Autobiography

John Milnor

I grew up in Maplewood, New Jersey, a suburban community where at least half of the adult male population took the train into New York City every weekday morning. My father, an electrical engineer with Western Union, was no exception. He worked particularly on undersea cable engineering,[1] and obtained many patents for devices which helped to optimize telegraphic transmission. My mother was an enthusiastic amateur artist. During the depression, she organized a Toy Lending Library to help parents who couldn't afford toys.

I was painfully shy, and socially backward as a youngster. It didn't help that my parents bought a farm an hours drive to the west when I was four years old. After that time, every summer and every weekend was spent on the farm. I certainly enjoyed the rolling countryside, and the animals. But it meant that I was isolated from anyone my own age much of the time.

My father and my brother Bob, who is seven years older, were both adept with tools, and were always happy building things. (There was a hydroponic garden in the barn, and a ten inch telescope, permanently mounted under a sliding shed out in the field.) Bob built an elaborate model railway system. I was fascinated by the relay switching circuits used to control it; but wasn't much help in actually constructing anything. With World War II looming, he took an accelerated degree in aeronautical engineering at the University of Michigan, and spent the rest of the war in the army, working on aircraft maintenance.

[1]See J. Willard Milnor, *Submarine cable telegraphy*, Transactions of the American Institute of Electrical Engineers **41** (1922) 20–38.

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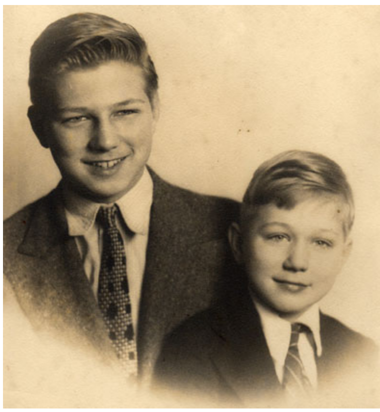
My father J. Willard Milnor and mother Emily Cox Milnor in 1928, a few years before I was born

We had a few cows and chickens, at least during the war years. I even learned how to milk a cow



With my brother Bob in the late 1930s

Thus I spent a great deal of time by myself, reading everything I could get my hands on. I loved Bertrand Russell's "History of Western Philosophy", and was quite intrigued by the few mathematics books that my father owned. These included a calculus text for engineers, and a translation of a very brief German text on complex function theory, which was fascinating but very mysterious. However, I certainly never thought of mathematics as a career at that point.

One memorable event was a cross-country road trip which Bob and I took in 1948. In particular, we both took an exciting lesson in rock climbing in Wyoming. We have both had a love of the mountains since that time.[2]

At age seventeen I enrolled in Princeton, and was almost immediately captivated by the mathematical world. It was not that I wasn't interested in other subjects; but I found everything else much harder. Furthermore, the friendly ambience in the mathematics department felt wonderful. There was a cosmopolitan atmosphere, created by many distinguished refugees from Nazi Germany and elsewhere in Europe.

One particularly memorable course was taught by Ralph Fox. The subject was point set topology in the manner of R.L. Moore. This meant that Fox provided the definitions and theorems, while we were required to find the proofs, without any help from books. Later Fox introduced me to 3-dimensional topology, which many people in the department then seemed to think of as a boring backwater, although the field has really come into its own in recent years. I wrote both my senior thesis and my doctoral thesis, under Fox's direction, on the theory of higher order linking invariants.

Another memorable course, completely opposite in style, was a totally polished presentation of algebraic number theory, by Emil Artin. Although the course was very enjoyable, I was chagrined to realize some time later that I had no idea what an "algebraic number" is. As far as I can remember, that particular topic had never been mentioned. Nevertheless, that course, and also contact with junior faculty members such as Serge Lang and John Tate, served me well in later years.

A third memorable class, on elementary differential geometry, was taught by Albert Tucker. In particular, Tucker introduced us to a problem of Karol Borsuk on the total curvature of knots, which I was happily able to solve.

