# Fast Wavelet Transform Based on Spiking Neural Network for Visual Images

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**Abstract.** The functionalities of spiking neurons can be applied to deal with biological stimuli and explain complicated intelligent behaviors of the brain. Wavelet transform is a powerful time-frequency analysis tool that can efficiently compress image and extract image features. In this article, a spiking neural network combined with the ON/OFF neuron arrays associated with the human visual system is proposed to perform the fast wavelet transform for visual images. The simulation results show that the spiking neural network can preserve the key features of visual images very well.

**Keywords:** Spiking neural network, human visual system, fast wavelet transform, visual image.

## 1 Introduction

Hodgkin-Huxley Spiking Neuron Model was proposed in 1952 [1]. But if this model is applied to a large scale network, the implementation will encounter a very high computational complexity. Therefore, the simplified conductance-based integrateand-fire model will be used for each neuron in Spiking Neuron Networks (SNNs) [2]. In the human visual system, there are various receptive fields from simple cells in the striate cortex to those of the retina and lateral geniculate nucleus [3-5]. The visual images are transferred among these neurons in the form of spiking trains through the ON or OFF pathways [6-7]. It is assumed that each neuron receives spike trains through excitatory synapse for ON neurons and through inhibitory synapse for the OFF neurons [8]. Different ON/OFF pathways are used to construct the specific network in a biological manner. On the other hand, wavelet transform can efficiently extract the key features of images [9-11]. In this paper, a SNN is proposed to mimic behaviors of spiking neurons in the human visual system for wavelet transform and extract the main features of visual images.

## 2 Spiking Neural Network Model for Fast Wavelet Transform

### 2.1 Fast Wavelet Transform

Mallat proposed fast wavelet transform (FWT) in 1987[12, 13]. The flow chart of two-dimensional FWT is shown in Fig. 1.



Fig. 1. Achieved 2D-FWT through the application of filter bank and down-sampling

As shown in Fig. 1, the input signal f(m,n) is passed through different filters  $h_0(m,n)$  and  $h_1(m,n)$  and down-sampled respectively and the four signals ultimately obtained are approximate coefficient *wA*, horizontal detail *wH*, vertical detail *wV* and diagonal detail *wD* of wavelet transform.

#### 2.2 Spiking Neural Network Model for Fast Wavelet Transform

Based on the Mallat algorithm and ON/OFF pathways mechanism in the visual system [8, 14, 15], an integrate-and-fire SNN model is proposed as shown in Fig. 2.

The dimension of the input neuron array is  $M \times N$ . Each pixel of the image corresponds to a receptor. Assume that  $G_{m,n}(t)$  represent the gray scale of an image pixel and each photonic receptor transfers the pixel brightness to a synapse current  $I_{m,n}(t)$  [16-18]. The  $I_{m,n}(t)$  and the neuron potential  $v_{m,n}(t)$  can be represented as follows:

$$\frac{dI_{m,n}(t)}{dt} = -\frac{1}{\tau} I_{m,n}(t) + \alpha G_{m,n}(t)$$
(1)

$$c\frac{dv_{m,n}(t)}{dt} = g_l(E_l - v_{m,n}(t)) + I_{m,n}(t) + I_0$$
<sup>(2)</sup>

where m=1,...,M and n=1,...,N,  $\alpha$ ,  $\tau$  are constants,  $g_l$  is the membrane conductance,  $E_l$  is the reverse potential, c represents the membrane capacitance and  $I_0$  is background noise. If the membrane potential passes threshold  $v_{th}$ , then the neuron generates a spike. Let  $S_{m,n}(t)$  represent the spike train generated by the neuron such as that:

$$S_{m,n}(t) = \begin{cases} 1 & \text{if neuron } (m,n) \text{ fires at time } t. \\ 0 & \text{if neuron } (m,n) \text{ does not fire at time } t. \end{cases}$$
(3)



