Stationary State Properties of a Microscopic Traffic Flow Model Mixing Stochastic Transport and Car-Following

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Abstract We study the stationary properties of a microscopic traffic flow model related to a continuous time mass transport process. It is a stochastic collision-free mapping of a classical deterministic first order car-following model calibrated by the targeted speed function and the driver reaction time. The stationary states of the model are analytically treated for vanishing reaction time. Some approximations are calculated, assuming a product form of the invariant measure. When the reaction time is strictly positive, the process is studied by simulation. A relation between the parameters and the propagation of kinematic *stop-and-go* waves is identified as identical to the well-known stability condition of the car-following model. The results underline a negative impact of the driver reaction time parameter on the homogeneity of the flow in stationary state.

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1 Introduction

Stochastic transport processes, extracted from the systems of interacting particles, can be used to model traffic flow [1]. The approach represents an alternative to the modelling of traffic flow by systems of differential equations and is an extension in continuous time of cellular automata models. Markovian jump processes such as *exclusion* [2, 3], *zero-range* [2, 4, 5], *random average* [6, 7] or *misanthrope* [8] processes can describe microscopic traffic flows in the totally asymmetric case and for specific interpretations of the model parameters.

The totally asymmetric simple exclusion process (TASEP) is a basic model that has been studied extensively and is often used as a theoretical tool. The TASEP with nearest-vehicle interaction (which can be mapped to a zero-range process [5, 9, 10]) as well as with Arrhenius interaction [11] are employed to model traffic flows. The zero-range process is used to model the evolution of platoons in [12]. Recently, the misanthrope process is applied to describe microscopic multi-lane traffic flow [13] or mesoscopic ones [14]. These approaches, based on particle systems, are defined on a discrete space. The totally asymmetric random average process (TARAP), studied in [15], allows the microscopic modelling of traffic flows in continuous space.

On the other hand, microscopic traffic models based on ordinary or delayed differential systems are developed since the 1950s. The approach assumes interactions of the vehicles with their predecessors. Fundamental parameters, initially estimated from statistical observations, of the targeted speed as a function of the distance gap and the driver reaction time are sufficient to reproduce reasonably the driver's behavior. Models such as the first order one by Newell [16] or the second order *Optimal Velocity* model [17] are well understood. Notably, conditions of linear stability of homogeneous configurations are known. The first one imposes the derivative of the targeted speed function to be strictly positive, the second one a derivative strictly positive and strictly less than the inverse of two times the reaction time [17].

A new model combining the two approaches is proposed. The model is a stochastic mass transport process mapping a discretisation scheme of the Newell car-following model. The stochastic process we use is close to the TARAP. It models the evolution of vehicles in continuous time on a continuous space.

The paper is organised as follows. In a first part, we study the basic model without reaction time. The model behaviour in the stationary state is described analytically with the help of results taken from the literature. In a second part, we define and study the mass transport model including a strictly positive driver reaction time parameter. The stationary state of the process is investigated by simulations. Its stability properties are underlined according to the values of the parameters. We conclude by comparing the properties of the model to empirical observations and classical traffic theories.

2 The Basic Model with No Reaction Time

Let us start by mapping the TARAP to the Newell first order car-following model with no delay.

2.1 Model Definition

The model represents the evolution in continuous time of the distance gaps of vehicles on a one-way road. Since the vehicle order remains the same, this representation is an exact mapping of a line of vehicles [9, 15]. When a vehicle moves, a part of its distance gap is transmitted to the following vehicle. This representation allows the use of zero-range processes in discrete space case or random average processes in the continuous space case that is studied here.

$\eta = (\Delta_i)_i \in E = \left(\mathbb{R}^+\right)^{\mathbb{Z}}$ denotes the process of the vehicles distance gaps on an infinite lane. The jump size and jump rate of the process are defined by a mapping of the Newell first order car-following model: $\frac{\mathrm{d}x_i}{\mathrm{d}t}(t) = V\big(\Delta_i(t)\big)$ where $x_i(t)$ is the position of the vehicle i at the instant t. $\Delta_i = x_{i+1} - x_i - \ell$, with the vehicle length ℓ, is the distance gap of the vehicle i that depends on the position of its predecessor $i+1$. $V : \Delta \mapsto \min\{V_{\max}, \Delta/T\}$ is the positive non-decreasing and piecewise linear targeted speed function. $V_{\max} > 0$ is the maximal desired speed in the free traffic state and $T > 0$ is the targeted time gap for a vehicle in the pursuit case. For a vehicle i the explicit eulerian discretisation scheme of its motion with time step $\delta t > 0$ is

