Reasoning over 2D and 3D Directional Relations in OWL: A Rule-Based Approach

Sotiris Batsakis

Department of Electronic and Computer Engineering
Technical University of Crete (TUC)
Chania, Greece
batsakis@intelligence.tuc.gr

Abstract. Representation of spatial information for the Semantic Web often involves qualitative defined information (i.e., information described using natural language terms such as "North"), since precise arithmetic descriptions using coordinates and angles are not always available. A basic aspect of spatial information is directional relations, thus embedding directional spatial relations into ontologies along with their semantics and reasoning rules is an important practical issue. This work proposes a new representation for directional spatial information in ontologies by means of OWL properties and reasoning rules in SWRL embedded into the ontology. The proposed representation is based on the decomposition of cone shaped directional relations (CSD-9) offering a more compact representation and improved reasoning performance over existing approaches. A 3D representation is proposed as well and both 2D and 3D representations and reasoning are evaluated.

1 Introduction

Ontologies are formal definitions of concepts their properties and their relations. They form the basis of knowledge representation required for materializing the Semantic Web vision. Semantic Web technologies are used for automating tasks handled manually by users, tasks such as organizing a trip. Understanding the meaning of Web information requires formal definitions of concepts and their properties, using the Semantic Web Ontology definition language OWL. OWL provides the means for defining concepts, their properties and their relations and allowing for reasoning over the definitions and the assertions of specific individuals using reasoners such as Pellet. Furthermore, reasoning rules can be embedded into the ontology using the SWRL rule language.

Spatial information is an important aspect of represented objects in many application areas. Spatial information in turn can be defined using quantitative (e.g. using coordinates) and qualitative terms (i.e., using natural language expressions such as "East"). Qualitative spatial terms have specific semantics which can be embedded into the ontology using reasoning rules. In previous work [1] such a representation is proposed for both bi-dimensional (2D) spatial and temporal information in OWL.

Current work deals with the case of directional spatial information and proposes a new representation for such information which is more compact then the representation used in [1]. Specifically, instead of asserting one directional relation between two

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points, such as “North-West”, two relations are asserted (e.g., “North” and “West”). The first relation represents the relative placement of points along the North-South axis and the second along the East-West axis. Both relations correspond to cone shaped regions in the plane and their definitions and semantics are introduced in the current work. Reasoning is applied on each set of relations separately, achieving a decomposition of cone-shaped directional relations. Using the proposed representation both the number of required relations and the corresponding reasoning rules are significantly reduced offering increased reasoning performance. Specifically the required number of OWL axioms and SWRL rules for 2D representation have been reduced to 106, compared to 964 in [1].

The compactness of representation and the increased reasoning performance allows for extension of the proposed representation for three-dimensional (3D) space. To the best of author’s knowledge this work is the first that proposes the optimized representation based on the decomposition of directional relations, and also the first that deals with 3D representation of directional relations in OWL ontologies.

Current work is organized as follows: related work in the field of spatial knowledge representation is discussed in Section 2. The proposed representation is presented at Section 3 and the corresponding reasoning mechanism at Section 4. The extension to three-dimensional space is presented at Section 5 followed by evaluation in Section 6 and conclusions and issues for future work in Section 7.

2 Background and Related Work

Definition of ontologies for the Semantic Web is achieved using the Web Ontology Language OWL[1]. The current W3C standard is the OWL 2[2] language, offering increased expressiveness while retaining decidability of basic reasoning tasks. Reasoning tasks are applied both on the concept and property definitions into the ontology (TBox) and the assertions of individual objects and their relations (ABox). Reasoners include among others Pellet[3], Fact++[4], RacerPro[5], KAON2[6] and Hermit[7]. Reasoning rules can be embedded into the ontology using SWRL[8]. To guarantee decidability, the rules are restricted to *DL-safe rules* [4] that apply only on named individuals in the ontology ABox. *Horn Clauses* (i.e., a disjunction of classes with at most one positive literal), can be expressed using SWRL, since Horn clauses can be written as implications (i.e., $\neg A \vee \neg B ... \vee C$ can be written as $A \wedge B \wedge ... \Rightarrow C$). The efficiency of reasoning over Horn clauses using forward chaining algorithms is a reason for choosing this form of rules. The antecedent (body) of the rule is a conjunction of clauses. Notice that, neither disjunction nor negation of clauses is supported in the body of rules. Also, the

[1] http://www.w3.org/TR/owl-ref/
[2] http://www.w3.org/TR/owl2-overview/
[3] http://clarkparsia.com/pellet/
[4] http://owl.man.ac.uk/factplusplus/
[5] http://www.racer-systems.com/
[6] http://kaon2.semanticweb.org/
[7] http://hermit-reasoner.com/
[8] http://www.w3.org/Submission/SWRL/

consequence (head) of a rule is one positive clause. Neither negation nor disjunction of clauses can appear as a consequence of a rule. These restrictions improve reasoning performance but complicate qualitative spatial reasoning, since disjunctions of clauses typically appear in the head of a spatial reasoning rule.

