked as $D_n(r,s)$ of assistant activity (r,s) of terminal node (n) is defined as free float FF_{rs} of the activity (r,s), viz. $D_n(r,s) = FF_{rs}$.
- Eigenvalue which marked as $D_r(t,u)$ of assistant activity (r,s) of any non-terminal node (r) is defined as sum of free float FF_{tu} and eigenvalue $D_r(r,s)$ of fundus activity (r,s) of activity (t,u), viz. $D_r(t,u) = FF_{tu} + D_r(r,s)$.
- Eigenvalue which marked as $D_r(u,v)$ of host activity (u,v) of any node (r) is defined as infinite, viz. $(u,v) \in \mu_r^*, D_r(u,v) = +\infty$.

12.2.2 Free Float Theorem

Theorem 12.1. *Margin of length of critical path μ^∇ minus length of any path μ is equal to sum of free floats of activities on the path μ, viz.*

$$FF_\mu = L(\mu^\nabla) - L(\mu). \tag{12.2}$$

Proof. Suppose any path marked as $\mu = (1) \to (a) \to (b) \to \cdots \to (e) \to (f) \to (n)$. There into node is start node and node (n) is terminal node. According to conception of path free float and Equation (12.1),

$$\begin{aligned} FF_{\mu} &= FF_{1a} + FF_{ab} + FF_{bc} + \cdots + FF_{ef} + FF_{fn} \\ &= (ET_a - ET_1 - T_{1a}) + (ET_b - ET_a - T_{ab}) + \cdots + (ET_n - ET_f - T_{fn}) \\ &= ET_n - ET_1 - (T_{1a} + T_{ab} + T_{bc} + \cdots + T_{fn}). \end{aligned}$$

In CPM network, $ET_1 = 0$ and $ET_n = L(\mu^{\nabla})$, then length of the path μ is $L(\mu) = T_{1a} + T_{ab} + T_{bc} + \cdots + T_{de} + T_{ef} + T_{fn}$, therefore $FF_{\mu} = L(\mu^{\nabla}) - l(mu)$. Equation (12.2) is correct. □

12.3 Simplification of Super Large-scale Network in Time-cost Tradeoff Problem

12.3.1 Description of Algorithm

For time-cost tradeoff problem, if we want to shorten total duration T to ΔT, we only need to shorten length of paths which are longer than $T - \Delta T$ in CPM network. If simplifying original complicated network to sub-network composed by paths whose lengths are bigger than $T - \Delta T$, then shorten total duration of the sub-network is equivalent to shorten total duration of original network. Therefore, Simplification of super large scale network in time-cost tradeoff problem is to delete path whose lengths are smaller than or equal to $T - \Delta T$ as more as possible. The process of simplification is described as follows (Ω^k represents muster):

Step 1. Find out critical path μ^{∇}, and find out assistant activity (i, j) of terminal node (n), and then compute eigenvalue $D_n(i, j)$ as follows: $D_n(i, j) = FF_{ij}$.

Step 2. Make $(k) = (n)$, and compare $D_k(i, j)$ and ΔT.

(1) If $D_k(i, j) < \Delta T$, put $D_k(i, j)$ into Ω^k;

(2) If $D_k(i, j) \geq \Delta T$, delete activity (i, j).

Step 3. Check.

(1) If $\Omega^k = \emptyset$, stop;

(2) If $\Omega^k \neq \emptyset$, turn to Step 4.

Step 4. Find out the minimal value $D_r(u_0, v)$ in Ω^k, and delete the value, and then find out fore main chain $\mu^*_{u_0}$ of node (u_0), viz. find out activities without free floats from predecessor activities of node (u_0).

Step 5. Find out each assistant activity (e, u_i) of node (u_0) from predecessor activities of node (u_i), $i = 0, 1, 2, \cdots, n$, and compute its eigenvalue as follows: $D_{u_0}(e, u_i) = FF_{eu_i} + D_r(u_0, v)$. Then make $(k) = (u_0)$, substitute $D_k(i, j)$ by $D_{u_0}(e, u_i)$, and turn to Step 2.