$$x_i(t + \delta t) = x_i(t) + \delta t \cdot V\big(\Delta_i(t)\big)$$

with the position x and the distance gap Δ to the predecessor. In this discrete time model the jump size of vehicle i is $\delta t V(\Delta_i)$ and the jump time is δt. One may consider a stochastic model with a jump rate equal to $1/\delta t$ (and the mean jump time δt) with jump size $\delta t V(\Delta_i)$. For this model, the generator is

$$\mathscr{L} f(\eta) = \sum_{i \in I_f} \frac{1}{\delta t}[f(\eta^i) - f(\eta)]\mathbb{1}_{\{\Delta_i > \delta t\, V(\Delta_i)\}} \tag{1}$$

with $\eta^i = (\Delta^i_j)_j$ such that $\Delta^i_j = \Delta_j$ if $j \neq i$ and $j \neq i-1$, $\Delta^i_i = \Delta_i - \delta t V(\Delta_i)$ and $\Delta^i_{i-1} = \Delta_{i-1} + \delta t V(\Delta_i)$. The generator is an operator of a function f depending on the finite set of coordinates I_f.

The Chapman-Kolmogorov equation allows to describe the marginal first order momentum of the process. As expected, one obtains the Newell car-following form applied to the distance gap ($(.)'$ denotes the time derivative):

$$\big(\mathbb{E}(\Delta_i(t))\big)' = \mathscr{L}\,\mathbb{E}(\Delta_i(t)) = \mathbb{E}\big(V(\Delta_{i+1}(t)) - V(\Delta_i(t))\big).$$

Vehicles trajectories are independent for the free case where $V(\Delta) = V_{\max}$ is constant. The model is a totally asymmetric random average process [7] when $V(\Delta) = \Delta/T$ is a non-decreasing linear function. The generator of a TARAP is

$$\mathscr{L} f(\eta) = \sum_{i \in I_f} \lambda \int p(\mathrm{d}u)\, [f(\eta^i(u)) - f(\eta)]\, \mathbb{1}_{\{\Delta_i > 0\}} \tag{2}$$

with $\eta^i = (\Delta^i_j)_j$ such that $\Delta^i_j = \Delta_j$ if $j \neq i$ and $j \neq i-1$, $\Delta^i_i(u) = u\Delta_i$ and $\Delta^i_{i-1}(u) = \Delta_{i-1} + (1-u)\Delta_i$. $p(\mathrm{d}u)$ is the distribution on [0, 1] of the distance gap fraction jumping, concentrated on $1 - \delta t/T$ in the traffic model, and $\lambda > 0$ is the constant jump rate corresponding to the inverse of the time step δt.

The vehicles jump successively and the jump times of each vehicle are continuous. More precisely, the jump time of a vehicle is an homogeneous poissonian process. The jump size does not exceeded the distance gap.

The quantity $\delta t > 0$ is not a physical parameter but a tool of the modelling that should be close to 0 (at least $\delta t \leq T$). In the following, one focuses on the limit case where $\delta t \to 0$ and targeted speed function $V(\Delta) = \Delta/T$.

2.2 *Stationary State Description*

When the jump size remains constant (free case), vehicle trajectories are independent. The stationary distribution of this process is a product form and the invariant marginal distributions are independent and identical distributed and exponential. This is not the case for the TARAP. In contrast to the zero-range process whose invariant distribution is a product form [2, 4, 5], the invariant distribution of the TARAP is not known for any p distribution. However, explicit formulas for the first and second moments of the marginal invariant distribution have been derived [7].