Qualitative spatial reasoning (i.e., inferring implied relations and detecting inconsistencies in a set of asserted relations) typically corresponds to Constraint Satisfaction problems which are NP, but tractable sets (i.e., solvable by polynomial algorithms) are known to exist [3]. Formal spatial representations have been studied extensively within the the Semantic Web community. Relations between spatial entities in ontologies can be topological, directional or distance relations. Furthermore, spatial relations are distinguished into qualitative (i.e., relations described using lexical terms such as “South”) and quantitative (i.e., relations described using numerical values such as “45 degrees North”)..

A representation of topological relations using OWL class axioms has been proposed in [6], but an alternative representation using object properties offered increased performance [5]. Embedding spatial reasoning into the ontology by means of SWRL rules applied on spatial object properties forms the basis of the SOWL model proposed at [1]. Based on the representation proposed at [1] the dedicated Pellet-Spatial reasoner [5] has been extended for directional relations in the CHOROS system [7] (Pellet-Spatial supports only topological relations). CHOROS achieved improved performance over the SOWL model but the spatial reasoner in not embedded into the ontology, thus requiring specific software which must be properly adjusted whenever modifications into the ontology occur. Furthermore, it does not offer support for 3D representation. SOWL on the other hand offers greater flexibility since it can be used and modified freely using only standard Semantic Web tools such as the Protégé editor and the Pellet reasoner[9]. In this work an improved representation of directional spatial relations based on decomposition of relations on each axis is proposed, analogously to the approach proposed for temporal interval relations in [2].

3 Spatial Representation

Directional relations in this work are represented as object properties between OWL objects representing points. For example if $Point1$ if $North\ Of\ Point2$ user asserts the binary relation $Point1\ North\ Point2$, or equivalently *North(Point1, Point2)*. This approach is similar to the approach used in [1] for directional relations as part of the SOWL model. In [1] between two points 9 different directional relations (*CSD-9* relations) can be defined, namely *North (N), NorthEast (NE), East (E), SouthEast (SE), South (S), SouthWest (SW), West (W), NorthWest (NW) and Identity*, corresponding to cone shaped regions (and the identity relation for identical points) in the two-dimensional (2D) space presented in Figure 1. This set of relations, known as CSD-9, is a special case of the modified star calculus presented in [8], when the lines separating the cone-shaped areas belong to only one of these areas. In this case reasoning over basic relations is decided by path consistency and it is tractable [8]. Also additional

[9] SWRL spatial reasoning rules and CHOROS are available on the Web at: `http://www.intelligence.tuc.gr/prototypes.php`

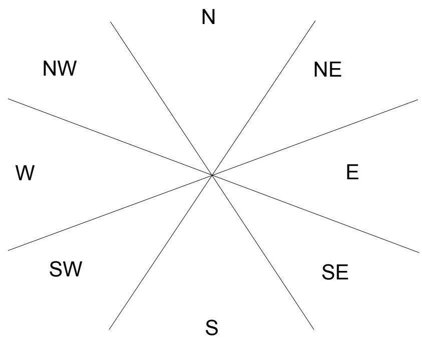

Fig. 1. Cone Shaped Directional Relations (CSD-9)

relations representing disjunctions of the above 9 relations are introduced in [1], since these additional relations are required for implementing reasoning rules similar to the rules proposed in Section 4. This leads to a complicated representation requiring 33 relations and 964 SWRL rules and OWL axioms [1].

