

# Perspectives of Dynamic Complexity

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**Abstract.** Many current data processing scenarios deal with about large collections of permanently changing data. In this context, it is often outright impossible to compute the answer for a query from scratch. Rather some auxiliary data needs to be stored that helps answering queries quickly, but also requires to be maintained incrementally. This incremental maintenance scenario can be studied in various ways, e.g., from the perspective of dynamic algorithms with the goal to reduce (re-) computation time. Other options are to study the scenario from the perspective of low-level parallel computational complexity [3] or parallelizable database queries [1]. As the “lowest” complexity class  $AC^0$  (with a suitable uniformity condition) and the core of the standard database query language SQL both coincide with first-order predicate logic, one naturally arrives at the question which queries can be answered/maintained dynamically with first-order predicate logic (DYNFO).

The most intensively studied query in this dynamic setting is the reachability query on graphs, arguably the “simplest recursive” query. It has been shown that it can be maintained in DYNFO on undirected [3] or acyclic directed graphs [1]. However, whether it can be maintained on general directed graphs is considered the main open question of the field.

Actually, it turned out that showing that a given query can *not* be maintained in DYNFO is a very challenging problem, for which currently no methods are available. Furthermore, even though  $AC^0$  is a small complexity class in the static setting, first-order logic is already quite powerful in the dynamic world. These two observations have recently led to the study of fragments of DYNFO, e.g., by restricting or forbidding quantification, with the idea to start developing inexpressibility tools there. A surprising result found along these lines is that on strings, quantifier free predicate logic can *exactly* maintain the regular languages [2]. The talk will give an introduction into dynamic complexity, survey some of its most important results, and report about recent work on fragments of DYNFO.

## References

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