First and Second Moment of the Marginal Invariant Distribution

If $\alpha \in E$ is the space-homogeneous initial distribution of the process, i.e. $\mathbb{P}(\eta_0 = \alpha) = 1$ with the initial system state η_0, such that $\mathbb{E}\alpha_i = D$ for all i and $\sum_i |\mathbb{E}\alpha_0\alpha_i - D^2| < \infty$, the first and second order moment of the marginal distribution satisfy [7]:

$$\mathbb{E}\Delta_i = D \qquad \forall i \forall t \qquad\qquad \text{and} \qquad\qquad \begin{aligned} &\lim_{t\to\infty} \mathbb{E}\Delta_i\Delta_j = D^2 && \forall i \neq j \\ &\lim_{t\to\infty} \mathbb{E}\Delta_i^2 = \frac{r}{s} D^2 && \forall i \end{aligned}$$

with $r = \int (1-x)p(x)\mathrm{d}x$ and $s = \int x(1-x)p(x)\mathrm{d}x$. The first moment is the mean distance gap that remains constant since the system is conservative. The covariance is nil, attesting that the invariant distribution of the process may have a product form.

In the car-following model given by generator (1) with $V(\Delta) = \Delta/T$, $r = \delta t/T$, $s = \delta t/T(1 - \delta t/T)$ and

$$\lim_{t\to\infty} \mathbb{V}\Delta_i = \lim_{t\to\infty} \mathbb{E}\Delta_i^2 - (\mathbb{E}\Delta_i)^2 = D^2 \frac{\delta t}{T - \delta t} \qquad \forall i. \tag{3}$$

The variability of the vehicles distance gap tends towards 0 in the stationary state (i.e. the flow is homogeneous) when $\delta t \to 0$. This aspect is observed in the Newell car-following model with differential equation. It is well-known that the homogeneous state of this model is a stable equilibrium state when V is non-decreasing (see for instance [14]).

Invariant Distribution

The stationary measure of the process, denoted $\pi : E \mapsto [0, 1]$, satisfies the invariant equation

$$\int_E \mathscr{L} f(\eta)\, \pi(\mathrm{d}\eta) = 0\,. \tag{4}$$

If one assumes that the stationary measure π admits a distribution ($\pi(\mathrm{d}\eta) = \pi(\eta) \prod_j \mathrm{d}\Delta_j$), the stationary equation is $\int_E \mathscr{L} f(\eta)\, \pi(\eta) \prod_j \mathrm{d}\Delta_j = 0$.

For the TARAP given by the generator (2), one obtains after substituting for all $i \in I_f$, Δ_{i-1} by $\Delta_{i-1} - (1-u)\Delta_i$ and Δ_i by Δ_i/u:

$$\int_E f(\eta) \prod_j \mathrm{d}\Delta_j \Bigg\{ \sum_{i\in I_f} \int_{\left(1+\frac{\Delta_{i-1}}{\Delta_i}\right)^{-1}}^{1} p(u) \frac{\mathrm{d}u}{u} \times \\ \pi\left(\ldots, \Delta_{i-2}, \Delta_{i-1} - \frac{1-u}{u}\Delta_i, \frac{\Delta_i}{u}, \Delta_{i+1}, \ldots \right) \Bigg\} = \int_E f(\eta) \prod_j \mathrm{d}\Delta_j \Big\{ \sum_{i\in I_f} \pi(\eta) \Big\}.$$

This equality is satisfied for any function f depending on $I_f \subset \mathbb{Z}$ with $\mathrm{card}(I_f) < \infty$ if for all $\eta \in E$ the following equality using $x = \frac{1-u}{u}\Delta_i$ holds:

$$\sum_{i\in I_f} \int_0^{\Delta_{i-1}} p\left(\frac{\Delta_i}{x + \Delta_i}\right) \frac{\mathrm{d}x}{x + \Delta_i} \pi\left(\ldots, \Delta_{i-2}, \Delta_{i-1} - x, \Delta_i + x, \Delta_{i+1}, \ldots\right) = \sum_{i\in I_f} \pi(\eta).$$

Assuming that the invariant distribution of the process has product form of homogeneous in space marginal distribution $\tilde{\pi} : \mathbb{R}^+ \mapsto [0, 1]$ such that $\int x\, \tilde{\pi}(x)\, \mathrm{d}x = D$, $\pi(\eta) = \prod_i \tilde{\pi}(\Delta_i)$, one obtains the equality (cf. [18] with p uniform):

$$\sum_{i\in I_f} \int_0^{\Delta_{i-1}} p\left(\frac{\Delta_i}{x + \Delta_i}\right) \frac{\mathrm{d}x}{x + \Delta_i} \frac{\tilde{\pi}\,(\Delta_{i-1} - x)\, \tilde{\pi}\,(\Delta_i + x)}{\tilde{\pi}(\Delta_{i-1})\tilde{\pi}(\Delta_i)} = \mathrm{card}(I_f).$$