One particularly enjoyable feature of the department was the commons room, which was open at all times. There was often some game such as Go or Kriegspiel in progress, usually surrounded by a crowd of kibitzers. In fact there was an active group in Princeton studying Game Theory, headed by Tucker, but also including younger people such as David Gale, Harold Kuhn, and John Nash. I became actively involved for a few years, and spent several summers at The Rand Corporation in California working in Game Theory. (They were particularly interested in this field because of its possible military applications.) However, I eventually lost enthusiasm for the subject, since it seemed to me that mathematics could play only a limited role. Any really important application would also involve questions of politics, sociology, and psychology, which were completely foreign to me.

[2]Some of my adventures (and misadventures) in the mountains are described in the dedication pages of my "Collected Papers III".

With a hyperbolic blackboard, Princeton 1963. Alfred Eisenstaedt/Time & Life Pictures/Getty Images. Reprinted with permission

During my time as a graduate student in Princeton, I took a year off to study geometry under Heinz Hopf in Zürich. The time in Switzerland, was useful and enjoyable, but the difference in style was amazing to me. When I acted like a Princeton student, and interrupted the lecture to ask a question, everyone turned around to stare at me as if I were totally crazy.

I returned to Princeton with a Swiss wife, Brigitte. Over the next ten years, we had three children. Stefan, the oldest, is a computer hardware engineer, who shares my love of the mountains. After many treks in Nepal, he and his wife Lisa adopted two Nepali children. Daniel runs a successful business as a commercial artist in Switzerland, and Gabrielle, the youngest, lives with her blacksmith partner and two horses in the California hills.

During the many years I spent at Princeton,[3] my primary focus was on the topology of manifolds, and the tools from algebraic topology needed to understand them. Here I benefited very much from the presence of Norman Steenrod and John Moore. This was a golden time in topology. The work of Jean-Pierre Serre had made homotopy groups accessible, and the work of René Thom had provided an unexpected and surprising relationship between homotopy theory and the study of smooth manifolds.[4] Furthermore Raoul Bott's work had provided an amazingly simple description for the stable homotopy groups of classical groups. The confluence of these new ideas, together with the well established techniques of cohomology theory, obstruction theory, fiber bundle theory, and characteristic classes led to solutions for many problems which had seemed completely intractable.

[3]For more about my mathematical life in Princeton, see: *Growing Up in the Old Fine Hall*, in "Prospects in Mathematics", edited by H. Rossi, AMS, 1998.

[4]One conversation with Thom, in which he described how to "kill" a homotopy class by a surgery construction, was particularly important to me.

With grandchildren Kavi and Deepa, in Provincetown

The power of these new methods was brought home to me when I discovered what I first thought was a contradiction in mathematics. I had been studying the classification problem for closed $2n$-dimensional manifolds which are $(n-1)$-connected. The homotopy theory of such a manifold is relatively easy to describe, since it has homology only in dimensions 0, n, and $2n$. In dimension eight, many such examples can be constructed by starting with a fiber bundle

$$D^4 \xrightarrow{\subset} E^8 \twoheadrightarrow S^4$$

over S^4 with the closed disk D^4 as fiber. Whenever the boundary manifold $M^7 = \partial E^8$ is a topological sphere, one can paste a copy of the disk D^8 onto E^8 to obtain the required closed 8-dimensional manifold. In fact, for many examples of smooth D^4-bundles over S^4, one can easily check that the boundary is a homotopy 7-sphere. Assuming the Generalized Poincaré Hypothesis, this seemed to lead to a contradiction. It was easy to compute the characteristic classes for the 8-manifold constructed in this way, but in many cases the results contradicted the restrictions on characteristic classes for smooth closed manifolds which followed from the work of Thom and Hirzebruch. My first thought was that the boundary was only a homotopy sphere, not a topological sphere, so that I had found a counterexample to the 7-dimensional Poincaré Hypothesis. However, a little careful analysis showed that the boundary was indeed a topological sphere. Thus I had actually constructed examples of smooth 7-dimensional manifolds which are homeomorphic, but not diffeomorphic, to the standard 7-sphere. In effect, I had answered a question which, as far as I know, no one had ever asked.

A year or so later, Michel Kervaire and I discovered that we were working on very similar ideas, and decided to combine forces, leading to our work on "Groups of Homotopy Spheres."

