Conjunctive Grammars in Greibach Normal Form and the Lambek Calculus with Additive Connectives

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Abstract. We prove that any language without the empty word, generated by a conjunctive grammar in Greibach normal form, is generated by a grammar based on the Lambek calculus enriched with additive ("intersection" and "union") connectives.

1 Conjunctive Grammars

Let Σ be an arbitrary finite alphabet, Σ^* is the set of all words, and Σ^+ is the set of all non-empty words over Σ.

We consider a generalisation of context-free grammars, introduced by Okhotin [9] (and earlier by Szabari [14]).

A *conjunctive grammar* is a quadruple $\mathcal{G} = \langle \Sigma, N, \mathcal{P}, S \rangle$, where Σ and N are two non-intersecting alphabets (Σ is the alphabet in which the language is being defined, its elements are called *terminal symbols*, and N is an auxiliary alphabet, consisting of *nonterminal symbols),* $S \in N$ (the *start symbol)*, and $\mathcal{P}$ is a finite set of *rules* of the form

$$A \to \beta_1 \& \ldots \& \beta_m,$$

where $A \in N$, $m \geqslant 1$, $\beta_1, \ldots, \beta_m \in (\Sigma \cup N)^*$.

We define the language generated by this grammar in terms of a formal deduction system associated with the grammar [10]. This formal system derives pairs of the form $[X, w]$, where $X \in \Sigma \cup N$ and $w \in \Sigma^*$. Axioms are pairs $[a, a]$, for all $a \in \Sigma$, and for every rule $A \to B_{11} \ldots B_{1m_1} \& \ldots \& B_{k1} \ldots B_{km_k} \in \mathcal{P}$, $B_{ji} \in \Sigma \cup N$, and for all strings $u_{ji} \in \Sigma^*$, $j \in \{1, \ldots, k\}$, $i \in \{1, \ldots, m_j\}$, that satisfy $u_{11} \ldots u_{1m_1} = \ldots = u_{k1} \ldots u_{km_k} = w$, there is a deduction rule

$$\frac{[B_{11}, u_{11}] \quad \ldots \quad [B_{km_k}, u_{km_k}]}{[A, w]} .$$

The formal system, associated with the grammar $\mathcal{G}$, is also denoted by $\mathcal{G}$. Define $\mathfrak{L}_{\mathcal{G}}(X) \leftrightharpoons \{w \mid \mathcal{G} \vdash [X, w]\}$ and $\mathfrak{L}(\mathcal{G}) \leftrightharpoons \mathfrak{L}_{\mathcal{G}}(S)$ ("$\leftrightharpoons$" here and further means "equals by definition"). $\mathfrak{L}(\mathcal{G})$ is the *language generated by* $\mathcal{G}$.

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Example 1. Consider the following conjunctive grammar (here small letters stand for terminal symbols, capital stand for nonterminal ones; S is the start symbol):

$$\begin{aligned}
&S \to aAB \,\&\, aDC\\
&A \to aA\\
&A \to a\\
&B \to bBc\\
&B \to b\\
&C \to cC\\
&C \to c\\
&D \to aDb\\
&D \to b
\end{aligned}$$

This grammar generates the language $\{a^{n+1}b^{n+1}c^n \mid n \geq 1\}$ as an intersection of two context-free languages. For example, the word $aaabbbcc = a^3b^3c^2$ is generated in the following way: first we derive $[S, aaabbbcc]$ from $[a,a]$, $[A, aa]$, $[B, bbbcc]$, $[a,a]$, $[D, aabbb]$, and $[C, cc]$. The pair $[a,a]$ is an axiom; the others are derived as follows:

$$\frac{[a,a] \quad \dfrac{[a,a]}{[A,a]}}{[A,aa]} \qquad \frac{[b,b] \quad \dfrac{[b,b] \quad \dfrac{[b,b]}{[B,b]} \quad [c,c]}{[B,bbc]} \quad [c,c]}{[B,bbbcc]}$$

$$\frac{[a,a] \quad \dfrac{[a,a] \quad \dfrac{[b,b]}{[D,b]} \quad [b,b]}{[D,abb]} \quad [b,b]}{[D,aabbb]} \qquad \frac{[c,c] \quad \dfrac{[c,c]}{[C,c]}}{[C,cc]}$$

For technical reasons we also consider an enlarged version of this deduction system, called $\mathcal{G}_{\text{cut}}$. We allow nonterminal symbols to appear in the second components of the pairs (derivable objects in it are of the form $[X, \omega]$, where $X \in \Sigma \cup N$ and $\omega \in (\Sigma \cup N)^*$) and add new axioms $[A, A]$ for all $A \in N$ and the cut rule:

$$\frac{[B, \tau] \quad [A, \omega_1 B \omega_2]}{[A, \omega_1 \tau \omega_2]}\,.$$