Fig. 2. Spiking neural network for fast wavelet transform

The first layer of the intermediate neuron array is composed of three  $M \times (N+1)$  neuron arrays as shown in Fig. 2. First two are the ON neuron arrays 1ON1(p, q), 1ON2(p, q) and the third is the OFF neuron array 1OFF(p, q), where  $p=1, \ldots, M$  and  $q=1, \ldots, N+1$ . The convolutions of FWT are corresponding to the accumulation of different neural arrays. Assume the spike trains are transferred to the ON/OFF neuron arrays through excitatory synapses  $W_{10N1(p,q)}$  and  $W_{10N2(p,q)}$  and inhibitory synapse  $W_{0FF(p,q)}$ . The synapse strength distribution can be set as follows.

$$W_{1ONi(p,k)} = a_{ON}f(p,q), \qquad \qquad W_{1OFF(p,q)} = a_{OFE}f(p,q)$$
(4)

where  $i=\{1,2\}, 1 \le p \le M$ ,  $1 \le q \le N$ . if i=1, k=q, else, k=q+1.  $a_{ON}=1/\sqrt{2}$ ,  $a_{OFF}=-1/\sqrt{2}$ . The synapse currents  $I_{1ON1(p,q)}(t)$ , and  $I_{1OFF(p,q)}(t)$  are governed by the current constraint equation:

$$\frac{dI_{1\sigma(p,q)}(t)}{dt} = -\frac{1}{\tau}I_{1\sigma(p,q)}(t) + \sum_{p=1}^{M}\sum_{q=1}^{N+1} W_{1\sigma(p,q)}\beta_{1}S_{\sigma(p,q)}(t)$$
(5)

where  $\sigma \in \{ON, OFF\}$ .  $S_{\sigma(p,q)}(t)$  represent a spike train. The neuron potential in the ON/OFF array is governed by the potential constraint equation.

The intermediate second layer of the neuron array is composed of two  $M \times (N+1)$  neuron arrays 2ON(p,q) and 2OFF(p,q). Each neuron receives spike trains through excitatory synapse  $W_{2ON(p,q)}$  and inhibitory synapse  $W_{2OFF(p,q)}$ , they are set as:

$$W_{2\text{ON}(p,q)} = \begin{cases} W_{1\text{ON}2(p,q)} - W_{1\text{OFF}(p,q)}, & \text{if } W_{1\text{ON}2(p,q)} - W_{1\text{OFF}(p,q)} > 0\\ 0, & \text{if } W_{1\text{ON}2(p,q)} - W_{1\text{OFF}(p,q)} \le 0 \end{cases}$$
(6)

$$W_{2\text{OFF}(p,q)} = \begin{cases} -(W_{1\text{ON}2(p,q)} - W_{1\text{OFF}(p,q)}), & \text{if } W_{1\text{ON}2(p,q)} - W_{1\text{OFF}(p,q)} < 0\\ 0, & \text{if } W_{1\text{ON}2(p,q)} - W_{1\text{OFF}(p,q)} \ge 0 \end{cases}$$
(7)

where  $1 \le p \le M$ ,  $1 \le q \le N+1$ . The synapse current and the neuron potential in the ON/OFF array are still governed by the current and potential constraint equation.

The third layer of the intermediate array is still composed of two  $M \times (N+1)$  neuron arrays. Neurons in these arrays are labeled with  $3\text{CN1}^*(p, q)$  and  $3\text{CN2}^*(p, q)$ . The synapses strength distribution can be calculated by the following expressions.

$$W_{3\text{CN1}^{*}(p,q)} = W_{1\text{ON1}(p,q)} + W_{1\text{ON2}(p,q)}, \qquad W_{3\text{CN2}^{*}(p,q)} = W_{2\text{ON}(p,q)} - W_{2\text{OFF}(p,q)}$$
(7)