Reducing the complexity of representation is necessary in order to improve performance and to allow for efficient 3D representation and reasoning. A representation based on projections on each axis and reasoning over the pairs of relations on these one-dimensional spaces, instead of cone shaped regions in bi-dimensional space, has been proposed as well in [1]. Note that this projection based representation has different semantics than the cone-shaped representation, thus it can not be consider as an alternative to it. For example, using the projection based approach, if a point is located far east relatively to another point and slightly north of it, following the projection based approach relations $East$ and $North$ will hold at the horizontal and the vertical axis respectively, thus and the $NorthEast$ relation. Following the cone-shaped approach only the relation $East$ holds which is conceptually right according to the way humans usually refer to directional relations.

In this work we follow the cone-shaped approach but relations are decomposed into two sets of relations, one for the East-West axis (horizontal) and one for the North-South axis (vertical) is case of 2D representation. Relations on each set are jointly exhaustive and pairwise disjoint but for each pair of points two relations, one from each set can hold. For example point A can be $North$ and $East$ of point B corresponding to the *North-East* CSD-9 cone-shaped relation.

The basic relations on each set are: *North, South, Equal-Vertical and Identical-Vertical* for the first set as presented in Figure 2 and *East, West, Equal-Horizontal and Identical-Horizontal* for the second set presented in Figure 3. Lines separating the cone-shaped regions belong to only one of the adjacent regions. By convention they belong to the $North$ and $South$ relations in the set of Figure 2 and to the $East$ and $West$ areas in in case of relations of Figure 3. Also relations *Identical-Horizontal* and *Identical-Vertical* are sub-properties of the $Identical$ property and also equivalent properties. Furthermore the implementation of the reasoning mechanism from Section 4 requires the definition of additional properties representing the disjunction of basic ones. These relations are the *Equal-North* (representing the fact that a point is equal vertically or north of another) and Equal-South (representing the fact that a point is equal vertically or south of another) in the first set. In the second set the additional relations are *Equal-East* and *Equal-West* representing disjunction of equality with relations $East$ and $West$ respectively. Notice that, in total 8 basic and 4 additional relations are required for representation and reasoning in this work, compared to 9 basic and 33 total relations for directly implementing 2D cone-shaped CSD-9 relations.

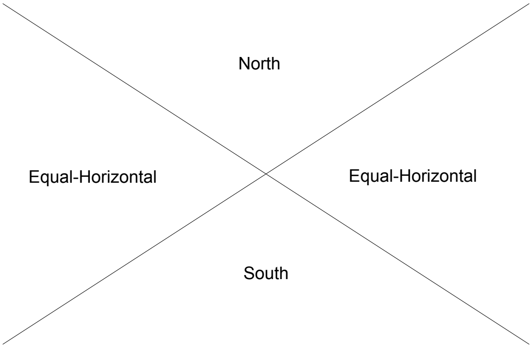

Fig. 2. North-South Relations

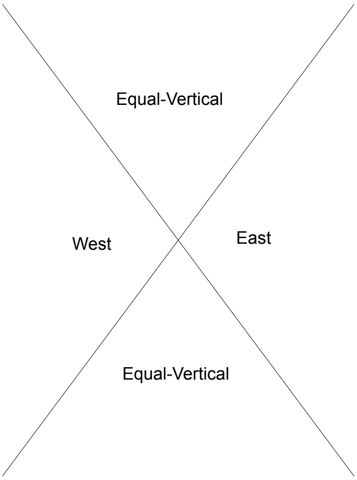

Fig. 3. East-West Relations

Additional OWL axioms required for the proposed representation; basic relations on each set are pairwise disjoint e.g., $North$ is disjoint with $South$. Also $North$ is $inverse$ of $South$ and $East$ is $inverse$ of $West$. Relations *Identical-Horizontal* and *Identical-Vertical* are symmetric. Relations *Equal-North*, Equal-South are the inverse of each other, and the same holds for relations *Equal-East* and *Equal-West*. Summarizing, the proposed representation is conceptually equivalent to the cone-shaped representation of [1]. By decomposing the relations into two different sets the required number of relations is reduced to 8 basic relations and 4 additional ones. Between each pair of points, (in case of regions the points represent their centroid) two basic relations can hold. Specifically, decomposition of CSD-9 relations into proposed relations is defined as follows:

$$\begin{aligned}
N_{CSD9}(x,y) &\equiv North(x,y) \wedge Equal\text{–}Vertical(x,y)\\
NE_{CSD9}(x,y) &\equiv North(x,y) \wedge East(x,y)\\
E_{CSD9}(x,y) &\equiv Equal\text{–}Horizontal(x,y) \wedge East(x,y)\\
SE_{CSD9}(x,y) &\equiv South(x,y) \wedge East(x,y)\\
S_{CSD9}(x,y) &\equiv South(x,y) \wedge Equal\text{–}Vertical(x,y)\\
SW_{CSD9}(x,y) &\equiv South(x,y) \wedge West(x,y)\\
W_{CSD9}(x,y) &\equiv Equal\text{–}Horizontal(x,y) \wedge West(x,y)\\
NW_{CSD9}(x,y) &\equiv North(x,y) \wedge West(x,y)\\
Identity_{CSD9}(x,y) &\equiv Identical\text{–}Horizontal(x,y) \wedge Identical\text{–}Vertical(x,y)
\end{aligned}$$