I was particularly lucky during these years to have a wonderful group of graduate students and junior faculty, who helped me convert some of my lectures into print. Thus my lectures on Morse Theory, inspired by Bott, were put into book form by

In Princeton, perhaps in the 1970s

Michael Spivak and Robert Wells. Similarly, my lectures on Stephen Smale's h-Cobordism Theorem were converted into published form by Larry Siebenmann and Jonathan Sondow, and my Characteristic Classes lectures by Jim Stasheff.

Contacts between American and European mathematicians always play a very important role. I was very grateful for the opportunity to visit the IHES near Paris many times during this period. The annual Arbeitstagung organized by Fritz Hirzebruch in Bonn also provided a wonderful opportunity for keeping up with the latest developments.

Unfortunately, my marriage to Brigitte fell apart. In 1967, I left Princeton, spending a year at UCLA and two years at MIT. Then in 1970, I returned to Princeton, but at the Institute for Advanced Study, where I spent 20 happy years. During this time, work in algebraic topology inevitably led to related problems in pure algebra. Jean-Pierre Serre's "Cours d'Arithmétique" provided a marvelously readable introduction to quadratic forms, which form an indispensable tool in studying the homotopy theory of manifolds. Michael Atiyah taught me the importance of K-theory, and I learned the related subject of Algebraic K-theory through the work of Hyman Bass. The group K_0A of a ring A can be thought of as a simplified description of the class of finitely generated projective modules over A, while K_1A is closely related to J.H.C. Whitehead's theory of the "simple homotopy type" of a finite simplicial complex. I was happy to find a useful definition of the group K_2A of a ring A, and made a completely ad hoc definition of what are now called the "Milnor K-groups" $K_n^M F$ of a field F for higher values of n. These groups are of interest because of their close relation with the theory of quadratic forms and with Galois cohomology, as proved later in the work of Kazuya Kato and Vladimir Voevodsky.[5] (A few years later, Daniel Quillen constructed a more generally useful theory of higher K groups.)

[5]Compare: *On the Milnor Conjectures, history, influence, applications*, by Albrecht Pfister, Jahresber. Deutsch. Math.-Verein, **102** (2000) 15–41.

With Dusa in the California High Sierra in 1980, and in Arizona, many years later

Another development that I was particularly happy about in this period was the book "Singular Points of Complex Hypersurfaces," which made a useful contribution to elementary algebraic geometry, even though I was completely untrained in that field.

During the late seventies, my attention drifted towards a different area. Under the influence of Bill Thurston, and later of Adrien Douady, I became very much interested in the theory of dynamical systems. In particular, the abundance of fascinating problems which can be directly visualized seemed very attractive.

The theory of dynamics in one complex variable had received a powerful start in the early 20-th century through the work of Pierre Fatou and Gaston Julia. But there was a long hiatus until their work was brought to new life in the late 20-th century through the work of many mathematicians such as Adrien Douady, John Hubbard, Bill Thurston, and Dennis Sullivan. The advent of computers which could bring the abstract mathematics to life with vivid illustrations, played an important role. One particularly interesting feature is that dynamics over the complex numbers, where all of the powerful classical machinery is available, often provides an essential tool in understanding problems in real dynamics which seem a priori much simpler.

After a second failed marriage, I met and married Dusa McDuff. We have one child Thomas who is now a graduate student in Vancouver. (She also has one child Anna, by a previous marriage.) A high point of every year is the family get together, which takes place at Daniel's vacation house in Provincetown.

In 1989 I left the Institute for Advanced Study, and moved to Stony Brook University. In part this was because I missed the regular contact with students. However, the fact Dusa had been in Stony Brook for some years was the deciding factor.

At Stony Brook, I organized a small "Institute for Mathematical Sciences", with the help of Misha Lyubich, who has now taken over as its director. Over the years, many young (and not so young) mathematicians have spent time at our Institute, many of them working in the field of dynamics.

In front of the Abel monument, Oslo 2011

Although I am now formally working only half-time at Stony Brook, I still feel that happiness consists of thinking about mathematical problems.