12.3.2 Analysis on Correctness of Algorithm

(1) Correctness of Step 4 could be proved by the conception of fore main chain.
(2) Correctness of Step 1 and 5 could be proved by the conception of eigenvalue.
(3) In Step 2-(2), now we prove that if deleting activity (i,j), lengths of disappeared paths are all smaller than or equal to $T-\Delta T$.
(a) According to conception, $D_k(i,j)=FF_{ij}+D_k(u_1,v_1)$, there into (u_1,v_1) is fundus activity of activity (i,j); in the same way, $D_k(u_1,v_1)=FF_{u_1v_1}+D_k(u_2,v_2)$, there into (u_2,v_2) is fundus activity of activity (u_1,v_1); $\cdots$; until to $D_k(k,v_n)=FF_{kv_n}$, there into node (v_n) is critical node, then: $D_k(i,j)=FF_{ij}+FF_{u_1v_1}+\cdots+FF_{kv_n}$. Suppose

$$\begin{aligned}\mu =\ & \mu_i^*+(i)\to(j)\to\cdots\to(u_1)\to(v_1)\to\cdots\to(u_2)\to(v_2)\\ & \to\cdots\to(u_{n-1})\to(v_{n-1})\to\cdots\to(k)\to(v_n)+\mu^{\nabla}_{v_n\to n},\end{aligned}$$

thereinto, for activities which locate between node (i) and (v_n) but don't be list in formula of μ, according to conception of host activity, they locate on fore main chains respectively and their free floats are all zero. Then we could deduce: $FF_\mu = FF_{ij}+FF_{u_1v_1}+\cdots+FF_{u_{n-1}v_{n-1}}+FF_{kv_n}$.

According to above Equation,

$$FF_\mu = D_k(i,j). \tag{12.3}$$

According to Step 2-(2), $D_k(i,j)\geq \Delta T$, then according to free float theorem, the difference of path lengths is: $L(\mu^\nabla)-L(\mu)=FF_\mu$. For $T=L(\mu^\nabla)$, then $L(\mu)=L(\mu^\nabla)-FF_\mu=T-D_k(i,j)\leq T-\Delta T$. If deleting (i,j), for path μ passes activity (i,j), then path μ disappears at the same time.
(b) Suppose any paths which pass activity (i,j) are:

$$\begin{aligned}\mu' =\ & (1)\to\cdots\to(u_1)\to(s_1)\to\cdots\to(u_t)\to(s_t)\to(i)\to(j)\to\cdots\to(e_1)\\ & \to(f_1)\to\cdots\to(e_2)\to(f_2)\to\cdots\to(e_m)\to(f_m)\to\cdots\to(w)\\ \mu'' =\ & (1)\to\cdots\to(i)\to(j)\to(e_1)\to(f_1)\to\cdots\to(e_2)\\ & \to(f_2)\to\cdots\to(e_m)\to(f_m)\to\cdots\to(w),\end{aligned}$$

therefore, free floats of activities (e_r,f_r) and (u_p,s_p) is nonzero, $r=1,2,\cdots,n; p=1,2,\cdots,t$, and free floats of other activities are all zero.

It is obvious that $FF_{\mu'}>FF_{\mu''}$, and according to free float theorem, the paths' lengths satisfy with:

$$\mu' < \mu''. \tag{12.4}$$

It is similarly with μ that:

$$\begin{aligned}D_{e_m}(i,j) &= FF_{ij}+D_{e_m}(e_1,f_1)\\ &= FF_{ij}+FF_{e_1f_1}+D_{e_m}(e_2,f_2)\\ &\quad \cdots\end{aligned}$$

$$
\begin{aligned}
&= FF_{ij} + FF_{e_1 f_1} + \cdots + FF_{e_{m-1} f_{m-1}} + D_w(e_m, f_m) \\
&= FF_{ij} + FF_{e_1 f_1} + \cdots + FF_{e_{m-1} f_{m-1}} + FF_{e_m f_m} \\
&= FF_{\mu''},
\end{aligned}
$$

viz.

$$FF_{\mu''} = D_{e_m}(i, j). \tag{12.5}$$

① If $D_{e_m}(i, j) \geq D_k(i, j)$, then $FF_{\mu''} > FF_{\mu}$. According to free float theorem, then $L(\mu'') \leq L(\mu)$.