4 Spatial Reasoning

Reasoning is realized by introducing a set of SWRL[10] rules operating on spatial relations. Reasoners that support DL-safe rules such as Pellet[11] can be used for inference and consistency checking over directional relations. Defining compositions of relations is a basic part of the spatial reasoning mechanism. Table 1 represents the result of the composition of two directional relations of Figure 2 (relations $North$, $South$, *Equal-Horizontal* and *Identical-Horizontal*, are denoted by "N","S","EqH", "IdH" respectively).

Table 1. Composition Table for North-South Directional Relations

Relations	N	S	EqH	IdH
N	N	N,S,EqH,IdH	N,EqH	N
S	N,S,EqH,IdH	S	S,EqH	S
EqH	N,EqH	S,EqH	N,S,EqH,IdH	EqH
IdH	N	S	EqH	IdH

Table 2 represents the result of the composition of two directional relation pairs of Figure 3 (relations $East$, $West$, *Equal-Vertical* and *Identical-Vertical*, are denoted by "E","W","EqV", "IdV" respectively).

[10] http://www.w3.org/Submission/SWRL/

[11] http://clarkparsia.com/pellet/

Table 2. Composition Table for East-West Directional Relations

Relations	E	W	EqV	IdV
E	E	E, W, EqV, IdV	E, EqV	E
W	E, W, EqV, IdV	W	W, EqV	W
EqV	E, EqV	W, EqV	E, W, EqV, IdV	EqV
IdV	E	W	EqV	IdV

Composition Table can be interpreted as follows: if relation R_1 holds between $point2$ and $point1$ and relation R_2 holds between $point3$ and $point2$, then the entry of the Table 1 corresponding to line R_1 and column R_2 denotes the possible relation(s) holding between $point3$ and $point1$. For example if $point2$ is $North$ of $point1$ and $point3$ is *Equal-Horizontal* to $point1$ then $point3$ is *North OR Equal-Horizontal* to $point1$. Entries in the above composition tables are determined using the following observation: composition of two relations corresponds to the addition of two vectors representing the relative placement of $point2$ to $point1$ and $point3$ to $point2$ forming angles θ_1 and θ_2 respectively with the horizontal axis. The resulting vector represents the relative placement of $point3$ to $point1$, i.e., the composition of two vectors, as illustrated in Figure 4. When adding the two vectors the resulting vector forms an angle θ with the horizontal axis such that $\theta_1 \leq \theta \leq \theta_2$. Angle θ defines the directional relation between $point1$ and $point3$. Using this observation it can be concluded for example that composing relations $North$ and *Equal-Horizontal* yields the disjunction of these two relations as a result.

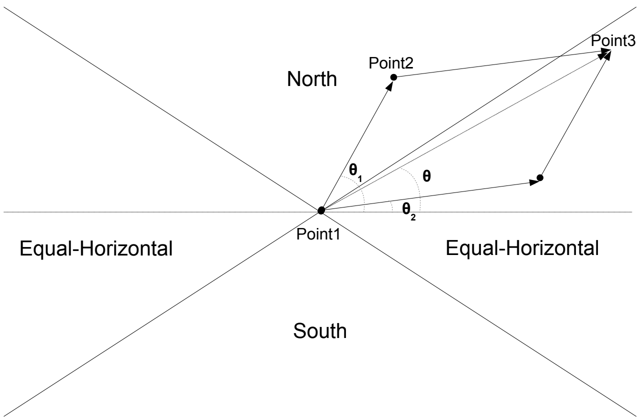

Fig. 4. Composition Example

A series of compositions of relations may yield relations which are inconsistent with existing ones (e.g., the above example will yield a contradiction if *point3 south of point1* has been also asserted into the ontology). Consistency checking is achieved by ensuring path consistency by applying formula:

$$\forall x, y, k\ R_s(x, y) \leftarrow R_i(x, y) \cap (R_j(x, k) \circ R_k(k, y))$$

representing intersection of compositions of relations with existing relations (symbol $\cap$ denotes intersection, symbol $\circ$ denotes composition and R_i, R_j, R_k, R_s denote directional relations). The formula is applied until a fixed point is reached (i.e., the application of the rules above does not yield new inferences) or until the empty set is reached, implying that the ontology is inconsistent. Implementing path consistency formula requires rules for both compositions and intersections of pairs of relations.

