Application of Grey-Markov Model in Forecasting the Variation Trend of Three Industrial Structures in Qingdao City

Ming-yi Li and Hui-ying Gao

Abstract Grey system model has been applied to a wide field, such as economy, agriculture, weather, engineering technology, etc. since it was born in 1982. In this article, GM (1, 1) is combined with Markov model to predict the variation trend of industrial structure in Qingdao over the next 8 years according to the proportion of primary, secondary, and tertiary industry in the past 13 years. In this way, we can provide useful advices to the government in the regulatory policies.

Keywords GM (1, 1) • Markov model • Three industrial structures • Variation trend

1 Introduction

Since the 2008 financial crisis, the economic growth has slowed down in many countries, including China. Except for Shandong province, the economic growth rates of five other provinces in China are lower than the national average growth rate over the past 2 years. In this economic context, taking countermeasures to adjust the industrial structure is becoming an imperative strategy faced by countries all over the world. But how do we know whether the policies we take will meet our expectations? How to predict the variation trend of industrial structure in order to provide reference for the government? Based on the Grey Model, we analyzed the proportion of primary, secondary, and tertiary industry in three industries of Qingdao in the past 13 years, and found its basic development trend. For the random factors, Markov model was used to revise the predicted value, so as to obtain a group of more accurate data.

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2 The Establishment of Grey–Markov Model

2.1 The Establishment of Grey Model

The Typical Grey forecasting model is GM (1, 1), which was proposed by Professor Ju-long DENG. Before dealing with the original random numerical observation data, the accumulated generating operation (AGO) technique is applied to reduce the randomization of the raw data (He-jin Xiong and Hua-zhong Deng 2005). Assume that $X^{(0)}$ is original historical time series data.

$$X^{(0)} = \left\{x^{(0)}\ (k)\,|k = 1, 2\ldots n\right\}$$

Construct X (1) by one time accumulated generating operation (1-AGO), $x^{(0)}$ was changed into $x^{(1)}$.

$$X^{(1)} = \left\{x^{(1)}(k)\,|k = 1, 2\ldots n\right\}$$

Where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \tag{1}$$

Furthermore, we can get the mean generation sequence $Z^{(1)}$ with consecutive neighbors.

$$Z^{(1)} = \left\{z^{(1)}\ (k)\,|k = 2, 3\ldots n\right\}$$

Where

$$Z^{(1)}(k) = \frac{1}{2}\left(x^{(1)}(k) + x^{(1)}\ (k-1)\right) \tag{2}$$

Define $\widehat{a} = [a, b]^T$ as a parameter matrix, and

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{3}$$

Establish the differential equation as:

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{4}$$

a, b are parameters to be identified, which can be solved through the least square method.

$$\widehat{a} = [a, b]^T = \left(B^T B\right)^{-1} B^T Y \tag{5}$$

The shadow equation of Eq. (4) is:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{6}$$

Solve (6) together with initial condition, and the particular solution is

$$\widehat{x}^{(1)}(k+1) = \left(x^{(1)}(0) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}\,(k = 1, 2..n) \tag{7}$$

Define $x^{(1)}(0) = x^{(0)}(1)$, Then the predicted value $\widehat{x}^{(0)}(k)$ at k step can be estimated by inverse accumulated generating operation (IAGO), which is defined as:

$$\widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \widehat{x}^{(1)}(k-1)\,(k = 1, 2..n) \tag{8}$$

Up to this point, the establishment of Grey model is completed (Si-feng Liu et al. 2004).

2.2 *Partition of States by Markov Forecasting Model*

The sequence $\widehat{x}^{(0)}(k)$ reflects the general forecasting trend of raw data, and $x^{(0)}(k)$ the actual trend. Due to the unknown information, the predicted values may not coincide with the actual values. Markov forecasting model provides a method to increase the forecasting precision of random fluctuating sequences.

Firstly, generate a new sequence $\varepsilon(k)$

Where

$$\varepsilon(k) = \frac{x^{(0)}(k)}{\widehat{x}^{(0)}(k)}\,(k = 1, 2 \ldots \mathrm{n}) \tag{9}$$

Secondly, divide the sequence $\varepsilon(\kappa)$ into a series of contiguous intervals according to the extent of its variation. Each interval represents a state. Such as $\theta_i = [\theta_{i1}, \theta_{i2}]$ (i = 1,2 . . . n), θ_{i1} and θ_{i2} are the minimum and maximum of interval θ_i respectively (Nai-fang Yin and Lei Sun 2012).

