How Much Rationality Tolerates the Shadow Economy? – An Agent-Based Econophysics Approach

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Abstract. We calculate the size of the shadow economy within a multi-agent econophysics model previously developed for the study of tax evasion. In particular, we analyze deviating behavior depending on the fraction of rational agents which aim to pursue their self interest. Two audit mechanisms are considered within our model, that are, (i) a constant compliance period which is enforced after black market activities of an agent have been detected and (ii) a backauditing method which determines the compliance period according to the particpation rate in the shadow economy within a previously preassigned time interval. We calibrate our simulation with respect to experimental evidence of tax compliance in France and Germany and give estimates for the percentage of selfish agents in these countries. This implies different policy recommendations that may work to fight the shadow economy, tax evasion, and the like.

Keywords: shadow economy; econophysics; multi-agent model.

1 Introduction

Theoretical approaches to account for shadow economy and tax compliance are often based on the seminal work of Allingham and Sandmo (1972) which incorporates potential penalties, tax rates and audit probabilities as basic parameters in order to evaluate the expected utility of tax payers. Obviously one of the reasons why people participate in the shadow economy is to circumvent the tax system. Vice versa the shadow economy is often taken as a proxy for the amount of tax evasion (Alm et al., 2012) although the former is naturally driven by additional factors. Buehn and Schneider (2012) define shadow economy "as all market-based legal production of goods and services that are deliberately concealed from public authorities ....". The size of this production is of course depending on efficient audit mechanisms and penalties which provides the link for the application of theories originally developed for the modelling of tax compliance.

In this regard agent-based models have been set up as a comparatively new tool for analyzing tax compliance issues. In fact, an essential feature of any agent-based model is the direct non-market based interaction of agents, which is combined with some process that allows for changes in individual behavior patterns[1]. Therefore, agent-based tax

[1] An exception is the work by Szabó et al. (2009, 2010).

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evasion models may be categorized according to the features of this individual interaction process. In fact, in econophysics models this process is driven by statistical mechanics using the Ising model (Ising, 1925) that is well known in physics and describes objects which can be in one of two states and interact on a given lattice structure. Examples include Zaklan et al. (2008, 2009), Lima and Zaklan (2008), and Lima (2010) which have identified the Ising states with compliant and non-compliant tax payers. In contrast, if the interacting process is driven by parameter changes that induce behavioral changes via a utility function and (or) by stochastic processes that do not have physical roots, these models belong to the economics domain. Examples include Mittone and Patelli (2000); Davis et al. (2003); Bloomquist (2004, 2008); Korobow et al. (2007); Antunes et al. (2007); Szabó et al. (2009, 2010); Hokamp and Pickhardt (2010); Méder et al. (2012); Nordblom and Žamac (2012); Andrei et al. (2013); Hokamp (2013); Pellizari and Rizzi (2013) of which some are summarized by Bloomquist (2006) and Pickhardt and Seibold (2013).

In agent-based tax evasion models of the econophysics type the Ising model is used to mimic conditional cooperation among agents (Zaklan et al., 2009). Yet, the actual patterns and levels of tax evasion in these models depend on two additional factors: the network structure of society and the tax enforcement mechanism. The network structure is implemented by alternative lattice types and tax enforcement consists of the two economic standard parameters audit probability and penalty rate. To this extent, rational behavior patterns are essentially reconstructed by means of statistical mechanics.

In previous work Pickhardt and Seibold (2013) and more recently Seibold and Pickhardt (2013) have extended the Ising-based econophysics approach to tax evasion toward the implementation of different agent types. This theory is able to reproduce results from agent-based economics models (Hokamp and Pickhardt, 2010) so that it should be also appropriate for a quantitative analyis. Following this idea we aim in the present contribution to apply the model to an analysis of the shadow economy in France and Germany with respect to the percentage of rational agents in both countries. Note that in the present paper we use the term 'shadow economy' synonymous with the participation in black market services.

In Sec. 2 we outline the basic ingredients of our econophysics model and exemplify the approach for a black market with homogeneous agents. We apply our model to the analysis of the shadow economies in Germany and France in Sec. 3 where we deduce the essential parameters entering the simulations from previous experimental and agent-based investigations in the literature. Finally, we discuss our results for different enforcement schemes and give policy recommendations to combat the shadow economy in Sec. 4.

