The Value at Risk Measure of the Yuan Against the Dollar

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Abstract In this article, we combined the methods of AR-GARCH and History Monte Carlo, with which we measured the exchange rate risk of the RMB. We mainly used AR-GARCH model to simulate and estimate the time series of the RMB exchange rate, with which we solved the problem of conditional heteroscedasticity, and get the residual series with zero mean, conditional heteroscedasticity and the conditional mean series and the autoregression equation, then which is used to be the basis of Monte Carlo simulation method to measure the value of the RMB exchange rate risk. The results show that this combination of both can effectively solve the problems of peak and fat tail in residual series, non-normality and caused error by estimating the parameters of the fitted distribution, and can effectively improve the credibility and accuracy of measurement of the exchange rate value at risk.

Keywords Exchange rate • Measure • Value at risk

1 Introduction

According to statistics, since exchange rate (Meng-Long Shih et al. 2008) reform from 1994 to the end of September 2011, the RMB against the dollar have appreciated cumulatively by 36.9 %. Calculated by caliber of bank for international settlements, up to the end of August 2011, the nominal and real effective exchange rate of the RMB against the major trading partners appreciate cumulatively by 33.4 % and 58.5 %, respectively. The increasing flexibility of the RMB exchange rate means that exchange rate impacts on Chinese economy more uncertainty. The

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widespread financial risk in 2008 and its great harm warn Chinese enterprises attention to the exchange rate value at risk.

Leptokurtic heavy tail (Creal et al. 2011) feature led to the problem that the residuals series of time series don't meet the normal distribution, in practice, this means that a random sample from such a distribution will have more extreme values, kurtosis of the normal distribution K(x) = 3, so K(x)-3 called as excess kurtosis, therefore, if a certain distribution has a positive excess kurtosis, then this distribution has a fat tail, the thick tail means that the distribution has more "quality" in its tail than that of the normal distribution. If we use the normal distribution to estimate the value at risk, it will cause the under pricing of risk. In this regard, many scholars at home and abroad solve it by using t distribution (Wang Meijin and Wang Hua 2002), GDM distribution (Liu Qingfu et al. 2007), mixed normal distribution (Zheng Yuhua and Cui Xiaodong 2009), the distribution of Brownian motion (Zhou Ying and Wu Zhehui 2007), Bayesian time-varying quantile forecasting (Gerlach et al. 2011), TGARCH (Xiaojia Wang 2013), combinatorial copula function (Chen Lin and Zhou Zongfang 2009) and so on, but these methods inevitably bring in some errors by fitting the distribution parameters estimation. So, in order to solve the problems all above, we propose a new method measuring the exchange rate value at risk to address the leptokurtic and fat tail problem, which is the combination of the AR-GARCH model and Monte Carlo simulation method.

2 Background and Proposed Method

2.1 VaR (Value at Risk) Theory

VaR (Jory 2010) means that, at a certain confidence level, financial assets (or portfolio) will have the potential loss of a specific time in the future. The mathematical idea is as follows:

If X Represents a random loss, $F_x(x)$ is the distribution function of X. VaR means that under a given significant level $\partial \in (0,1)$, the greatest possible loss is $P_r\{X \leq VaR_\partial(X)\} = \partial$, that is $VaR_\partial(X) = F_X^{-1}(\partial)$. Among which $F_X^{-1}(\partial)$ expressed quantile fractile at the significant level ∂ that is the value-at-risk. The generated VaR is the product of JP Morgan Company which is used to measure the market risk. Different means from that of measurement of traditional risk, VaR is a risk measurement technique, which is absolutely based on the statistical analysis.

The VaR calculations usually need to consider three factors: the size of the confidence level, length of the holding period, and the characteristics of the distribution of the return on future assets. Its key is the distribution of Yield. How to accurately describe it and estimate the yield distribution characteristics, especially that of the tail, which is key influencing factors to accurately define the VaR value (Tsay 2009).

