Comparing Schedules in the SINR and Conflict-Graph Models with Different Power Schemes

Tigran Tonoyan

TCS Sensor Lab
Centre Universitaire d'Informatique
Route de Drize 7, 1227 Carouge, Geneva, Switzerland
tigran.tonoyan@unige.ch

Abstract. The scheduling problem in wireless networks asks to split a set of links (transmission requests) into the minimum number of "feasible" subsets. In this paper the theoretical gap between the schedule length in the SINR model and in the corresponding conflict-graph model is evaluated, considering three different power schemes. It is shown that this gap depends largely on the power scheme used and on the metric space where the network is located. While in metric spaces of small doubling dimension (such as Euclidean metrics) certain upper bounds can be proven for the difference between the two models, in general metrics the difference can be arbitrary.

1 Introduction

The topic of interference resolution in wireless networks has a rich history of research. A fundamental problem in this research is the scheduling problem: given a set of transmission requests, how can one organize them into subsets, as few as possible, so that the transmissions in each subset can be done simultaneously. This problem has been considered in several models such as simple conflict-graph models and models incorporating path-loss and fading. In this paper we work with two models: the path loss model with SINR (signal-to-interference-and-noise ratio) describing the interference and an approximation of this model, the Conflict-Graph model. The Conflict-Graph model can be described in terms of a conflict graph, where two links are adjacent or conflicting if they cannot transmit simultaneously with respect to the SINR model, i.e. at least one of them makes too much interference for the other one. The scheduling problem in this simplified model becomes equivalent to the well-known vertex coloring problem in the corresponding graph. A motivation for considering this variant of scheduling problem is the localized nature of graph-based models and simplicity of treatment of the problem, i.e. in order to resolve the interference for a given link one needs to consider only the adjacent links, which is convenient when considering e.g. topology-related problems [1]. However, the over-simplified structure resulting from graph-based models has the following two problems. First, the graphs

J. Cichoń, M. Gębala, and M. Klonowski (Eds.): ADHOC-NOW 2013, LNCS 7960, pp. 317–328, 2013.
© Springer-Verlag Berlin Heidelberg 2013

can miss the possibility of spatial reuse because of the rigid assumptions on the interference; this problem is investigated in [2], [3] by software simulations and examples. It has been found that the network throughput can suffer because of the lack of spatial reuse in case of graph-based models. This problem seems to not induce dramatic differences (at least asymptotically) between schedules in two models. The second problem, that can induce a larger gap between the two models, is that graph-based models do not take into account the accumulative nature of interference, i.e. the schedules based on graphs can be too optimistic. This problem is investigated experimentally in [4], where it is shown that a significant fraction of links in a schedule that is feasible in a graph-based model, has too much cumulative interference in SINR model. Another study of this problem for the uniform power scheme is done in [5] in a slightly different context, but their proofs seem problematic (in particular, the proof of Proposition 5.3).

In this paper we investigate the asymptotic gap between the two models from the scheduling aspect. The author is not aware of any publication that systematically treats this question. The main goal of this paper is to collect some known results together with new results that deal with the mentioned problem. Some of these results somehow appeared in publications, but the relation between conflict-graphs and SINR was considered merely as a tool towards solution of SINR-scheduling problems, while we believe that this relation is interesting on its own and is worth to be presented to the attention of the scientific community. The main question that we consider is how well does the Conflict-Graph model approximate the SINR model and how does the difference scale with the network size and topology. The answer to this question is different for different network settings. It has been shown in [6] that the difference is only a constant factor, when the lengths of all the links are close to each other. We extend this result by considering arbitrary sets of links and different power assignment schemes. In particular, we show that the extent of the gap depends on what power assignment is in use (considering the same network topology). The gap can also drastically depend on the properties of the metric space where the network nodes are located. For doubling metric spaces, we show that the difference between two models can be bounded by a factor of $\log \Delta$ for the *uniform* and *linear* power schemes (to be defined in Section 2.3) and by a factor of $\min\{\log n, \log \Delta\}$ for the *mean* power scheme, where Δ is the ratio of the lengths of longest and shortest links and n is the number of links. Hence, the mean power scheme is more scalable/flexible from this viewpoint. We also show that in general metric spaces the gap can be arbitrarily large. The main technical framework addressing the mean power scheme has been developed in [7] in the context of *power control* (not related to model comparison).

2 Preliminaries

2.1 The Path-Loss Model

Let $L = \{1, 2, \ldots, n\}$ denote the set of n links in the network that need to be scheduled. Each link represents a transmission request between two wireless nodes that are located on a metric space. Let the sender node of each link i be assigned a power level $P(i)$ according to some assignment policy or *power scheme*.