2 The Agent-Based Econophysics Approach

Our considerations are based on the Ising model hamiltonian

$$H = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i B_i S_i \tag{1}$$

where J describes the coupling of Ising variables (spins) $S_i = +1, -1$ between adjacent lattice sites denoted by $\langle ij \rangle$. In the present context $S_i = +1(-1)$ is interpreted as a participant in the white (black) market. The following results are not sensitive to the specific lattice geometry and we implement the model on a two-dimensional square lattice with dimension 1000×1000. Eq. (1) contains also the coupling of the spins to a local magnetic field B_i which can be associated with the morale attitude of the agents and corresponds to the parameter γ_i in the theory of Nordblom and Žamac (2012). In addition, our model contains a local temperature T_i which measures the susceptibility of agents to external perturbations (either influence of neighbors of magnetic field). We then use the heat-bath algorithm [cf. Krauth (2006)] in order to evaluate statistical averages of the model. The probability for a spin at lattice site i to take the values $S_i = \pm 1$ is given by

$$p_i(S_i) = \frac{1}{1 + \exp\{-[E(-S_i) - E(S_i)]/T_i\}} \tag{2}$$

and $E(-S_i) - E(S_i)$ is the energy change for a spin-flip at site i. Upon picking a random number $0 \leq r \leq 1$ the spin takes the value $S_i = 1$ when $r < p_i(S_i = 1)$ and $S_i = -1$ otherwise. One time step then corresponds to a complete sweep through the lattice.

Following Hokamp and Pickhardt (2010) we consider societies which are composed of the following four types of agents: (i) *selfish a-type agents*, which take advantage from black market activities ($S_i = -1$) and, thus, are characterized by $B_i/T_i < 0$ and $|B_i| > J$; (ii) *copying b-type agents*, which conform to the norm of their social network and thus copy the behavior with respect to black or white market participation from their neighborhood. This can be modelled by $B_i << J$ and $J_i/T_i \gtrsim 1$; (iii) *ethical c-type agents*, which have large moral doubts about participating in black market services and thus are parametrized by $B_i/T_i > 0$ and $|B_i| > J$; (iv) *random d-type agents*, which act by chance, within a certain range, due to some confusion about the attribution of services to the black or white market. We implement this behavior by $B_i << J$ and $J/T_i << 1$. The parameters distinguishing the different agent types are taken from Pickhardt and Seibold (2013) and Seibold and Pickhardt (2013).

Furtheron we implement different enforcement schemes into our model. Here we first consider the case where the detection of a black market participating agent enforces its compliance over the following h time steps. This is the procedure which has been invoked in Zaklan et al. (2008, 2009); Lima (2010); Pickhardt and Seibold (2013) and also implemented in a randomized variant in Lima and Zaklan (2008). Second, we also study lapse of time effects, i.e. the situation where a detected agent is also screened over several years in the past by the (tax) authorities (i.e. backaudit). This variant has been studied within an econophysics tax compliance model in Seibold and Pickhardt (2013). If tax evasion is detected in the current time period, the backaudit comprises also an inspection of the preceding b_p time steps. Denote with n_e the number of time steps over which the agent was evading within the backaudit plus current period. Then the period k over which the agent is reinforced to be compliant is set to $k = n_e * h$. For example, for a convicted agent in the current time step, inspection of the preceding $b_p = 5$ time steps reveals three periods where he was evading. Setting $h = 2$ yields a number of $(3+1) * 2 = 8$ periods where he is forced to be compliant. Thus the above limit of fixed compliance period h is recovered in the limit of zero backaudit $b_p = 0$ since then $k = (0+1) * h = h$.