2.3 Calculate the Transition Probability Matrix

The transition probability from state θ_i to state θ_{j} can be calculated as follows:

$$P_{ij}(m) = \frac{M_{ij}(m)}{M_i} (i, j = 1, 2..n) \tag{10}$$

Where m is the number of transition steps each time, M_{ij} (m) is the number of raw data of state θ_i transferred from state θ_{j} for "m" steps; M_i is the number of raw data points in state θ_i. After calculating all the P_{ij}(m), a transition probability matrix P(m) is obtained then (Xiang-yong Li et al. 2003; Yong-gang Chen and Hai-feng Ren 2007; Ping Tao and Zhong-ying Qi 2012).

$$P(m) = \begin{bmatrix} p_{11}(m) & p_{12}(m) & \ldots & p_{1n}(m) \\ p_{21}(m) & p_{22}(m) & \ldots & p_{2n}(m) \\ \vdots & \vdots & \ldots & \vdots \\ p_{n1}(m) & p_{n2}(m) & \ldots & p_{nn}(m) \end{bmatrix} \tag{11}$$

It is enough to consider one step P (1) in general. But two or more steps are calculated in order to get more reliable data (Nai-fang Yin and Lei Sun 2012). Assume the subject to be predicted is in state θ_k, and then the row k in matrix P(m) should be considered. If $\max_j p_{kj} = p_{k1}$, and then the system will most probably transfer from state θ_{k} to state θ_1 at the next moment (Xiang-yong Li et al. 2003; Yong-gang Chen and Hai-feng Ren 2007; Hong-yan Dong and Shu-yuan Chen 1995).

2.4 Revise the Forecasting Data

The future state of the system can be obtained according to the matrix (11), and then the difference between predicted value and actual value will be confirmed. For instance, if the difference is located in the interval $[\theta_{i1}, \theta_{i2}]$, the predicted value should be revised as below (Duncan et al. 1998; Jian-bo Yuan and Xing Li 2011):

$$Y(k) = \frac{1}{2}(\theta_{i1} + \theta_{i2})\,\widehat{x}^{(0)}(k) \tag{12}$$

3 Forecast the Variation Trend of Three Industrial Structures in Qingdao

The proportion of the primary/ secondary/ tertiary industry (we call them P, S, T for short respectively) in three industrial structures is influenced by many factors, some are clear, some are vague. So the time sequence of these proportions shows random fluctuations. Table 1 shows the ratios of P, S and T in three industries of Qingdao from 2000 to 2012 (National Bureau of Statistics Investigation Team in Qingdao 2013; Fu-ying Xu 2012).

3.1 *Deal with the Raw Data According to the GM (1, 1) Grey Forecasting Model*

Take the data of primary industry as an example. Base on the formulas (1), (2), (3), sequences can be worked out as follow:

$$X^{(0)} = \{11.8, 10.5, 9.3, 8.0, 7.2, 6.6, 5.8, 5.4, 5.1, 4.7, 4.9, 4.6, 4.4\}.$$

$$X^{(1)} = \{11.8, 22.3, 31.6, 39.6, 46.8, 53.4, 59.2, 64.6, 69.7, 74.4, 79.3, 83.9, 88.3\}$$

$$Z^{(1)} = \{17.05, 26.95, 35.6, 43.2, 50.1, 56.3, 61.9, 67.15, 72.05, 76.85, 81.6, 86.1\}$$

Table 1 Ratios of P, S and T in three industries of Qingdao (2000～2012) (unit: %)

Ratio	Three industries		
Year	P	S	T
2000	11.8	46.6	41.6
2001	10.5	47	42.4
2002	9.3	47.9	42.8
2003	8	49.4	42.6
2004	7.2	50.7	42.1
2005	6.6	51.8	41.6
2006	5.8	52.4	41.8
2007	5.4	51.6	43
2008	5.1	50.8	44.1
2009	4.7	49.9	45.4
2010	4.9	48.7	46.4
2011	4.6	47.6	47.8
2012	4.4	46.6	49

Note: Data from Qingdao Statistical Yearbook

$$Y = \begin{bmatrix} 10.5 \\ 9.3 \\ 8.0 \\ 7.2 \\ 6.6 \\ 5.8 \\ 5.4 \\ 5.1 \\ 4.7 \\ 4.9 \\ 4.6 \\ 4.1 \end{bmatrix}, B = \begin{bmatrix} -17.5 & 1 \\ -26.95 & 1 \\ -35.6 & 1 \\ -43.2 & 1 \\ -50.1 & 1 \\ 56.3 & 1 \\ -61.9 & 1 \\ -67.15 & 1 \\ -72.05 & 1 \\ -76.85 & 1 \\ -81.6 & 1 \\ -86.1 & 1 \end{bmatrix}$$