Compositions of relations R_1, R_2 yielding a unique relation R_3 as a result are expressed in SWRL using rules of the form:

$$R_1(x, y) \wedge R_2(y, z) \rightarrow R_3(x, z)$$

The following is an example of such a composition rule:

$$North(x, y) \wedge North(y, z) \rightarrow North(x, z)$$

Rules yielding a set of possible relations cannot be represented directly in SWRL since, disjunctions of atomic formulas are not permitted as a rule head. Instead, disjunctions of relations are represented using new relations whose compositions must also be defined and asserted into the knowledge base. For example, the composition of relations $North$ and *Equal-Horizontal* (EqH) yields the disjunction of two possible relations (*North* and *Equal-Horizontal*) as a result:

$$North(x, y) \wedge EqH(y, z) \rightarrow (North \vee EqH)(x, z)$$

If the relation N_EqH represents the disjunction of relations *North* and *EqH*, then the composition of $North$ and EqH can be represented using SWRL as follows:

$$North(x, y) \wedge EqH(y, z) \rightarrow N\_EqH(x, z)$$

A set of rules defining the result of intersecting relations holding between two points must also be defined in order to implement path consistency. These rules are of the form:

$$R_1(x, y) \wedge R_2(x, y) \rightarrow R_3(x, y)$$

where R_3 can be the empty relation. For example, the intersection of relations $North$ and $South$ yields the empty relation, and an inconsistency is detected:

$$North(x, y) \wedge South(x, y) \rightarrow \perp$$

Intersection of relations $North$ and N_EqH (representing the disjunction of $North$ and *Equal-Horizontal* yields relation $North$ as a result:

$$North(x, y) \wedge N\_EqH(x, y) \rightarrow North(x, y)$$

Thus, path consistency is implemented by defining compositions and intersections of relations using SWRL rules and OWL axioms for inverse relations as presented in Section 3.

Another important issue for implementing path consistency is the identification of the additional relations, such as the above mentioned N_EqH relation, that represent disjunctions. Specifically the *minimal* set of relations required for defining compositions and intersections of all relations that can be yielded when applying path consistency on the basic relations of Figure 2 is identified. The identification of the additional relations is required for the construction of the corresponding SWRL rules.

In this work the closure method [3] of Table 3 is applied for computing the minimal relation sets containing the set of basic relations: starting with a set of relations, intersections and compositions of relations are applied iteratively until no new relations are yielded forming a set closed under composition, intersection and inverse. Since compositions and intersections are constant-time operations (i.e., a bounded number of table lookup operations at the corresponding composition tables is required) the running time of closure method is linear to the total number of relations of the identified set.

Applying the closure method over the set of basic North-South relations yields a set containing 7 relations. These are the four basic relations of Figure 2 and the relations *NorthEqualHorizontal* (denoted by *N_EqH*), representing the disjunction of relations $North$ and *EqualHorizontal*, *SouthEqualHorizontal* (denoted by *S_EqH*), representing the disjunction of relations $South$ and *EqualHorizontal*, and *N_S_EqH_IdH* or All denoting the disjunction of all relations. Applying the closure method over the set of basic South-West relations also yields a set containing 7 relations. These are the four basic relations of Figure 3 and the relations *EastEqualVertical* (denoted by *E_EqV*), representing the disjunction of relations $East$ and *EqualVertical*, *WestEqualVertical* (denoted by *W_EqV*), representing the disjunction of relations $West$ and *EqualVertical*, and *E_W_EqV_IdV* or All denoting the disjunction of all relations.