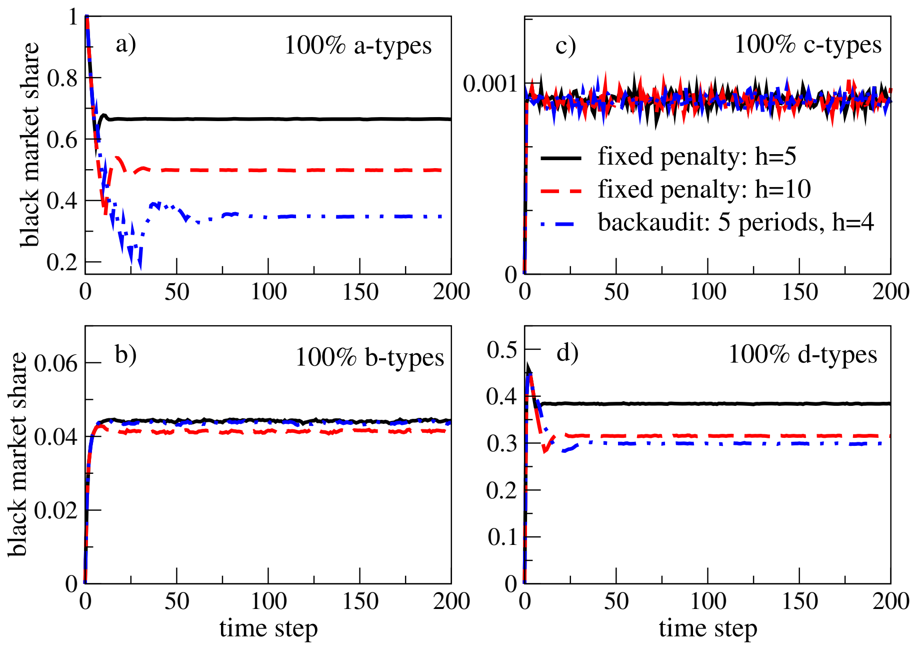

Fig. 1. Time evolution for the participation in black market services for a society consisting of 100% a-type (panel a), 100% b-type (panel b), 100% c-type (panel c), and 100% d-type (panel d) agents. Results are reported for different enforcement mechanisms: fixed compliance period $h = 5$ (solid, black), $h = 10$ (dashed, red), and backaudit (dashed-dotted, blue). Audit probability is $p_a = 10\%$ in each case.

Before analyzing the heterogeneous agent model it is instructive to consider first the case of a black market with all agents being of the same type. The resulting percentage of black market share as a function of time is shown in Fig. 1 where we also compare the different enforcement schemes.

The first case of endogenous non-compliant selfish agents is shown in Fig. 1a. At time step zero we correspondingly set the share of black market participation to $p_{bm} = 1$. Due to the enforcement mechanisms the black market share is significantly reduced because at each time step a certain percentage of the remaining non-compliant agents are forced to become compliant. Before reaching a stationary value small oscillations are observed since after h time steps the first detected agents can become non-compliant again. Notably, the black market share is reduced strongest for the backaudit mechanism

Fig. 1b reports the result for copying b-type agents. As initial condition all agents are set to 'compliant'. Since b-types tend to copy the behavior of their social network only few of them change their behavior and the equilibrium value for the black market share for all enforcement mechanisms approaches a rather small value between 4% and 5%. It should be noted that the equilibrium value is independent of the initial condition. If we would have set all agents to non-compliant at time step zero, the audits would have reduced the black market share to the same equilibrium value than shown in Fig. 1.

The time evolution for ethical agents is reported in Fig. 1c. Here the initial black market participation is set to $p_{bm} = 0$ and there is only a very small probability that one of the agents becomes non-compliant. Since ethical agents avoid black market participation the results are also almost independent of the audit probability. Hence, any positive audit probability would be inefficient in this case.

Finally, Fig. 1d shows the black market share for d-type agents. Since these agents act by chance their participation in black market activities would be of the order of $\sim 50\%$ without any audit. The different enforcement mechanisms then lead to a further reduction of this equilibrium value where similar to the a-types the backaudit mechanism is most effective.

3 Exploring the Shadow Economy

We now turn to the analysis of the shadow economies in France and Germany within our model which requires the estimation of the corresponding specific agent compositions. In general, tax experiments provide average data and do not provide individual compliance data of tax payers. The work by Bazart and Pickhardt (2011) is a notable exception and allows to extract the percentage of fully compliant individuals (i.e. essentially c-type agents in our terminology). Subject to the small group sizes of 5 subjects Bazart and Pickhardt (2011) obtain a full compliance ratio of 5% and 20% for Germany and France, respectively[2].