Based on the (5), the two parameters to be identified can be worked out:

$[a, b]^{\mathrm{T}} = (B^T B)^{-1} B^{\mathrm{T}} Y = [0.08785, 11.3155]^{\mathrm{T}}$

The shadow equation of GM (1, 1) is:

$$\frac{dx^{(1)}}{dt} + 0.08785x^{(1)} = 11.3155$$

The particular solution of the shadow equation is:

$$\widehat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \tfrac{b}{a}\right) e^{-ak} + \frac{b}{a}$$
$$= (11.8–11.3155/0.08785)\,\mathrm{e}^{-0.08785\mathrm{k}} + 11.3155/0.08785$$
$$= -117.0048\mathrm{e}^{-0.08785\mathrm{k}} + 128.805$$

After the IAGO, the predicted value $\widehat{x}^{(0)}(k)$ is obtained.

$$\widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \widehat{x}^{(1)}(k-1)\,.$$
$$\widehat{x}^{(0)}(13) = \widehat{x}^{(1)}(13) - \widehat{x}^{(1)}(12)$$
$$= \left(-117.0048\mathrm{e}^{-0.08785*13} + 128.805\right)$$
$$- \left(-117.0048\mathrm{e}^{-0.08785*12} + 128.805\right)$$
$$= 91.46–88.03 = 3.43$$

All of the calculating data is listed in the Table 2.

Table 2 Calculating data of the primary industry

Year	*k*	$x^{(0)}(k)$	$x^{(1)}(k)$	$\hat{x}^{(1)}(k)$	$\hat{x}^{(0)}(k)$	$\varepsilon(k)$
2000	1	11.8	11.8	11.8	11.8	1
2001	2	10.5	22.3	21.64	9.84	1.067
2002	3	9.3	31.6	30.65	9.01	1.032
2003	4	8	39.6	38.91	8.26	0.969
2004	5	7.2	46.8	46.47	7.56	0.952
2005	6	6.6	53.4	53.39	6.92	0.954
2006	7	5.8	59.2	59.74	6.35	0.913
2007	8	5.4	64.6	65.54	5.8	0.931
2008	9	5.1	69.7	70.86	5.32	0.959
2009	10	4.7	74.4	75.74	4.88	0.963
2010	11	4.9	79.3	80.2	4.46	1.099
2011	12	4.6	83.9	84.29	4.09	1.125
2012	13	4.4	88.3	88.03	3.74	1.177
2013	14			91.46	3.43	

Table 3 State of each year from 2000 to 2012

Year	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
k	1	2	3	4	5	6	7
State	2	3	2	2	1	1	1
Year	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	
k	8	9	10	11	12	13	
State	1	1	2	3	3	3	

3.2 *Build the Markov Model to Revise the Predicted Data*

3.2.1 Partition of States

In Table 2, the value of ε(k) varies from 0.913 to 1.177. Three states and contiguous intervals are divided as follows:

State 1: predicated value is a little higher than actual value. The interval of State 1 is $\theta_1 = [0.90, 0.96]$;

State 2: predicated value is approximate to the actual value. The interval of State 2 is $\theta_2 = [0.96, 1.05]$;

State 3: predicated value is a little lower than actual value. The interval of State 1 is $\theta_3 = [1.05, 1.2]$;

List the state of each year in the Table 3.

Table 4 State transitions

Initial state	Numbers of transition	To state 1	To state 2	To state 3
3	3	1/4	0	0
3	2	1/2	1/4	1/4
3	1	0	1/3	2/3

3.2.2 Calculate the Transition Probability Matrix

Transition probability matrix can be calculated according to the Eq. (10) as follows:

$$P(1) = \begin{bmatrix} 4/5 & 1/5 & 0 \\ 1/4 & 1/4 & 1/2 \\ 0 & 1/3 & 2/3 \end{bmatrix}, P(2) = \begin{bmatrix} 3/5 & 1/5 & 1/5 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/4 & 1/4 \end{bmatrix},$$

$$P(3) = \begin{bmatrix} 2/5 & 1/5 & 2/5 \\ 1/2 & 1/4 & 1/4 \\ 1/4 & 0 & 0 \end{bmatrix}.$$

3.2.3 Revise the Forecasting Data

Data of the past 3 years are chosen to revise the forecasting data of 2013. The state of 2010–2011 can be transferred to the state of 2013 via 1–3 steps. Then the probabilities of state in 2013 could be listed as the Table 4.