Table 3. Closure method

```
Input: Set S of tractable relations
Table C of compositions
WHILE S size changes
        BEGIN
            Compute C:Set of compositions of relations in S
            S=S ∪ C
            Compute I:set of intersections of relations in S
            S= S ∪ I
        END
RETURN S
```

A reduction to required relations and rules can be achieved by observing that the disjunction of all basic relations when composed with other relations yields the same relation, while intersections yield the other relation. Specifically, given that All represents the disjunction of all basic relations and, R_x is a relation in the supported set then the following holds for every R_x:

$$All(x, y) \wedge R_x(x, y) \rightarrow R_x(x, y)$$
$$All(x, y) \wedge R_x(y, z) \rightarrow All(x, z)$$
$$R_x(x, y) \wedge All(y, z) \rightarrow All(x, z)$$

Since relation All always holds between two points, because it is the disjunction of all possible relations, all rules involving this relation, both compositions and intersections, do not add new relations into the ontology and they can be safely removed. Also, all rules yielding the relation All as a result of the composition of two supported relations R_{x1}, R_{x2}:

$$R_{x1}(x,y) \wedge R_{x2}(y,z) \rightarrow All(x,z)$$

can be removed as well. Thus, since intersections yield existing relations and the fact that the disjunction over all basic relations must hold between two points, all rules involving the disjunction of all basic relations and consequently all rules yielding this relation can be safely removed from the knowledge base. After applying this optimization the required number of axioms for implementing path consistency over the set of directional relations of Figure 2 or Figure 3 is reduced to 52, while the combined implementation for relations of both Figure 2 and Figure 3 requires 106 axioms and rules, compared to the 964 axioms and rules required for reasoning over the cone-shaped directional relations of Figure 1 [1].

Reasoning over CSD-9 relations can be reduced to reasoning over the proposed 2D relations. This can be proved by decomposing CSD-9 relations into pairs of corresponding 2D relations, composing the resulting relations and checking if the resulting relations correspond to reasoning over CSD-9 relations using the composition table defined in [1,8]. All possible CSD-9 compositions are checked in order to establish the equivalence of the representations. Due to space limitations only a composition example will be provided, but all possible 81 compositions of CSD-9 basic relations can be redefined equivalently. For example the composition of CSD-9 relations N and NE yields the disjunction of relations N and NE as a result [1]. Specifically:

$$N_{CSD9}(x,y) \wedge NE_{CSD9}(y,z) \rightarrow N_{CSD9}(x,z) \vee NE_{CSD9}(x,z)$$

Using the proposed representation the composition of the above relations yields the same result; The corresponding 2D representation as defined in section 3 yields the compositions of relations *North (N)* and *North (N)* of Figure 2 and *Equal-Vertical (EqV), East(E)* of Figure 3. Composing these relations using compositions of Table 1 and Table 2 yields the same relation as the direct composition of the CSD-9 relations. Specifically:

$$N_{CSD9}(x,y) \wedge NE_{CSD9}(y,z) \equiv (N(x,y) \wedge EqV(x,y)) \wedge (N(y,z) \wedge E(y,z))$$
$$\equiv (N(x,y) \wedge N(y,z)) \wedge (EqV(x,y) \wedge E(y,z)) \Rightarrow N(x,z) \wedge (EqV(x,z) \vee E(x,z))$$
$$\equiv ((N(x,z) \wedge EqV(x,z)) \vee (N(x,z) \wedge E(x,z)) \equiv N_{CSD9}(x,z) \vee NE_{CSD9}(x,z)$$

Thus, composing the CSD-9 North and NorthEast relations using the corresponding 2D representation of Section 3 is equivalent to the composition defined in [1,8]. This equivalence also holds for intersections and inverses, thus the two representations are equivalent. An example of inverse operator, applied on the CSD-9 $North$ relation and yielding the desired CSD-9 $South$ relation using the equivalent 2D representation is the following:

$$N_{CSD9}(x,y) \equiv N(x,y) \wedge EqV(x,y) \equiv S(y,x) \wedge EqV(y,x) \equiv S_{CSD9}(y,x)$$

5 Three-Dimensional Representation and Reasoning

Representing points in three dimensional space is achieved by adding a third relation between two points (in addition to relations of Figure 3 and Figure 2). The basic relations on this additional set presented in Figure 5 are: *Up, Down, Equal-Height and Identical-Height*. Relation *Identical-Height* and relations *Identical-Horizontal* and *Identical-Vertical* are sub-properties of the $Identical$ property and also equivalent properties. These relations correspond to cone-shaped regions on a plane that two points belong, a plane that id perpendicular to the plane that 2D relations of section 3 are defined. The implementation of the reasoning mechanism (as in the 2D case of Section 4) requires the definition of additional properties representing the disjunction of basic ones. These relations which are detected using the closure method are the *Equal-Up* (representing the fact that a point has equal height or is higher than another point) and Equal-Down (representing the fact that a point has equal height or is lower than another point). Combined with existing relations for the 2D representation a total of 8 basic and 6 additional relations are required for representation and reasoning for 3D space.