The synapse currents are set as the following equations:

$$\frac{dI_{3CN1^{*}(p,q)}(t)}{dt} = -\frac{1}{\tau}I_{3CN1^{*}(p,q)}(t) + \sum_{p=1}^{M}\sum_{q=1}^{N+1}W_{1ON1(p,q)}\beta_{2}S_{p,q}(t) + \sum_{p=1}^{M}\sum_{q=1}^{N+1}W_{1ON2(p,q)}\beta_{2}S_{p,q}(t)$$
(8)

$$\frac{dI_{3CN2^{*}(p,q)}(t)}{dt} = -\frac{1}{\tau}I_{3CN2^{*}(p,q)}(t) + \sum_{p=1}^{M}\sum_{q=1}^{N+1}W_{2ON(p,q)}\beta_{2}S_{p,q}(t) - \sum_{p=1}^{M}\sum_{q=1}^{N+1}W_{2OFF(p,q)}\beta_{2}S_{p,q}(t)$$
(9)

where  $\beta_1$ ,  $\beta_2$  is a constant.

After the accumulation of signals, only the neurons of the even-numbered columns of the 3CN1 and 3CN2 neuron layer generate spikes, while the neurons of the odd-numbered columns do not fire. Then two new neuron arrays are obtained which are labeled with 3CN1<sup>\*</sup> and 3CN2<sup>\*</sup>. Synapse strength distribution can be set as follow.

$$W_{3CNi(p,q)} = W_{3CNi^{*}(p,2k)}$$
(10)

where  $i = \{1, 2\}, k = 1, 2, \dots, N/2, 1 \le p \le M, 1 \le q \le N/2$ .

Thereafter, the remaining synapse strength distribution of the network can be set in a similar iteration and down-sampling manner, and eventually we will obtain four neuron array *OUT*1, *OUT*2, *OUT*3 and *OUT*4 as the bottom layer and the firing rate for these layers is calculated by the following expression:

$$r_{OUT\{j\}(m,n)}(t) = \frac{1}{T} \sum_{t}^{t+T} S_{OUT\{j\}(m,n)}(t)$$
(11)

where  $j = \{1, 2, 3, 4\}$ ,  $S_{OUT\{j\}(m,n)}(t)$  represent spike train generated by the output array.

### **3** Simulation Results

This network model is simulated by using the Euler method with a time step of 0.1 ms by Matlab. The following parameters were used in the experiments corresponding to biological neurons.  $v_{th} = -60$  mv.  $E_l = -70$  mv.  $g_l = 1.0 \ \mu s/mm^2$ .  $c = 8 \text{ nF/mm}^2$ .  $\tau = 16$ ms. T=400ms.  $\alpha = 0.02$ .  $\beta_1 = 4.3$ .  $\beta_2 = 5.1$ .  $I_0 = 7\mu$ A. These parameters can be adjusted to get a good quality output image.

The Lena image  $(512\times512)$  is used to test the network model. Since the image exceeds the Matlab predetermined matrix dimension, therefore the image has been divided into  $32\times32$  blocks and each block contains  $16\times16$  pixels. Fig. 3(a)-(d) show the four coefficients of the wavelet transform obtained by Mallat method. Fig. 3(e)-(h) display the similar results obtained by SNN. In Fig. 3, the dimensions of all of the images are  $8\times8$  and the resolution of these results is a quarter of the original image.



Fig. 3. Wavelet transform by Mallat method (a-d) and by SNN (e-h) of Lena

It can be seen that, although the visual image signals pass through a complex spiking neural network and lost a lot of details, but ultimately still retain all the main information and achieve the purpose of the feature extraction.

### 4 Discussion

In this paper, we propose an integrate-and-fire spiking neuron network combining visual ON/OFF neuron pathways and synapse current mechanism to extract features from a visual image. In the process of building the model, the accumulation between different neuron arrays are used to perform the convolutions of FWT, while the firing neurons is selected instead of the down-sampling algorithm. The simulation results show that the SNN is able to perform FWT. The key information can be obtained when the visual image signals pass through a complex spiking neural network.

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