The maximum value in Table 4 is 2/3. Therefore, the state of 2013 is most probably at State 3. Next year, the proportion of primary industry in three industrial structures of Qingdao is revised as follows:

$$\begin{aligned} Y(13) &= \frac{1}{2}(\theta_{31} + \theta_{32})\,\widehat{x}^{(0)}(13) \\ &= \frac{1}{2}(1.05 + 1.2) \times 3.43\ \% = 3.9\ \% \end{aligned}$$

Deal with the data of secondary and tertiary industry in the similar way, and we can get their predicted values as Table 5. Comparing the predicted value with actual value, and we get the Fig. 1.

The predicted values gotten from the Grey model well reflect the variation trend of three industrial structures in Qingdao to some extent. Due to unpredictable information, the difference between predicted and actual values shows random fluctuations. In this case, given the data of recent years, Markov model can be used to modify the state of the next year, and then a more precise data can be obtained as a result.

Table 5 The predicted values of three industrial structures of Qingdao from 2000 to 2020

Year / Ratio(%)		2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Three industries	P	11.8	9.84	9.01	8.26	7.56	6.92	6.35	5.8	5.32	4.88
	S	46.6	49.86	49.78	49.73	49.65	49.59	49.52	49.45	49.38	49.32
	T	41.6	40.82	41.39	41.97	42.54	43.13	43.73	44.33	44.95	45.57
Year / Ratio(%)		**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
Three industries	P	4.09	3.74	3.43	3.14	2.88	2.63	2.42	2.2	2.03	1.85
	S	49.18	49.12	49.05	48.99	48.92	48.85	48.79	48.72	48.66	48.59
	T	46.84	47.49	48.15	48.81	49.49	50.17	50.87	51.58	52.28	53.01

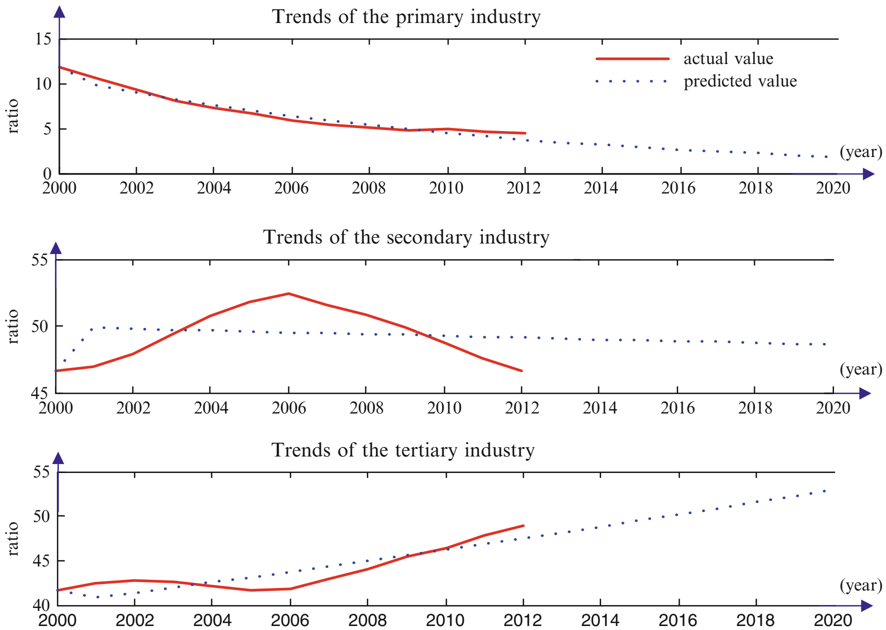

Fig. 1 Comparison of variation trend in three industrial structures of Qingdao between the curves of predicted value and actual value from 2000 to 2020

4 Conclusion

Many theories of industrial structure, such as Hoffman's law, the law of Petty-Clark and the research of trend variation by Kuznets indicate that with the development of industrial structure, the proportion of primary industry shows a declining trend in both output value and employment figure; (Grossman and Kreuger 1995; Yue-jun and Xiao-feng 2006) the share of the secondary industry is mushrooming at first, then stabilized; for the tertiary industry, the proportion is increasing all the time (Xiao-wen Wang and Shu-chao Li 2013). The forecasting trend of industrial structure of Qingdao is conform to these theories, which manifests that Grey–Markov model can be applied to imitate the possible variation trend of three industrial structures in Qingdao. Nowadays, since the economic growth has been slowing down, adjusting the economic structure and transforming the mode of development attract more attention from government sector. However, while adjusting the economic structure, we should not only follow the objective law in the market, adjust measurements according to local condition, but also have insight into whether the economic development will meet our anticipated goal under the current measures we taken. Grey–Markov model can provide a useful forecasting method.

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