② If $D_{e_m}(i, j) < D_k(i, j)$, according to Equation (12.4), $D_{e_m}(e_1, f_1), D_{e_m}(e_2, f_2), \cdots$, $D_1(e_m, f_m)$ are all smaller than $D_{e_m}(i, j)$, therefore:

$$
\begin{cases}
D_{e_m}(e_1, f_1) < D_k(i, j), \\
D_{e_m}(e_2, f_2) < D_k(i, j), \\
\cdots\cdots \\
D_n(e_m, f_m) < D_k(i, j).
\end{cases} \tag{12.6}
$$

For $D_n(e_m, f_m) < D_k(i, j), D_k(i, j) < \Delta T$, then $D_n(e_m, f_m) < \Delta T$. According to Step 1 and 2, $D_n(e_m, f_m) \in \Omega^k$.

i. If $D_n(e_m, f_m)$ dose not be chosen all along in process of choosing the minimal value $D_r(u, v_0)$ in Step 4, thus it dose not be deleted all along and is still in Ω^k. Then in this choosing, for $D_n(e_m, f_m) < D_k(i, j)$, $D_n(e_m, f_m)$ should be chosen to replace $D_k(x, y)$, which is dissociable.

ii. If $D_n(e_m, f_m)$ being chosen in process of choosing the minimal value $D_r(u, v_0)$ in Step 4, according to Equation (12.6), $D_{e_m}(e_{m-1}, f_{m-1}) < D_k(i, j) < \Delta T$.

According to Step 5 and 2-(1), $D_{e_m}(e_{m-1}, f_{m-1}) \in \Omega^k$, then $D_{e_m}(e_{m-1}, f_{m-1}) < D_k(i, j)$. Therefore, according to Step 5, $D_{e_m}(e_{m-1}, f_{m-1})$ should be chosen to replace $D_k(x, y)$, which is also dissociable.

Similarly, If $D_{e_m}(e_{m-1}, f_{m-1})$ is chosen, according to Equation (12.5), $D_{e_m}(e_{m-2}, f_{m-2}) < D_k(i, j) < \Delta T$.

According to Step 2-(1), $D_{e_m}(e_{m-2}, f_{m-2}) \in \Omega^k$, but $D_{e_m}(e_{m-2}, f_{m-2}) < D_k(i, j)$, and according to Step 5, choosing $D_k(x, y)$ is also dissociable.

Deducing in turn similarly, until $D_{e_m}(i, j) < D_k(i, j)$, and choosing $D_k(x, y)$ is still dissociable.

From above analysis, $D_{e_m}(i, j) < D_k(i, j)$ is not correct, therefore $D_{e_m}(i, j) \geq D_k(i, j)$.

Then according to conclusion which proved in ①), $\mu'' \leq \mu . \mu' < \mu''$ has been proved, therefore $\mu' < \mu$.

For arbitrariness of μ', μ is the longest path which pass activity (i, j). If deleting (i, j), the lengths of disappeared paths are all no longer than μ. And for $L(\mu) \leq T - \Delta T$, the lengths of disappeared paths are all not bigger than $T - \Delta T$, therefore Step 2-(2) is correct.

(4) The algorithm is to reserve all paths whose lengths are bigger than $T - \Delta T$.

According to Equation (12.3), $FF_{\mu} = D_k(i, j)$. If $D_k(i, j) < \Delta T$, then $FF_{\mu} < \Delta T$. According to free float theorem, $L(\mu^{\nabla}) - L(\mu) = FF_{\mu}$. And for $L(\mu^{\nabla}) = T$, then

$L(\mu) = T - FF_{\mu} > T - \Delta T$. In Step 2-(1), if $D_k(i, j) < \Delta T$, then reserve the value in Ω^k, which means the paths longer than $T - \Delta T$ are reserved.