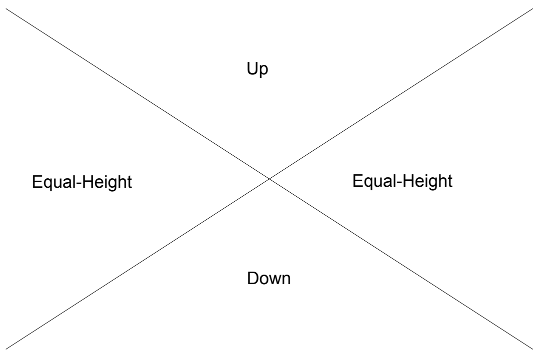

Fig. 5. Up-Down Relations

Table 4 represents the result of the composition of two directional relation pairs of Figure 3 (relations Up, $Down$, *Equal-Height* and *Identical-Height*, are denoted by "U","D","$EqHe$", "$IdHe$" respectively).

Table 4. Composition Table for Up-Down Directional Relations

Relations	U	D	$EqHe$	$IdHe$
U	U	$U, D, EqHe, IdHe$	$U, EqHe$	U
D	$U, D, EqHe, IdHe$	D	$D, EqHe$	D
$EqHe$	$U, EqHe$	$D, EqHe$	$U, D, EqHe, IdHe$	$EqHe$
$IdHe$	U	D	$EqHe$	$IdHe$

Reasoning rules in SWRL implementing path consistency for 3D directional relations, have been defined as well. These rules are almost identical to the rules presented in Section 4, but they apply on properties of Figure 5. Also relation Up is the inverse of $Down$ and *Identical-Height* and *Equal-Height* are symmetric. New relations, rules and OWL axioms are combined with existing 2D representation requiring a total of 158 axioms and rules for 3D representation and reasoning. These are considerably fewer than the 964 axioms and rules required for 2D representation using the relations of Figure 1. Three-dimensional representation using directly an extension of CSD-9 relations, instead of their decomposition as proposed in the current work, will require thousands of rules and axioms and it will be impractical.

6 Evaluation

In the following the proposed representation and reasoning mechanism is evaluated both theoretically and experimentally.

6.1 Theoretical Evaluation

The required expressiveness of the proposed representation is within the limits of OWL 2 expressiveness. Reasoning is achieved by employing DL-safe rules expressed in SWRL that apply on named individuals in the ontology ABox, thus retaining decidability. Furthermore, since the proposed representation is equivalent to the CSD-9 representation, reasoning using the polynomial time path consistency algorithm is sound and complete, as in the case of CSD-9 relations.

Specifically, any point can be related with every other point with two basic directional relations (one of each set presented in Figures 2 and 3), because relations of each set are mutually exclusive, between n points, at most $2n(n-1)$ relations can be asserted (in case of 3D representation one additional relation belonging to the set presented in Figure 5 can be also be asserted leading to an upper limit of $3n(n-1)$). Furthermore, path consistency has $O(n^5)$ time worst case complexity (with n being the number of points). In the most general case where disjunctive relations are supported in addition to the basic ones, any point can be related with every other point by at most k relations, where k is the size of the set of supported relations (containing four additional relations for 2D and six for 3D besides the basic ones). Therefore, for n points, using $O(k^2)$ rules, at most $O(kn^2)$ relations can be asserted into the knowledge base.

Applying the *closure method* over the proposed directional relations the total number of relations required for 2D representation is 14 (or 12 if the disjunction of all relations for each set are eliminated as proposed in Section 4) compared to 33 for the representation presented at [1]. The required number of axioms is 106 compared to 964 at [1]. In case of 3D representation reasoning the required number of relations is 21 (or 18 after applying the optimizations proposed in Section 4) and the number of required axioms and rules is 158.