2.2 AR-GARCH Model

GARCH (Nelson 1990) model has a requirement for residual series $\{\varepsilon_t\}$: $\{\varepsilon_t\}$ is a sequence with zero mean and pure random and heteroscedasticity. But sometimes regression function $f(t, x_{t-1}, x_{t-2}, \ldots)$ cannot fully extract the information in the original series, and residual series may be relevant, rather than random. To solve the problem at this time, first, we need to fit from the autoregression model, and then consider homogeneity of variance of autoregressive residuals sequence $\{\eta_t\}$, If $\{\eta_t\}$ is heteroscedasticity, then we use GARCH model to fit it, such a tectonic model as the AR(m)-GARCH(p,q) model. Considering the heteroscedasticity and related issues of residual series, we use AR(m)-GARCH(p,q) model (Wang Yan 2005).

2.3 History Monte Carlo Simulation Method

Monte-Carlo simulation method is the most effective method to measure the VaR up to date, the method is very flexible and can include time-varying variance or expected revenue, fat tail and extreme cases and so on, the actual value of the data can be approximated through Repeating simulation of the original data. Here, we use the history Monte-Carlo simulation method to calculate the VaR of the exchange rate. This method differs from the general Monte-Carlo simulation method selecting the random variable from the hypothetical distribution, but it selects from the historical data sample, because the future data and the data is similar, including the environmental impact of data changes, so do may be more in line with the actual situation.

Step 1 The random number generated, along with residual series of the conditional mean of the exchange rate coming from the historical data constitute the sampling set $\{\eta_t\}$, then each of random sample from the space takes a sample of data; Step 2 Use sample to generate pseudo-random variables $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_n$, and then by a formula:

$$\mu_{t+i} = \varphi_1\mu_{t+i-1} + \varphi_2\mu_{t+i-2} + \varphi_3\mu_{t+i-3} + \varphi_4\mu_{t+i-4} + \varepsilon_i \quad (i = 1, 2, \cdots n)$$

to get the conditional mean sequence $\mu_{t+1}, \mu_{t+2}, \ldots \mu_{t+n}$ of RMB exchange rate (Wann-Jyi Horng and Chi-Ming Kuan 2009) and the final value of the conditional mean sequence; Step 3 Calculate the final value of the target time T, getting the conditional mean sequence $\mu_{t+n} = \mu_T$ of RMB exchange rate; Step 4 Repeat the second and third step m times, resulting in the distribution of the conditional mean of the RMB exchange rate. The quantile fractile ∂ of the distribution calculated based on the distribution is called VaR.

2.4 Monte Carlo Simulation of the Exchange Rate Risk Based on AR-GARCH Model

In the process of simulation and estimation using AR-GARCH model to fit the exchange rate time series, often exists leptokurtic heavy tail of residual sequence. The result is that the assumption of the AR-GARCH model residual series does not follow normal distribution. To this end, we will combine AR-GARCH model and the Monte Carlo model to get the improved method and steps.

First, AR-GARCH model estimation, get the autoregression equation of the conditional mean sequence $\mu_t = f(\mu_{t-1}, \mu_{t-2}, \cdots, \mu_{t-n}) + \eta_t$. And calculate the residual series $\{\eta_t\}$ of historical sample. Second, generate random numbers, random sample from a sampling set constituted by residual series $\{\eta_t\}$, each taking a sample of data. Third, generate pseudo-random variables $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_n$, and then by a formula:

$$\mu_{t+i} = \varphi_1 \mu_{t+i-1} + \varphi_2 \mu_{t+i-2} + \varphi_3 \mu_{t+i-3} + \varphi_4 \mu_{t+i-4} + \varepsilon_i \quad (i = 1, 2, \cdots n) \tag{1}$$

Get the conditional mean sequence of RMB exchange rate $\mu_{t+1}, \mu_{t+2}, \ldots \mu_{t+n}$. Fourth, calculate the final value of the target time T; get the conditional mean sequence of RMB exchange rate $\mu_{t+n} = \mu_T$. Fifth, repeat the third and fourth steps m times, resulting in the distribution of the conditional mean of the RMB exchange rate. According to the distribution, calculate the quantile fractile ∂ of the distribution, which is VaR.