This equality holds when one has for all $i \in I_f$:

$$\int_0^{\Delta_{i-1}} p\left(\frac{\Delta_i}{x+\Delta_i}\right)\frac{\mathrm{d}x}{x+\Delta_i}\frac{\tilde{\pi}\left(\Delta_{i-1}-x\right)\tilde{\pi}\left(\Delta_i+x\right)}{\tilde{\pi}(\Delta_{i-1})\tilde{\pi}(\Delta_i)} = 1. \tag{5}$$

In the case of the car-following model (1), this equation does not admit explicit solution for the marginal $\tilde{\pi}$. In the next paragraph we show that the process has an invariant product form distribution with gamma marginal solution for particular beta distributions of p.

Beta Distribution for *p*

We assume that p has a beta distribution on $[0, 1]$ with parameters $m > 0$ and $n > 0$:

$$p(u) = \frac{1}{\beta(m,n)}u^{m-1}(1-u)^{n-1}\mathbb{1}_{[0,1]}(u)$$

with $\beta(m,n) = \int_0^1 u^{m-1}(1-u)^{n-1}\mathrm{d}u$.

To keep the meaning of the traffic model, one assumes that the expected value of p is equal to $1-\delta t/T$. One denotes $\sigma = \frac{1}{K}\frac{\delta t}{T}\left(1-\frac{\delta t}{T}\right)$ with $K > 1$ the variance of the p distribution. One obtains $m = (1-\delta t/T)\,(K-1)$ and $n = \delta t/T\;(K-1)$. The beta distribution tends towards a deterministic one concentrated on $1-\delta t/T$ when K tends towards infinity. It becomes bi-modal concentrated on $\{0, 1\}$ when K tends towards 1.

If we refer to formulas given in [7], the variability of the distance gap in stationary state for a beta distribution of p in stationary state is:

$$\lim_{t\to\infty} \mathbb{V}\Delta_i = D^2\frac{T+\delta t(K-1)}{(T-\delta t)\,(K-1)} \qquad \forall i. \tag{6}$$

The variance tends towards infinity for $\delta t \to T$ or $K \to 1$. The distance gap variance tends towards $D^2/(K-1)$ when $\delta t \to 0$ and towards $D^2\delta t/(T-\delta t)$ when $K \to \infty$. Therefore, the variance tends towards 0 for $\delta t \to 0$ and $K \to \infty$ (that corresponds to the deterministic case of p described previously).

When we assume a beta distribution for p and a measure product form with marginal gamma distribution with parameter γ and θ:

$$\tilde{\pi}(x) = x^{\gamma-1}\frac{\exp(-x/\theta)}{\Gamma(\gamma)\theta^\gamma}\mathbb{1}_{[0,\infty)}(x)$$

with $\Gamma(\gamma) = \int_0^\infty u^{\gamma-1}e^{-u}\mathrm{d}u$, the condition of invariance (5) leads to

$$\int_0^{\Delta_{i-1}} \frac{1}{\beta(m,n)}\left(\frac{\Delta_i}{x+\Delta_i}\right)^{m-1}\left(\frac{x}{x+\Delta_i}\right)^{n-1}\frac{\mathrm{d}x}{x+\Delta_i}\left[\,(\Delta_{i-1}-x)\,(\Delta_i+x)\,\right]^{\gamma-1} = \left[\Delta_{i-1}\Delta_i\right]^{\gamma-1}$$

for all $i \in I_f$ By substituting x by $u \cdot \Delta_{i-1}$, one obtains after simplifications for all $i \in I_f$:

$$\frac{\Delta_i^{m-\gamma} \Delta_{i-1}^n}{\beta(m,n)} \int_0^1 (\Delta_i + \Delta_{i-1}u)^{\gamma-m-n}\, u^{n-1}(1-u)^{\gamma-1}\mathrm{d}u = 1. \tag{7}$$

If one considers the gamma distribution with $\gamma = m + n = K - 1$ (and $\theta = (K-1)/D$ to be sure that the expected value is equal to D), the equation of invariance is:

$$\left(\frac{\Delta_{i-1}}{\Delta_i}\right)^n \frac{\beta(\gamma,n)}{\beta(m,n)} = \left(\frac{\Delta_{i-1}}{\Delta_i}\right)^n \frac{(\Gamma(\gamma))^2}{\Gamma(\gamma+n)\Gamma(\gamma-n)} = 1\,. \tag{8}$$