Unfortunately no data are available on the percentage of d-type agents. However, Andreoni et al. (1998) report that about seven percent of U.S. households overpaid their taxes in 1988. If we anticipate that about the same amount of people underpays their taxes we arrive at a percentage of $\sim 15\%$ of d-type agents [cf. also Hokamp and Pickhardt (2010)]. For simplicity, this percentage is adopted in equal measure for France and Germany[3].

In the following we will denote the parameters derived for France (i.e. 20% c-types and 15% d-types) as parameter set 'F' and the parameters derived for Germany (i.e. 5% c-types and 15% d-types) as parameter set 'G'. The aim is then to determine the fraction of a- and b-type agents which participate in the shadow economy in these countries. Calculations are performed for two different enforcement schemes which both are based on the probability p_a for an audit at a given lattice site (agent). For both, France and Germany we set $p_a = 0.1$. Concerning the backaudit enforcement we consider a backaudit period of $b_p = 5$ time steps which is compatible with the limitation period of 5 years in 2006/2007 where the experiments by Bazart and Pickhardt (2011) have been conducted.

In order to obtain values on the shadow economies we adopt the values from Buehn and Schneider (2012) which have estimated the shadow economy of 162 countries.

[2] Note that the compliance ratio only specifies the percentage of subjects which behaved fully compliant in each of the rounds of the experiment. The average compliance rate which determines tax evasion is larger.

[3] One may of course expect that due to different tax pressure this number differs between France, Germany and the U.S.. Future experiments can help to resolve this element of uncertainty for the percentage of d-type agents.

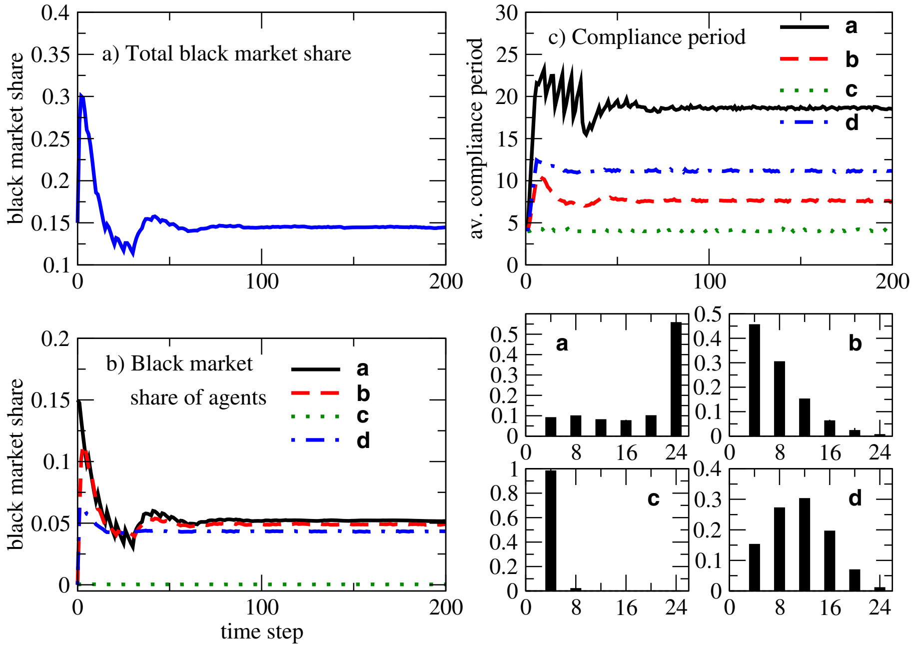

Fig. 2. Time evolution for the participation in black market services for a society consisting of 15% a-type, 50% b-type, 20% c-type, and 15% d-type agents. The upper left panel compares the extent and dynamics of black market participation for backaudit periods $b_p = 5$. The lower left panel breaks down the participation probability to the individual agent types. The upper right panel displays the average forced compliance period (or penalty) for the individual agent types and the lower right panels show the corresponding distribution. Audit probability is $p_a = 10\%$ in each case.

According to this analysis the size of the shadow economy in France and Germany is 15% and 16% of the official GDP, respectively, averaged over the period 1999 - 2007. These numbers are supported by the data of Elgin and Öztunali (2012) which report 16.53% for OECD EU-countries in the period from 2001-2009.