3 Measures Validation and Results

3.1 Sample Selection and Data Sources

Because the exchange rate of RMB against the dollar is at the core of the foreign exchange market in China, their changes will all impact the economic entities of all levels in China. It becomes a priority to study the risk of fluctuations in the exchange rate of the RMB against the dollar naturally. In view of several major reforms of the RMB exchange rate formation mechanism since July 21, 2005. As well as the impact of the financial crisis since 2008, we have chosen a broader data to reduce the impact of special events and improve stability of the data. So, we select the central parity of yuan against the dollar from August 1, 2005 to September 30, 2012 as the sample of data analysis, a total of 1,743 samples of observations. These data are derived from the People's Bank of China possessing higher authority (*Sample sources:* http://www.pbc.gov.cn/publish/zhengcehuobisi/637/index.html).

Table 1 AR-GARCH model parameters estimation

GARCH estimate			
SSE	622.120055	Observations	1,747
MSE	0.35611	Uncond VaR	.
Log likelihood	−842.48614	Total R-square	0.9999
SBC	1744.69753	AIC	1700.97228
Normality test	298.6866	Pr > ChiSq	<0.0001

Table 2 AR-GARCH model parameters estimation

Variable	DF	Estimate	Standard error	t Value	Approx Pr > |t|
Intercept	1	634.5588	1.5054	421.52	<.0001
AR1	1	−1.0477	0.0229	−45.69	<.0001
AR2	1	0.0732	0.0300	2.44	0.0146
AR3	1	−0.1066	0.0288	−3.71	0.0002
AR4	1	0.0810	0.0207	3.91	<.0001
ARCH0	1	0.0000123	7.7094E−6	1.60	0.1099
ARCH1	1	0.2105	0.0102	20.63	<.0001
GARCH1	1	0.8096	0.005781	140.05	<.0001

3.2 *Selecting Basis and Given Order of AR-GARCH Model*

The time series of exchange rate of the RMB against the dollar have characteristics: non-stationarity, conditional heteroscedasticity, autocorrelation, non- white noise and so on. It suggests that we can use the time-series model to simulate and estimate the time series of exchange rate of the RMB against the dollar, but the AR model and ARCH model does not apply to it, so we select the AR-GARCH model.

After several times of fitting AR-GARCH model, considering the AIC criteria and SBS criteria and Total R -Square in Table 1, the goodness of fit of the model parameters in Table 2, we have adopted the AR(4)-GARCH(1,1) model, fitting results shown in Fig. 1, you can visually see that the fitting effect is very good, but by the normality test $p < 0.0001$ in Table 2, we should reject the null hypothesis (H_0: normality exists), so the residual series does not follow a normal distribution, we need to improve the AR-GARCH model.

Among them, X represents central parity of exchange rate of RMB against the dollar; starting time from the September 30, 2012.

3.3 *Monte Carlo Simulation Model's Specific Operation Based on AR-GARCH*

Step 1. Estimation of AR(4)-GARCH(1,1) model's parameters.

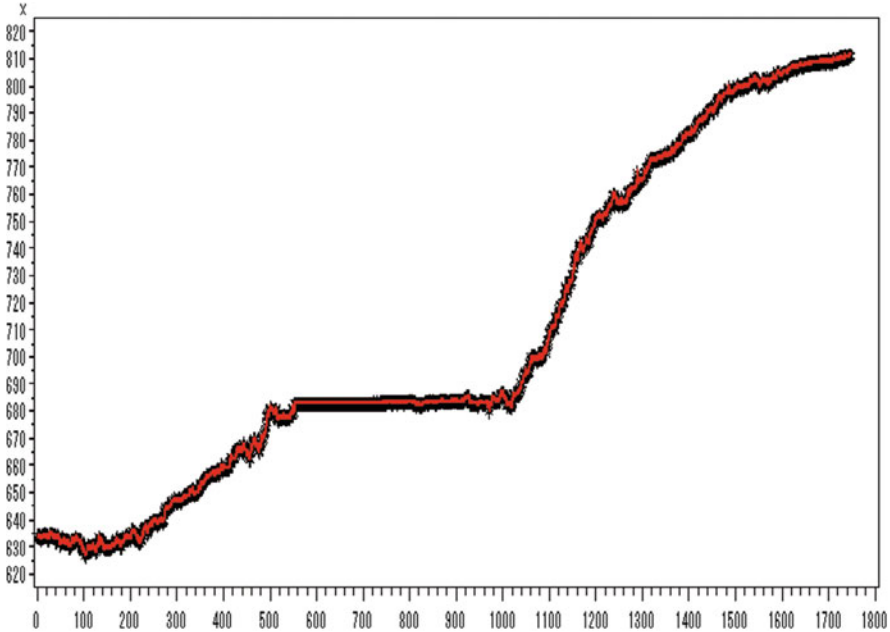

Fig. 1 The fitting effect AR(4)-GARCH(1, 1) model *black* '*star*' for raw data graphics, *red* '*line*' for fitting data graphics (Color figure online)