The $O(n^5)$ upper limit for path consistency running time referred to above is obtained as follows: At most $O(n^2)$ relations can be added in the knowledge base. At each such addition step, the reasoner selects 3 variables among n points which corresponds to $O(n^3)$ possible different choices. Clearly, this upper bound is pessimistic, since the overall number of steps may be lower than $O(n^2)$ because an inconsistency detection may terminate the reasoning process early, or the asserted relations may yield a small number of inferences. Also, forward chaining rule execution engines employ several optimizations (e.g., the Rete algorithm employed at the SWRL implementation of Pellet), thus the selection of appropriate variables usually involves fewer than $O(n^3)$ trials. Nevertheless, since the end user may use any reasoner supporting SWRL, a worst case selection of variables can be assumed in order to obtain an upper bound for complexity. Nevertheless retaining control over the order of variable selection and application of rules yields an $O(n^3)$ upper bound for path consistency [5].

6.2 Experimental Evaluation

Measuring the efficiency of the proposed reasoner requires a spatial ontology, thus a data-set of 200 to 1000 points generated randomly was used for the experimental evaluation. Reasoning response times of the spatial reasoning rules are measured as the average over 5 runs. Pellet 2.2.0 running as a plug-in of Protégé 4.2 was the reasoner used in the experiments. All experiments run on a PC, with Intel Core 2 Duo CPU at 3.00 GHz, 4 GB RAM, and Windows 8.

Measurements illustrate that the proposed representation offers faster reasoning performance. Measurements over 2D points (using the decomposition to North-South and East-West relations) of Section 3 and 3D points of Section 5 are presented in Table 5. They are compared to measurements over the CSD-9 representation of [1]. Since the number of basic relations of CSD-9 relations is 9 (Figure 1) and because all possible disjunctions appearing in the supported set must also be supported, the CSD-9 representation is particularly involved. On the other hand, the CSD-9 representation requires only one relation between points while the proposed 2D representation requires two relations between points. Thus, the definition of n directional relations between n points, requires n assertions in case of CSD-9 relations, $2n$ in case of the proposed 2D representation and $3n$ in case of the proposed 3D representation. The reasoner will have to handle 964, 106 and 158 rules and axioms for the CSD-9 and the proposed 2D and 3D representations respectively.

In the following experiment, we measure the performance of reasoning in the cases of both the proposed 2D and 3D representations and the CSD-9 based representation, and their performance is discussed. In all cases, n random points and n random directional relations between them were asserted (using $2n$ and $3n$ assertions in the proposed 2D and 3D representations respectively), and reasoning times using Pellet are measured. Measurements of the time required by each approach for producing all inferred relations from a data set of random points are reported in Table 5. Each entry in the table is the average over 5 runs of the reasoner corresponding to 5 random instantiations of the ontology.

The evaluation indicated that the proposed 2D representation clearly outperforms the CSD-9 based approach, although the number of asserted relations is twice that of the CSD-9 approach. The proposed representation requires fewer rules and axioms (106) applied on a largest set of relations ($2n$) compared to the CSD-9 approach which requires n relation assertions and 964 OWL axioms and SWRL rules. The increased performance allows for a practical 3D representation and reasoning mechanism with performance equal to that of CSD-9 bi-dimensional representation.

Table 5. Average reasoning time for directional relations as a function of the number of points

Number of Points	Reasoning Time (ms)		
	2D	3D	CSD-9
200	299.8	268.6	386.0
400	405.8	685.8	782.4
600	865.2	1099.0	1066.4
800	1053.6	1407.2	1396.4
1000	1526.2	2261.4	2380.2

Summarizing, reasoning over the proposed 2D representation is approximately 30% faster over the CSD-9 based representation, due to the small number of axioms involved. This allows for an efficient and compact 3D cone-shaped directional representation and reasoning mechanism as well. This is the first such representation for 3D directional relations for the Semantic Web.

7 Conclusions and Future Work

In this work a representation framework for handling directional spatial information in ontologies is introduced. The proposed framework handles both, 2D and 3D information using an inference procedure based on path consistency. The proposed representation based on decomposition of CSD-9 relations offers increased performance over existing approaches [1]. Both the proposed and the existing representations are presented and evaluated.

The proposed representation is fully compliant with existing Semantic Web standards and specifications which increases its applicability. Being compatible with W3C specifications the proposed framework can be used in conjunction with existing editors, reasoners and querying tools such as Protégé and Pellet without requiring specialized additional software. Therefore, information can be easily distributed, shared and modified. Directions of future work include the development of real world applications based on the proposed mechanism. Such applications will combine temporal and topological spatial representations with the proposed directional representation and reasoning mechanism.

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