Fig. 2a shows the dynamics of black market participation for the parameter set 'F' appropriate to France and 15(50)% a(b)-type agents. Initial conditions are chosen such that all agents but the a-types are set to white market participants so that the black market share at time step 'zero' just reflects the percentage of selfish a-types. Note, however, that the equilibrium result for large time steps does not depend on these initial conditions. As can be seen from Fig. 2b the initial increase in the first time step is due to the b-types which copy the black market participation from the a-type agents. In the following periods black market participation is reduced to the (back)audit and approaches an equilibrium value after passing a transient regime. For this parameter set a-, b- and d-type agends contribute equally with $\approx 5\%$ to the total black market share. It is also instructive to monitor the average compliance period of the convicted agents as shown in Fig. 2c and the actual distributions which are displayed in the lower right panels of

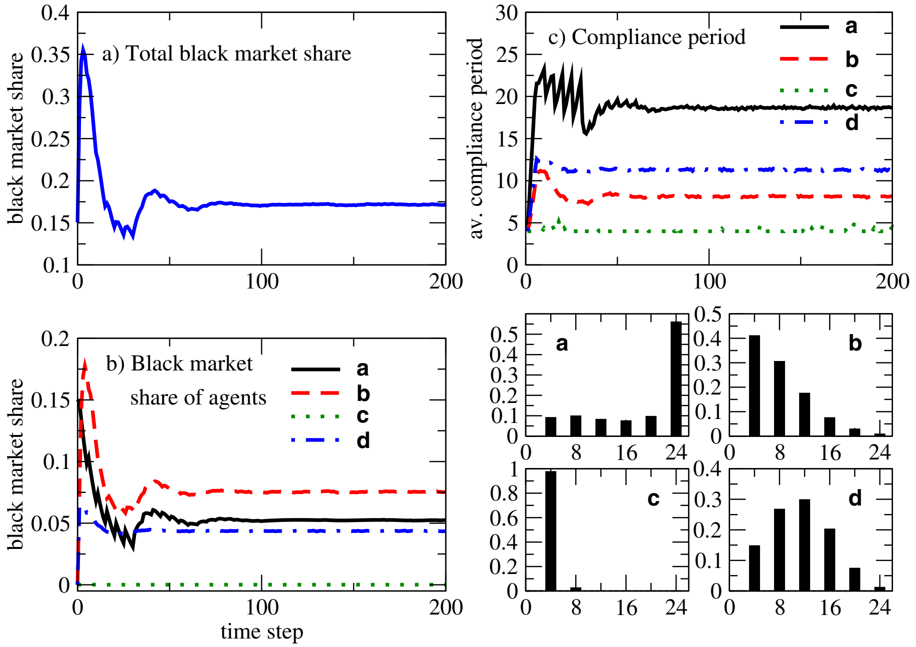

Fig. 3. The same as Fig. 2 but for a society consisting of 15% a-type, 65% b-type, 5% c-type, and 15% d-type agents

Fig. 2. Naturally most a-types are penalized with the maximum compliance period of $k = 6 * 4 = 24$ time steps. Only those which have been convicted also in previous time steps (and therefore have to stay compliant) are penalized with a reduced compliance period. On the other hand, the (few) convicted c-type agents are penalized with a compliance period of $k = 4$ since their probability of repeatedly being non-compliant within the backaudit period is vanishingly small.

Fig. 3 shows the analogous results for the parameter set 'G' derived for Germany and also 15% of a-type agents. The reduced percentage of (compliant) c-types as compared to France is compensated by the increased number of b-type agents which enhance the black market participation as becomes apparent from Fig. 3a. In fact, the largest contribution is now from the b-types (cf. Fig. 3b) in contrast the previous parameter set. Note especially that the percentage of convicted b-types with a compliance period of 4 time steps is reduced from $\sim 45\%$ in Fig. 2 to $\sim 40\%$ in Fig. 3 due to the concomitant reduction of c-types which define the social norm of adjacent b-type agents.