In view of the history Monte-Carlo simulation method, it mainly simulates and estimates the potential loss of the RMB exchange rate by means of a large number of historical data. Although it is widely used in practice, the assumptions of its consistency on historical data have ignored the combined effect, the thick tail effect, time-varying variance effect of the distribution of the RMB exchange rate. Therefore, we use the AR-GARCH model to process the conditional mean of residual series $\{\eta_t\}$, in order to more accurately portraying the volatility and heteroscedasticity of the conditional mean of residual series, and provide more accurate data for the application of the history Monte-Carlo simulation to simulate the conditional mean μ_t, thereby better conform to the reality.

By the autoregression process of SAS system, we use conditional least squares method to estimate parameters (see Table 2), and get the model's caliber:

$$\begin{cases} x_t = 634.5588 + \mu_t \\ \mu_t = 1.0477\mu_{t-1} - 0.0732\mu_{t-2} + 0.1066\mu_{t-3} - 0.0810\mu_{t-4} + \eta_t \\ \eta_t = \sqrt{h_t}\upsilon_t, \upsilon_t \sim \mathrm{N}\,(0, 0.35611) \\ h_t = 0.0000123 + 0.2105\eta_{t-1}^2 + 0.8096h_{t-1} \end{cases} \tag{2}$$

We take the residual series as a time series $\{\eta_t\}$ with zero mean and heteroscedasticity.

The formulas of conditional mean and residual of the sample:

$$\mu_t = x_t - 634.5588 \tag{3}$$

$$\eta_t = \mu_t - 1.0477\mu_{t-1} + 0.0732\mu_{t-2} - 0.1066\mu_{t-3} + 0.0810\mu_{t-4} \tag{4}$$

But the residual series $\{\upsilon_t\}$ does not follow a normal distribution, we combined the model above with Monte Carlo simulation to measure VaR and test the effects.

Step 2. AR(4)-GARCH(1, 1) model combined with Monte Carlo simulation to measure VaR.

Taking into account the inevitable sampling error in the simulation of the time, it is subject to restrictions on the sample and the times of simulation and which results in differences of estimations, but more repetitions can get a more accurate estimate. Therefore, in order to obtain efficient and accurate data, we simulated 10,000 data (completed by MATLAB software programming).

First, to calculate historical sample of the residual series $\{\eta_t\}$ and the conditional mean series $\{\mu_t\}$ from AR(4)-GARCH(1,1) model, we randomly select 100 pseudo-random variables ε_i $(i = 1 \ldots 100)$ from the residual series.

Second, 100 simulation values of the conditional mean obtained by the formula:

$$\mu_t = 1.0477\mu_{t-1} - 0.0732\mu_{t-2} + 0.1066\mu_{t-3} - 0.0810\mu_{t-4} + \eta_t \tag{5}$$

And the initial value of simulation is sample conditional mean of the last 4 days, then get the final value of the future target period T.

Third, simulate 10,000 times by Monte Carlo simulation, and get the distribution of the simulation value of the conditional mean: $\mu_T^1, \mu_T^2, \cdots \mu_T^{10000}$

According to the quantile fractile of experience distribution, we calculated that the maximum possible VaR were 0.6082, 0.8604, 1.6274, respectively, under the significance level of 10 %, 5 %, 1 %.