This equation is satisfied for any system state $\eta \in E$ in the limit case $n \to 0$. This limit is reached when $\delta t \to 0$ or if $K \to 1$. Yet, the invariant marginal gamma distribution depends on K but not on δt. For a given $K > 1$, the gamma distribution is the exact asymptotic invariant marginal distribution for $\delta t \to 0$. For given $\delta t > 0$ and $K > 1$, the product of gamma distributions is an approximation of the invariant measure of the system. The variance of the gamma distribution is equal to $D^2/(K-1)$ while the exact value given previously is $D^2 \frac{T+\delta t(K-1)}{(T-\delta t)(K-1)}$ (cf. Eq. (6)). For any value of δt, the variability of the approximation is less that the exact one.

Some simulation experiments are undertaken to evaluate the precision of the gamma product approximation. 100 vehicles with a length of 5 m on a ring of length 2 km are considered. The targeted speed function is exclusively the pursuit one (i.e. $V_{\max} = +\infty$). The samples are obtained after a simulated time of 2 h which is sufficient to reach the stationary state. The initial configurations are uniform. The sample size for each value of the parameters is equal to $5 \cdot 10^5$ observations.

Figure 1 shows the distributions obtained. The lines represent the empirical distance gap distributions for various values of the parameter δt (varying from 1 to 0.05 s). The dotted lines are the asymptotic distributions $\Gamma(K-1, D/(K-1))$. Different values for K are compared (2, 10 and 50). The empirical distributions tend towards the asymptotic ones when $\delta t \to 0$ in all cases. For a given value of δt, the differences between the exact and the gamma distributions increase when K tends towards infinity.

The form of the distributions varies significantly in the limit case where δt tends towards 0 with the value of K. The form is exponential when K is less than 2 and monomial otherwise. The exponential case corresponds to the formation of platoon in stationary state in the system, while the flow is homogeneous when the distribution is monomial. On the Fig. 2, vehicles trajectories on a ring, from uniform initial conditions, are plotted for two values of the parameter K. The trajectories are perturbated and vehicles platooning occurs for K close to 1 (Fig. 2 at left). When K is high, p variability is low and the flow is homogeneous (Fig. 2 at right).

The formation of platoons observed when K tends towards 1 is due to the increase of noise in the system through the distribution p. For the deterministic

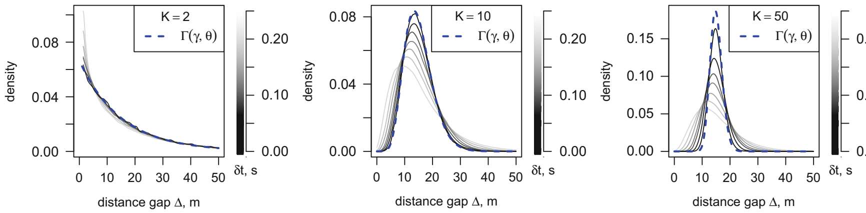

Fig. 1 Comparison of the asymptotic gamma marginal distribution with simulation data on a ring ($T = 1.2$ s, $V_{\max} = +\infty$ and $D = 15$ m)

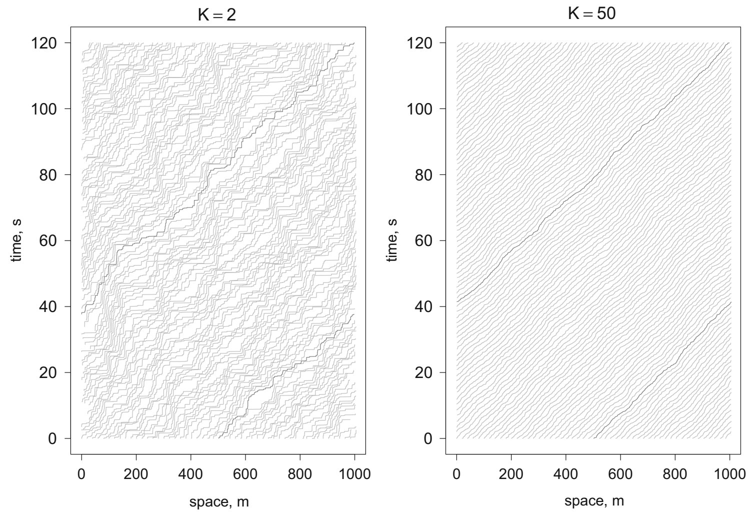

Fig. 2 Trajectories of 50 vehicles evolving on a ring ($\delta t = 0.01$ s, $T = 1.2$ s, $V_{\max} = +\infty$ and $D = 15$ m)

p distribution, the flow is homogeneous for δt small. In the next section, a model including a reaction time produces kinematic waves at the limit case when δt tends towards 0 and for the deterministic p distribution.