We are now in the position to evaluate the black market share as a function of a-type agents for the two parameter sets 'F' and 'G' derived above. Fig. 4 reports the corresponding results for different audit schemes, i.e. fixed compliance period with $h = 5, 10$ and backauditing with backwards auditing period of $b_p = 5$ and scaling factor $h = 4$ (cf. above). Within all auditing schemes we find a larger black market share for the parameter set as deduced for Germany due to the smaller (larger) percentage of c-(b-) type agents. The horizontal lines in Fig. 4 indicate the size of the shadow

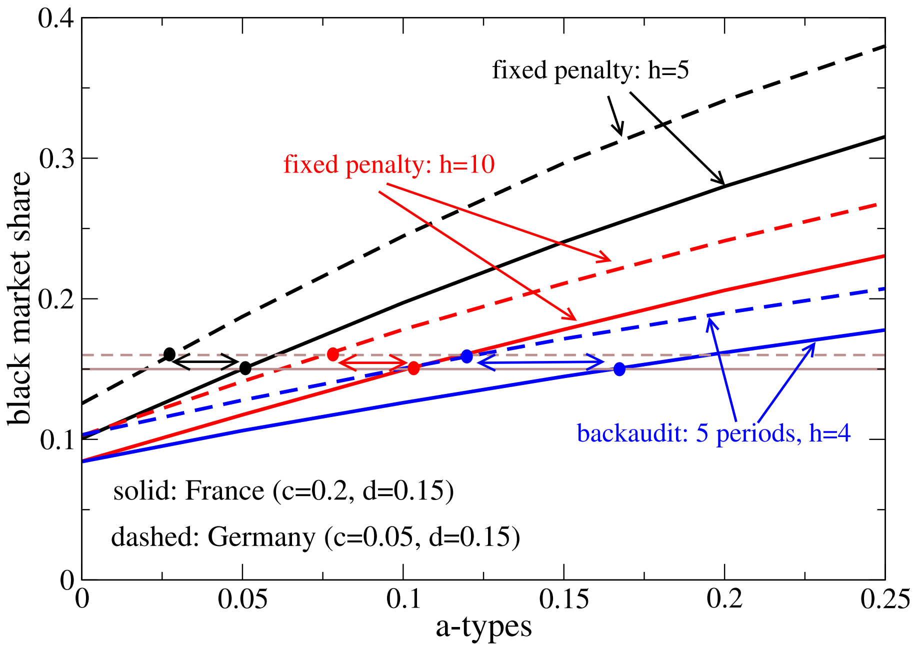

Fig. 4. Black market share for the two parameter sets derived for France (solid lines) and Germany (dashed lines) as a function of a-type agents. The horizontal lines indicate the size of the shadow economy in both countries and the intersections (indicated by dots) fix the respective percentage of a-types. Results are reported for different enforcement mechanisms: fixed compliance period $h = 5$ (black), $h = 10$ (red), and backaudit (blue).

economy for France and Germany as reported by Buehn and Schneider (2012) and the intersections with the tax evasion curves are indicated by dots for each audit mechanism. We find that the difference of a-types between both countries which is compatible with these values (as illustrated by arrows) is between 3% and 5% with the large difference obtained for the backaudit scheme. Since backauditing is more efficient in reducing black market services it is compatible with a larger percentage of a-type agents than audit schemes with fixed compliance period.

4 Policy Recommendations and Discussion

The larger percentage of rational a-types resulting for the parameter set 'F' suggests that audit mechanisms to combat the shadow economy are more efficient in France than in Germany. It is also interesting that the difference between rational a-types in both countries can be traced back to the much larger percentage of c-types in the parameter set 'F'. Inspection of the experiment by Bazart and Pickhardt (2011) reveals that this difference is due to the larger full compliance ratio for french female subjects (41%) than for german females (10%) whereas the full compliance ratio between male (France: 4.8%, Germany: 3.7%) does not differ significantly. Note, however, that the group sizes in these investigations were rather small so that further experiments are required in order to substantiate the corresponding data.

In our econophysics model we have assumed a constant (over time) distribution of agent types. On the other hand, Nordblom and Žamac (2012) have set up a economic model which describes how social beings update their personal norms. Implementing these mechanisms in our model would allow for the transformation between different agent types and is an interesting perspective for future research.

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