A trivial "cut elimination theorem" holds:

Lemma 1. *If $A \in N \cup \Sigma$, $w \in \Sigma^*$, then $\mathcal{G}_{\text{cut}} \vdash [A, w]$ if and only if $\mathcal{G} \vdash [A, w]$.*

Proof. The "if" part is obvious. For the "only if" part, we prove that every pair, derivable in $\mathcal{G}_{\text{cut}}$, is derivable without applying the cut rule (therefore, as w does not contain nonterminal symbols, they do not occur in the derivation, thus this derivation is valid in the original system). Let $[B, \tau]$ and $[A, \omega_1 B \omega_2]$ be derivable without applying the cut rule. Prove that $[A, \omega_1 \tau \omega_2]$ also has a cut-free proof. Proceed by induction on the derivation of $[A, \omega_1 B \omega_2]$. If it is an axiom, then ω_1 and ω_2 is empty, $B = A$, and our goal coincides with the left premise, $[B, \tau]$. If $[A, \omega_1 B \omega_2]$ is derived using an inference rule, then we can perform the substitution of τ for B in the premises of this rule, and apply the induction hypothesis.

2 Greibach Normal Form

Consider only languages without the empty word.

A conjunctive grammar is in *Greibach normal form* (a generalisation of Greibach normal form for context-free grammars [3]), if all the rules are of the form $A \to a\beta_1 \& \ldots \& a\beta_k$, $a \in \Sigma$, $\beta_j \in N^+$ or of the form $A \to a$, $a \in \Sigma$.

The question remains open, whether every conjunctive grammar can be transformed into this form. However, it is true for languages over the one-letter alphabet, as shown by Okhotin and Reitwießner [11]. Therefore, conjunctive grammars in Greibach normal form can capture some languages that are not context-free or even finite intersections of those, since the language $\{a^{4^n} \mid n \geq 1\}$ is generated by a conjunctive grammar found by Jeż [4].

Example 2. The grammar from Example 1 can be easily transformed into Greibach normal form:

$$
\begin{aligned}
&S \to aAB \,\&\, aDC \\
&A \to aA \\
&A \to a \\
&B \to bBU \\
&B \to b \\
&U \to c \\
&C \to cC \\
&C \to c \\
&D \to aDV \\
&D \to b \\
&V \to b
\end{aligned}
$$

3 Multiplicative-Additive Lambek Calculus

In this section we define an extension of the Lambek calculus (introduced in [7]) with two new connectives, *additive conjunction* and *disjunction*. The additive (intersective) conjunction was already introduced by Lambek [8], and the whole calculus was considered by Kanazawa [5]. We shall call this calculus **MALC**, as in [6], but use the Lambek-style notation for connectives.

A countable set $\Pr = \{p_1, p_2, p_3, \ldots\}$ is called the set of *primitive types. Types* of **MALC** are built from primitive types with five binary connectives: $\cdot$ (multiplication, product conjunction), $\backslash$ (left division), $/$ (right division), $\cap$ (intersection, additive conjunction), $\cup$ (union, additive disjunction). We denote types with capital Latin letters and their finite sequences (possibly empty) with capital Greek ones; Λ stands for the empty sequence. *Sequents* (derivable objects) of **MALC** are of the form $\Pi \to C$.

Axioms: $A \to A$.

Rules of inference:

$$\frac{A\,\Pi \to B}{\Pi \to A \setminus B}\ (\to \setminus),\ \Pi \neq \Lambda; \qquad \frac{\Pi \to A \quad \Gamma\, B\, \Delta \to C}{\Gamma\, \Pi\, (A \setminus B)\, \Delta \to C}\ (\setminus \to);$$

$$\frac{\Pi\, A \to B}{\Pi \to B \,/\, A}\ (\to /),\ \Pi \neq \Lambda; \qquad \frac{\Pi \to A \quad \Gamma\, B\, \Delta \to C}{\Gamma\, (B \,/\, A)\, \Pi\, \Delta \to C}\ (/ \to);$$

$$\frac{\Gamma \to A \quad \Delta \to B}{\Gamma\, \Delta \to A \cdot B}\ (\to \cdot); \qquad \frac{\Gamma\, A\, B\, \Delta \to C}{\Gamma\, (A \cdot B)\, \Delta \to C}\ (\cdot \to);$$