3 The Model Including a Reaction Time

The proposed model is a stochastic mapping of the delayed Newell car-following model. It is a stochastic mass transport process distinct from the TARAP.

3.1 Model Definition

As previously, the model is defined from an explicit eulerian discretisation of the differential equation with $T^r > 0$: $\frac{dx_i}{dt}(t) = V\big(\Delta_i(t - T^r)\big)$. If a linear development over the time is applied to the delayed distance gap to manipulate synchronous variable, the discretisation scheme for the motion of the vehicle i:

$$x_i(t + \delta t) = x_i(t) + \delta t \cdot V\big(\Delta_i(t) - T^r(v_{i+1}(t) - v_i(t))\big)$$

with the position x, the distance gap Δ to the predecessor and the speed v which is a new variable introduced in the system. Substituting the speed by the targeted speed function, a mass transport process corresponding to this discretisation can be characterised by the generator

$$\mathscr{L} f(\eta) = \sum_{i \in I_f} \frac{1}{\delta t}[f(\eta^i) - f(\eta)]\mathbb{1}_{\{\Delta_i > \delta t\, s_i\}} \tag{9}$$

with $s_i = V\big(\Delta_i - T^r(V(\Delta_{i+1}) - V(\Delta_i))\big)$ and $\eta^i = (\Delta^i_j)_j$ such that $\Delta^i_j = \Delta_j$ if $j \neq i$ and $j \neq i - 1$, $\Delta^i_i = \Delta_i - \delta t\, s_i$ and $\Delta^i_{i-1} = \Delta_{i-1} + \delta t\, s_i$.

With this form, the model is not a random average process even if V is linear since the jump size depends both on the distance gap and on the predecessor distance gap. If $V(\Delta) = \Delta/T$, one has the smallest sufficient condition on the time step parameter $\delta t \leq T/(1 + T^r/T)$.

3.2 Stationary State Description

The stationary state of the model including a reaction time is obtained by simulation. The simulation of this kind of stochastic process is easy and does not require to define a discretisation scheme. Each vehicle has an exponential clock giving the time of its next jump. The simulation is event driven by actualizing successively the vehicle with minimum jump time.

From the simulation results, two types of stationary states for the system on a ring are clearly identified for congested density levels, according to the value of the reaction time. the fist is a homogeneous stationary state with monomial and symmetric distributions of the vehicles distances gap. This state is observed when the reaction time is zero or not too large. The second is a heterogeneous stationary state with kinematic wave propagation and bi-modal distributions of vehicles distance gaps which occurs for sufficiently large reaction times.

In Fig. 3, vehicle trajectories are shown for a uniform initial configuration. The reaction time vanishes for the left system and the stationary state is homogeneous. For the right system the reaction time, equal to 1 s, is enough for convergence

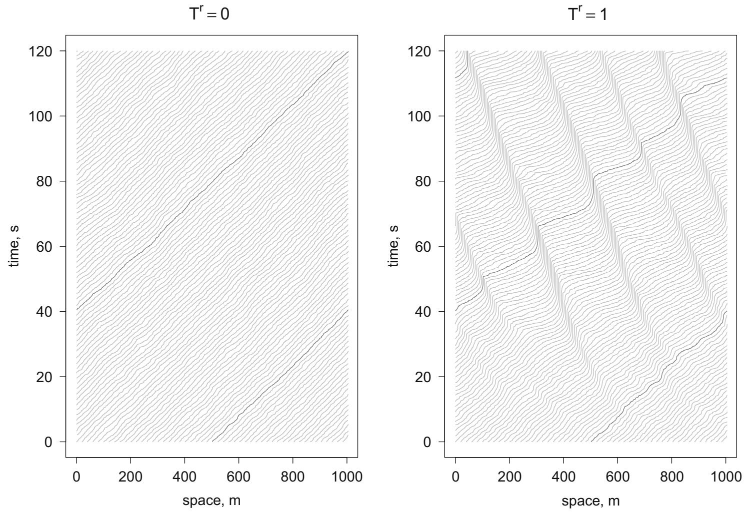

Fig. 3 Trajectories of 50 vehicles evolving on a ring ($\delta t = 0.01$ s, $T = 1.2$ s and $V_{\max} = +\infty$)