Therefore, according to Step 2-(2), if $D_k(i, j) \geq \Delta T$, delete paths whose lengths are smaller than or equal to $T - \Delta T$. According to Step 2-(1), if $D_k(i, j) < \Delta T$, put $D_k(i, j)$ into Ω^k, then reserve all paths whose lengths are bigger than $T - \Delta T$. According to Step 4, delete one $D_k(i, j)$ from Ω^k every time. According to Step 3-(2), if $\Omega^k \neq \emptyset$, the process need continue, and delete all paths whose lengths are smaller than or equal to $T - \Delta T$ by using Step 2-(2). And according to Step 3-(1), if $\Omega^k = \emptyset$, the process should stop. Therefore, by the process of simplifying, the paths whose lengths being smaller than or equal to $T - \Delta T$ are deleted as more as possible, and the network is simplified to the simplest sub-network equivalently.

12.4 Illustration

The CPM network planning of one project engineering could be showed as Fig. 12.1. If we want to shorten total duration of the project by 30 days, try to simplify the network to the simplest sub-network for solving the time-cost tradeoff problem equivalently.

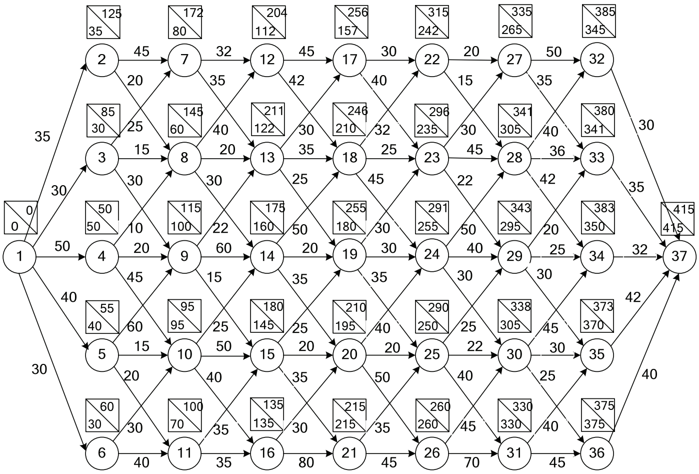

Fig. 12.1 CPM network planning

Step 1. Find out critical path μ^{∇}, $\mu^{\nabla} = (1) \rightarrow (4) \rightarrow (10) \rightarrow (16) \rightarrow (21) \rightarrow (26) \rightarrow (31) \rightarrow (36) \rightarrow (37)$.

Step 2. According to Equation (12.1), for immediate predecessor activities of critical nodes, $FF_{35,37} = 3 < 30$, $FF_{20,26} = 15 < 30$, eigenvalues of other immediate predecessor activities of critical nodes are all bigger than or equal to 30. Delete these activities except activities $(20,26)$ and $(35,37)$, and delete activities which don't connect other activities. Put $FF_{35,37} = 3$ and $FF_{20,26} = 15$ into Ω^{37}, and get Fig. 12.2.

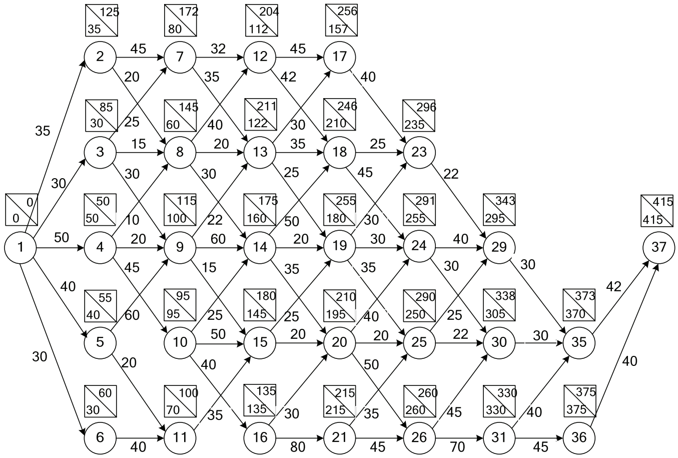

Fig. 12.2 The network after running Step 2

Step 3. Choose minimal eigenvalue $D_{37}(35,37) = 3$ from Ω^{37}.
Step 4. Make activity (35,37) as fundus activity, and find out its fore main chain μ_{35}^*, $\mu_{35}^* = (1) \rightarrow (4) \rightarrow (10) \rightarrow (16) \rightarrow (21) \rightarrow (26) \rightarrow (31) \rightarrow (35)$. Free float of each activity on μ_{35}^* is zero.