$$\frac{\Gamma \to A_1 \quad \Gamma \to A_2}{\Gamma \to A_1 \cap A_2}\ (\to \cap); \qquad \frac{\Gamma\, A_i\, \Delta \to C}{\Gamma\, (A_1 \cap A_2)\, \Delta \to C}\ (\cap \to)_i,\ i = 1, 2;$$

$$\frac{\Gamma \to A_i}{\Gamma \to A_1 \cup A_2}\ (\to \cup)_i,\ i = 1, 2; \qquad \frac{\Gamma\, A_1\, \Delta \to C \quad \Gamma\, A_2\, \Delta \to C}{\Gamma\, (A_1 \cup A_2)\, \Delta \to C}\ (\cup \to);$$

$$\frac{\Pi \to A \quad \Gamma\, A\, \Delta \to C}{\Gamma\, \Pi\, \Delta \to C}\ (\text{cut}).$$

The cut rule is eliminable using the standard technique [7].

The fragment without $\cap$ and $\cup$ is the ordinary (multiplicative) Lambek calculus, called **MLC** or **L**. We also consider fragments of **MALC** with other restrictions of the set of connectives: **MALC**$(/, \cap)$, **MALC**$(/, \cdot, \cap)$, **MLC**$(/)$.

4 Categorial Grammars

A **MALC**-*grammar* is a triple $\mathscr{G} = \langle \Sigma, H, \rhd \rangle$, where Σ is a finite alphabet, $H \in \mathrm{Tp}$, and $\rhd$ is a finite correspondence between Tp and Σ ($\rhd \subset \mathrm{Tp} \times \Sigma$). The *language generated by* $\mathscr{G}$ is the set of all nonempty words $a_1 \dots a_n$ over Σ for which there exist types $B_1, \dots, B_n$ such that $\mathbf{MALC} \vdash B_1 \dots B_n \to H$ and $B_i \rhd a_i$ for all $i \in \{1, \dots, n\}$. We denote this language by $\mathfrak{L}(\mathscr{G})$.

The notions of **MALC**$(/, \cap)$-, **MALC**$(/, \cdot, \cap)$-, **MLC**-, and **MLC**$(/)$-grammar are defined similarly.

As shown by Gaifman [1] and Buszkowski [2], any context-free language without the empty word is generated by an **MLC**$(/)$-grammar. On the other hand, any language generated by an **MLC**-grammar is context-free (Pentus [12]).

Kanazawa [5] proved that any finite intersection of context-free languages is generated by a **MALC**$(/, \cap)$-grammar (therefore such grammars go beyond context-free). No generalisation of Pentus' theorem for **MALC** is yet known.

Theorem 1. *If a language without the empty word is generated by a conjunctive grammar in Greibach normal form, then this language is generated by a* **MALC**$(/, \cdot, \cap)$*-grammar.*

5 The Construction

Given a conjunctive grammar $\mathcal{G} = \langle N, \Sigma, \mathcal{P}, S\rangle$ in Greibach normal form, we shall construct a **MALC**$(/, \cdot, \cap)$-grammar $\mathscr{G}$, such that $\mathfrak{L}(\mathscr{G}) = \mathfrak{L}(\mathcal{G})$.

In order to avoid notation collisions, further we shall use the following naming convention (all these letters can also be decorated with numerical or other indices):

Letter	*Range*
A, B, S	N (nonterminal symbols of $\mathcal{G}$)
a	Σ (terminal symbols)
x	$N \cup \Sigma$
w, u	Σ^* (strings of terminal symbols)
β	N^+ (strings of nonterminal symbols)
τ, ω	$(N \cup \Sigma)^*$
p	Pr (primitive types of **MALC**)
E, F, G, P	Tp (types of **MALC**)
Γ, Φ, Ψ	Tp* (sequences of types)

With every $A \in N$ we associate a distinguished primitive type p_A. For $\beta = B_1 \dots B_m$ let $P_\beta \leftrightharpoons p_{B_1} \cdot \ldots \cdot p_{B_m}$ (multiplication is associative, so we can omit the brackets).

Since intersection in **MALC** is commutative and associative, we can use intersections of nonempty sets of types, not bothering about order and brackets: $\bigcap_{j=1}^{k} E_j$ stands for $E_1 \cap \ldots \cap E_k$, and if $\mathcal{M} = \{E_1, \ldots, E_k\}$, then $\bigcap \mathcal{M} \leftrightharpoons E_1 \cap \ldots \cap E_k$. If $\mathcal{M} = \{E\}$, then $\bigcap \mathcal{M} \leftrightharpoons E$.