towards a heterogeneous stationary state. The formation of kinematic *stop-and-go* waves which propagate at constant speed can be observed. This phenomenon is well-known in real traffic. At a given density, the vehicles mean speeds are similar for the two stationary states although these states are different. The presence of kinematic waves is not observed in the model without reaction time, even if it produces locally vehicles platoons when the noise level is important.

Figure 4 presents the distribution of vehicle distance gaps and its mean and standard deviation for different values of the parameters. In the left part of the figure, distributions are obtained by simulations for different values of the time step parameter δt. One observes bi-modal distributions of the distance gap when δt is sufficiently close to 0. This reflects the presence of kinematic waves. At the center, the mean and standard deviation of the distributions are plotted for different values of the reaction time. A relation is observed, linking the reaction time and the targeted pursuit time with the emergence and propagation of waves in the stationary state. Various simulation experiments lead to the following condition for an homogeneous system in stationary state: $T > 2T^r$.

It is the same as the stability condition of the Newell delayed car-following model [14, 17]. It underlines a negative impact of the driver reaction time parameter on the homogeneity of the flow. To the right, one observes that the maximal speed parameter $V_{\max}$ induces a critical flow density threshold for kinematic waves to emerge. It refers to free and congested traffic states. For densities less that the critical density, vehicle speeds are close to the maximal value. Beyond the critical density threshold, vehicle speeds are regulated, with a mean less than the maximal value, and kinematic waves may emerge and propagate. Various simulation experiments show that the critical density is close to $1/(V_{\max}T + \ell)$ or equivalently for a given

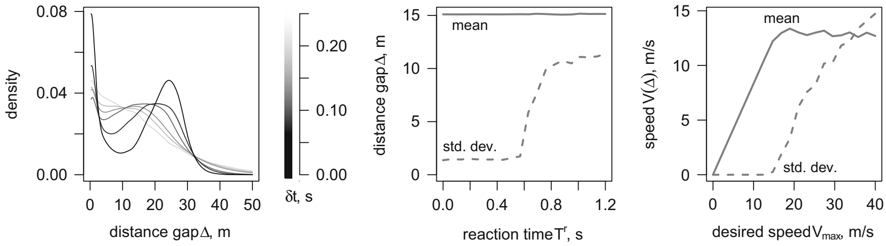

Fig. 4 Distributions in stationary state of the distance gap for different values of δt (*left*) and their mean and standard deviation for different values of, respectively, T^r and $V_{\max}$. ($\delta t = 0.01$ s, $T^r = 1$ s, $V_{\max} = 25$ m/s, $T = 1.2$ s and $D = 15$ m)

density with mean distance gap D, the critical maximal speed is $D/T = 12.5$ m/s in the figure.

4 Conclusion

We have presented results for a stochastic mass transport process calibrated by classical parameters of driver reaction time and targeted speed function, inspired by the deterministic car-following theory. The model, defined in continuous time, describes the evolution on a continuous space of the distances between vehicles without any collision. Its form allows analytical investigation in particular cases and simulations without requiring the use of a discretisation scheme.

When the reaction time vanishes, the introduction of randomness in the model induces the formation of vehicle platoons in the stationary state. Analytically, explicit approximations for the stationary distribution of the process are obtained. When the reaction time parameter is strictly positive, simulations exhibit a critical value beyond which kinematic stop-and-go waves propagate for congested densities in the stationary state. A relation linking the model parameters with the formation of waves is identified similar to the classical well-known stability conditions of car-following models.

The results underline the role of the two fundamental parameters of driver reaction time and targeted pursuit speed function. The reaction time, that is recognized and measured in real experiments, is identified as a negative factor on the homogeneity of the flow. Yet, it is known that the drivers are able to anticipate in order to deflect to the reaction time. They may increase the stability of the flow, but these aspects are not taken into account in the model.

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