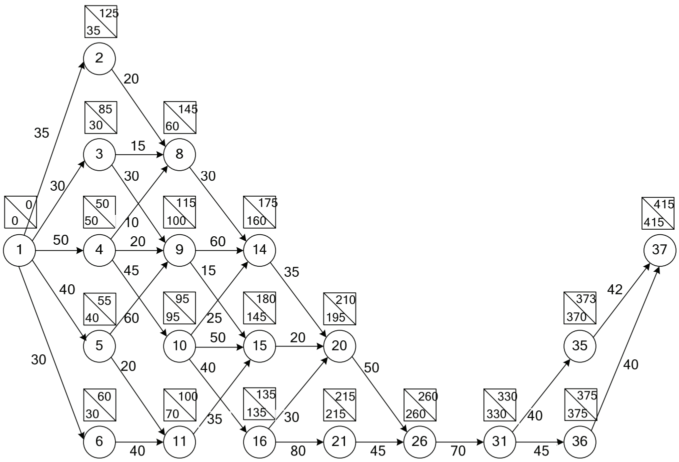

Fig. 12.3 The network after running Step 5

Step 5. For immediate predecessor activities of nodes on μ_{35}^{*}, according to Equation (12.1), $FF_{29,357} = 45 > 30$, $FF_{30,35} = 35 > 30$. Delete the two activities, and delete activities which don't connect other activities, then get Fig. 12.3.
Step 6. Choose the rest activity $(20,26)$ of Ω^{37} as new fundus activity, then $\mu_{20}^{*} = (1) \rightarrow (5) \rightarrow (9) \rightarrow (14) \rightarrow (20)$. For $FF_{3,9} = 40 > 30$, $FF_{4,9} = 30$, $FF_{8,14} = 70 > 30$, $FF_{10,14} = 40 > 30$, $FF_{15,20} = 30, FF_{16,20} = 30$, then delete these activities, and get Fig. 12.4.

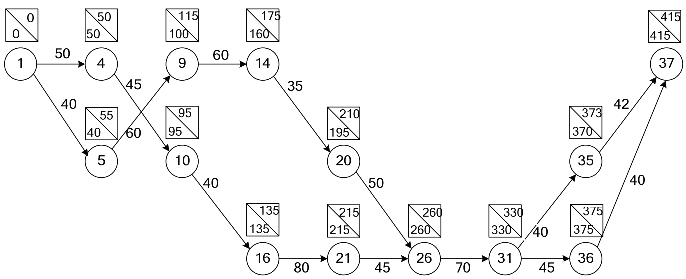

Fig. 12.4 The network after running Step 6

Step 7. Here $\Omega^{37} = \emptyset$, then stop. Fig. 12.4 is the simplest equivalent sub-network of original network.

It is equivalent to shorten total duration by 30 days in Fig. 12.1 and Fig. 12.4, but it is obvious that Fig. 12.4 is simpler than Fig. 12.4.

12.5 Conclusions

In this paper, according to the idea that simplify object of problem is equivalent to simplify any algorithms to solve the problem, problem of how to simplify super large-scale network to simple sub-network equivalently in time-cost tradeoff problem is analyzed mostly, which for realizing the purpose of any algorithms could be simplified to solve time-cost tradeoff problem, and final result would not be affected.

In the paper, firstly, the properties of free float are analyzed, the relations between free float of activity and length of path are found out, the free float theorem is deduced, and then the algorithm is designed to simplify super large-scale network when solving time-cost tradeoff problem by using the theory. The algorithm is simply and applied, and realizes the effect of simplifying object of problem and all correlative algorithms, which could decrease computation of solving time-cost tradeoff problem.

Free float theorem is important basic theory to study and apply CPM network planning, and help to study inherent rule of CPM network and properties of float more deeply. As direction of studying in future, we will deeply study the theory, open out inherent rule of CPM network, analyze and solve more correlative problems by combining these theories with practices.

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