For every $a \in \Sigma$ let

$$\mathcal{M}_a \leftrightharpoons \{p_A \,/ \Big(\bigcap_{j=1}^{k} P_{\beta_j}\Big) \mid (A \to a\beta_1 \& \ldots \& a\beta_k) \in \mathcal{P}\} \cup \{p_A \mid (A \to a) \in \mathcal{P}\}.$$

Let $G_a \leftrightharpoons \bigcap \mathcal{M}_a$. For $A \in N$ let $G_A \leftrightharpoons p_A$. The following holds due to the $(\cap \to)$ rule:

Lemma 2. *If* $E \in \mathcal{M}_a$ *and* **MALC** $\vdash \Phi\, E\, \Psi \to F$, *then* **MALC** $\vdash \Phi\, G_a\, \Psi \to F$.

For $\omega = x_1 \dots x_n \in (N \cup \Sigma)^+$ let $\Gamma_\omega \leftrightharpoons G_{x_1} \dots G_{x_n}$.

Lemma 3. *If* $\mathcal{G} \vdash [A, w]$, *then* **MALC** $\vdash \Gamma_w \to p_A$.

Proof. We proceed by induction on the length of w. The base case ($w = a$) corresponds to an application of a rule of the form $A \to a$ to the $[a, a]$ axiom (this is the only way to derive $[A, a]$). In this case we have $p_A \in \mathcal{M}_a$, therefore by Lemma 2 we get **MALC** $\vdash G_a \to p_A$, and $\Gamma_w = G_a$.

Now let w contain at least two symbols and the last step of the derivation of $[A, w]$ be an application of the rule $A \to a\beta_1 \& \ldots \& a\beta_k$. Then $w = aw'$, and for every $j \in \{1, \dots, k\}$, if $\beta_j = B_{j1} \dots B_{jm_j}$, then $w' = u_{j1} \dots u_{jm_j}$ and for every $i = \{1, \dots, m_j\}$ we have $\mathcal{G} \vdash [B_{ji}, u_{ji}]$. Therefore, by induction hypothesis, $\mathbf{MALC} \vdash \Gamma_{u_{ji}} \to p_{B_{ji}}$, whence $\mathbf{MALC} \vdash \Gamma_{w'} \to P_{\beta_j}$ for every j. Applying the $(\to \cap)$ rule k times we get

$$\mathbf{MALC} \vdash \Gamma_{w'} \to \bigcap_{j=1}^{k} P_{\beta_j},$$

and, finally, by $(/ \to)$,

$$\mathbf{MALC} \vdash p_A \,/ \left(\bigcap_{j=1}^{k} P_{\beta_j} \right) \Gamma_{w'} \to p_A.$$

Since $p_A \,/ (\bigcap_{j=1}^{k} P_{\beta_j}) \in \mathcal{M}_a$, by Lemma 2 we have $\mathbf{MALC} \vdash G_a \, \Gamma_{w'} \to p_A$, and $G_a \, \Gamma_{w'} = \Gamma_w$.

Before proving the inverse statement, we shall prove two technical lemmata:

Lemma 4. $\mathbf{MALC} \vdash \Phi \to \bigcap_{j=1}^{k} P_{\beta_j}$ *if and only if* $\mathbf{MALC} \vdash \Phi \to P_{\beta_j}$ *for every* $j \in \{1, \dots, k\}$.

Proof. The "if" part is just k applications of $(\to \cap)$. The "only if" part is proved using the cut rule (for every j_0):

$$\frac{\Gamma \to \bigcap_{j=1}^{k} P_{\beta_j} \quad \bigcap_{j=1}^{k} P_{\beta_j} \to P_{\beta_{j_0}}}{\Gamma \to P_{\beta_{j_0}}} \ (\text{cut})$$

Lemma 5. *If* $\omega \in (N \cup \Sigma)^+$, $\beta = B_1 \dots B_m \in N^+$, *and* $\mathbf{MALC} \vdash \Gamma_\omega \to P_\beta$, *then there exist such* $\tau_1, \dots, \tau_m \in (N \cup \Sigma)^+$, *that* $\omega = \tau_1 \dots \tau_m$ *and* $\mathbf{MALC} \vdash \Gamma_{\tau_i} \to p_{B_i}$ *for every* $i \in \{1, \dots, m\}$.

Proof. We can rearrange the derivation, so that the applications of $(\to \cdot)$ will be in the bottom (they are interchangeable with $(\cap \to)$ and $(/ \to)$, and these two are the only ones that can be applied below $(\to \cdot)$). Now the statement of the lemma is obvious.