3.4 Checking the Results of Measurement

The accuracy of the VaR can be evaluated by the test method based on the failure rate. There are T samples in a given T time window, if we define the number of failure as N which is the number of sample values over the predictive value of the corresponding period of the sample, that is, the number of abnormal samples, and then larger in the T, N should be close to the T $(1 - \alpha)$, and the failure rate is N/T, where α is the confidence level. We use the method which is proposed by Kupiec (1995) to build the log-likelihood ratio statistic:

$$\mathrm{LR} = -2\ln\left[(1-\alpha)^{\mathrm{T-N}}\alpha^{\mathrm{N}}\right] + 2\ln\left[(1-\mathrm{N/T})^{\mathrm{T-N}}(\mathrm{N/T})^{\mathrm{N}}\right] \tag{6}$$

Table 3 Results of VaR test

Confidence	T = 1,743 day (failure rate, LR values)	Non-rejection region of LR
α = 90 %	Failure rate = 0.078026, LR = 10.03811	(0.003932, 3.8414591)
α = 95 %	Failure rate = 0.039013, LR = 4.775885	(0.000982, 5.023886)
α = 99 %	Failure rate = 0.006311, LR = 2.757384	(0.000039, 7.879439)

By the above Table 3, under the significance level of 10 %, the LR value is in the rejection region, and LR = 10.03811 > 3.8414591, this shows that under the significance level of 10 %, the measurement of value at risk is a bit high, but under other significance levels, LR value is in the non-rejection region. Therefore, the VaR has very good accuracy and credibility at the significance levels of 5 and 1 %.

4 Conclusions

The empirical results show that, although VaR is estimated a bit high under the significance level of 10 % through simulating VaR of exchange rate by history Monte Carlo simulation based on the AR-GARCH model, it has good reliability and accuracy under the significant level of, respectively, 5 % and 1 %. To some extent, it solved the problems in the conditional heteroscedasticity and the leptokurtic heavy tail feature of residual series of the RMB exchange rate, and avoided error estimating parameters of fitted distribution, and solved the issue that the assumption that history can repeat itself can't satisfy the reality in historical simulation method, and improved the credibility and accuracy of measurement of VaR overall. Although we verified the effect of measurement of VaR of the exchange rate on base of the AR-GARCH model and Monte Carlo simulation through empirical, but the actual operability remains to be further inspection.

References

Chen Lin, Zhou Zongfang (2009) A measure on joint default risk based credit rating information and combinatorial copula function. J Converg Inf Technol 4(1):39–46

Creal D, Koopman SJ, Lucas A (2011) A dynamic multivariate heavy-tailed model for time-varying volatilities and correlations. J Bus Econ Stat 29(4):552–563

Gerlach RH, Chen CWS, Chan NYC (2011) Bayesian time-varying quantile forecasting for value-at-risk in financial markets. J Bus Econ Stat 29(4):481–492

Jory P (2010) Value at risk, 3rd edn. CITIC Press, China (in Chinese)

Kupiec PH (1995) Techniques for verifying the accuracy of measurement models. J Deriv 3:73–80

Liu Qingfu, Zhong Weijun, Hua Renhai, Liu Xiaoxing (2007) Application of EGARCH-GED model for calculating value at risk in Chinese futures market. J Ind Eng/Eng Manag 21(1):117–121 (in Chinese)

Meng-Long Shih, Shu-Hua Hsiao, Fang-Sheng Chen (2008) The association of China stock index with Japan and US. J Converg Inf Technol 3(2):13–22

Nelson DB (1990) Stationarity and persistence in the GARCH (1,1) model. Econom Theory 6: 318–334
Tsay RS (2009) Financial time series analysis, 2nd edn. Posts and Telecom Press, China (in Chinese)
Wang Meijin, Wang Hua (2002) Analysis of shanghai stock market value at risk based on GARCH-t model. J Quant Tech Econ 3:106–109 (in Chinese)
Wang Yan (2005) The application of time series analysis. China Renmin University Press, China (in Chinese)
Wann-Jyi Horng, Chi-Ming Kuan (2009) A DCC analysis of two exchange rate market returns volatility with an Japan dollars factor: study of Taiwan and Korea exchange rate markets. J Converg Inf Technol 4(4):7–14
Xiaojia Wang (2013) Prediction analysis for short term load based on GM (1, 1) and TGARCH error correction model in smart grid. J Converg Inf Technol 8(1):111–117
Zheng Yuhua, Cui Xiaodong (2009) Value-at-risk measure based on mixed normal distribution. Res Financ Econ Issues 5:68–73 (in Chinese)
Zhou Ying, Wu Zhehui (2007) Research on dynamic margin of physical futures based on risk control. Res Financ Econ Issues 9:67–71 (in Chinese)