Lemma 6. *If* $\mathbf{MALC} \vdash \Gamma_\omega \to p_A$, *then* $\mathcal{G}_{\text{cut}} \vdash [A, \omega]$.

Proof. Induction by the length of ω. If $\omega = a$, then the only possible case is $p_A \in \mathcal{M}_a$. Then $(A \to a) \in \mathcal{P}$, and $\mathcal{G}_{\text{cut}} \vdash [A, a]$.

Now let ω contain at least two letters. Consider the lowest application of $(/ \to)$ in the derivation of $\Gamma_\omega \to p_A$. Beneath this application there are only applications of $(\cap \to)$—the ones that open the type to which $(/ \to)$ is applied, and the ones that deal with other types in Γ_ω. We can transform the derivation so that the latter will be applied before the application of $(/ \to)$. Then we have $\omega = \omega_1 a \tau \omega_2$, $p_{A'} \,/ (\bigcap_{j=1}^{k} P_{\beta_j}) \in \mathcal{M}_a$, and the derivation step looks as follows:

$$\frac{\Gamma_\tau \to \bigcap_{j=1}^{k} P_{\beta_j} \quad \Gamma_{\omega_1} \, p_{A'} \, \Gamma_{\omega_2} \to p_A}{\Gamma_{\omega_1} \, p_{A'} \,/ (\bigcap_{j=1}^{k} P_{\beta_j}) \, \Gamma_\tau \, \Gamma_{\omega_2} \to p_A} \ (/ \to)$$

Then, by Lemma 4, $\mathbf{MALC} \vdash \Gamma_\tau \to P_{\beta_j}$ for every $j \in \{1, \ldots, k\}$. By Lemma 5, if $\beta_j = B_{j1} \ldots B_{jm_j}$, $\tau = \tau_{j1} \ldots \tau_{jm_j}$, and $\mathbf{MALC} \vdash \Gamma_{\tau_{ji}} \to p_{B_{ji}}$ (for every j and i in the ranges). By induction hypothesis, $\mathcal{G}_{\mathrm{cut}} \vdash [B_{ji}, \tau_{ji}]$, and, adding $[a, a]$, we can apply the rule for $A' \to a\beta_1 \& \ldots \& a\beta_k$, therefore $\mathcal{G}_{\mathrm{cut}} \vdash [A', a\tau]$.

By induction hypothesis for the right premise of the $(/ \to)$ rule, $\mathcal{G}_{\mathrm{cut}} \vdash [A, \omega_1 A' \omega_2]$. Finally, applying the cut rule to $[A', a\tau]$ and $[A, \omega_1 A' \omega_2]$, we get $[A, \omega_1 a\tau\omega_2] = [A, \omega]$.

Now we are ready to define $\mathscr{G} = \langle \Sigma, \rhd, H \rangle$. Let $H = p_S$, and $E \rhd a$ if and only if $E = G_a$. If $w \in \mathfrak{L}(\mathcal{G})$, then $\mathcal{G} \vdash [S, w]$, and, by Lemma 3, $\mathbf{MALC} \vdash \Gamma_w \to p_S$, whence $w \in \mathfrak{L}(\mathscr{G})$. Conversely, if $w \in \mathfrak{L}(\mathscr{G})$, then $\mathbf{MALC} \vdash \Gamma_w \to p_S$. By Lemma 6 we get $\mathcal{G}_{\mathrm{cut}} \vdash [S, w]$, and by Lemma 1 $\mathcal{G} \vdash [S, w]$. Hence, $w \in \mathfrak{L}(\mathcal{G})$.

Note that in $\mathscr{G}$ every $a \in \Sigma$ is associated with only one type (such grammars are called *grammars with single type assignment* or *deterministic grammars).* Having the intersection connective, it is usually easy to make our grammar deterministic (cf. [5]); for the pure Lambek calculus the fact that any context-free language is generated by a deterministic **MLC**-grammar is not obvious, but still valid, as shown by Safiullin [13].

Example 3. This construction gives the following **MALC**-grammar equivalent to the grammar from Example 2:

$$a \rhd p_A \cap (p_A \,/\, p_A) \cap (p_D \,/ (p_D \cdot p_V)) \cap \big(p_S \,/ ((p_A \cdot p_B) \cap (p_D \cdot p_C))\big)$$
$$b \rhd p_B \cap p_D \cap p_V \cap (p_B \,/ (p_B \cdot p_U))$$
$$c \rhd p_C \cap p_U \cap (p_C \,/\, p_C)$$

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