According to the path-loss propagation model [8], the receive signal of each link i is $P_i = P(i)/l_i^\alpha$, where l_i is the distance between the sender and the receiver of i and $\alpha > 0$ is the path-loss exponent. Similarly, the interference caused by link j at the receiver of link i is $I_{ji} = P(j)/d_{ji}^\alpha$, where d_{ji} denotes the metric distance from the sender node of link j to the receiver node of link i. We assume that the transmission of a link i is successful if and only if the SINR(signal-to-interference-and-noise ratio) is greater than a certain threshold $\beta \geq 1$:

$$\frac{P_i}{\sum_{j \in S \setminus \{i\}} I_{ji} + N} \geq \beta, \tag{1}$$

where the constant $N \geq 0$ denotes the noise and S is the set of links transmitting in the same time slot as i.

We call a subset S of L *feasible* if (1) holds for each link $i \in S$. Each partition of L into feasible subsets is called a *schedule*, and the number of subsets in such a partition is the *length* of the schedule. The *scheduling problem* asks to find a minimum length schedule for a given set of links.

In this paper we assume that $N = 0$. Note, however, that this assumption can be avoided if one is allowed to scale the power levels by a small multiplicative factor in the range $(1, 2]$. A formal proof of this statement is presented in [9].

For some of the results we also need the assumption that *doubling dimension* of the metric space be less than α (Theorem 2 and Lemma 3). The exact definition of doubling dimension can be found in [10]. Here we only need the fact that in a metric space with doubling dimension m, each ball of radius r contains at most $C \cdot (r/r')^m$ disjoint balls of a smaller radius r' where C is an absolute constant. It is known that the m-dimensional Euclidean space has doubling dimension m (see [10]), so for the euclidean plane we assume $\alpha > 2$, which is a common assumption in practice [8].

Now we can write the SINR condition as follows:

$$A(S, i) = \sum_{j \in S \setminus i} \min\{1, I_{ji}/P_i\} \leq 1/\beta. \tag{2}$$

Indeed, note that, since $\beta \geq 1$, $A(S, i) \leq 1/\beta$ if and only if (1) holds without the noise. This form of SINR condition has been considered in a number of papers (e.g. [11]) because it has the following additivity property: if there are two disjoint sets S_1 and S_2 then $A(S_1 \cup S_2, i) = A(S_1, i) + A(S_2, i)$.

Following [12] we say that a set S is a *p-signal set* if $A(S, i) \leq 1/p$. Similarly, a partition of the set of links is a p-signal partition if each subset is a p-signal set.

The following theorem shows that one can vary the threshold of the SINR condition without changing the schedule length much.

Theorem 1. *[12] There is a polynomial-time algorithm that takes a p-signal schedule and refines into a p'-signal schedule, for $p' > p$, increasing the number of slots by a factor of at most $\lceil 2p'/p \rceil^2$.*

2.2 The Conflict-Graph Model

We call two links i and j *q-adjacent* if

$$\text{either } A(\{i\}, j) \geq 1/q^\alpha \text{ or } A(\{j\}, i) \geq 1/q^\alpha,$$

otherwise we say that i and j are *q-independent.*

Using this definition we can define the q-adjacency graph $G_q(L)$ of the set of links L where the set of vertices of $G_q(L)$ is L and two links i and j are adjacent in $G_q(L)$ if and only if they are q-adjacent.

Note that if two links are q-adjacent then at least one of them interferes with the other one "too much", so $G_q(L)$ is a natural approximation of the stricter SINR model. The scheduling problem in the model associated with $G_q(L)$ is the famous vertex coloring problem in graphs, where the problem is to split the set of vertices of $G_q(L)$ into the smallest number of independent subsets. Following the standard notation, we denote that number $\chi(G_q(L))$.

The following lemma immediately follows from the definition of q-independence. It highlights an obvious relation between schedules in the two models.

Lemma 1. *A set of links that belong to the same q^α-signal slot in some schedule is q-independent.*

In the rest of this paper we study the relationship between the optimum schedule length (in the SINR model) and $\chi(G_q(L))$.

2.3 The Three Power Schemes

The motivation for considering power schemes is the fact that they are computable in a localized manner and do not depend on the whole network topology which makes them well-suited for decentralized wireless networks as opposed to complex power control algorithms.

We consider the following three power schemes. When the *uniform* power scheme is used, all the links are assigned the same power levels; hence, the sending power of each sender node is the same. When the *linear* power scheme is used, each link i is assigned a power level cl_i^α for a constant c. In this case the receive power of all links is constant (according to path-loss formula). When the *mean* power scheme is used, each link is assigned the power level $cl_i^{\alpha/2}$ for a constant c. Our analysis will be concentrated on these three power schemes.

3 The Uniform and Linear Power Schemes

Since the uniform power scheme and the linear power scheme are quite similar, we analyze them together. In this section we assume that the wireless nodes are placed on a doubling metric space of dimension $m < \alpha$ (this is used in Theorem 2).

Plugging the two power schemes in (2) we get that in the case of the uniform power scheme $A(S,i) = \sum_{j\in S\setminus i}(l_i/d_{ji})^{\alpha}$ and in the case of the linear power scheme $A(S,i) = \sum_{j\in S\setminus i}(l_j/d_{ji})^{\alpha}$.

It follows that two links i and j are q-independent with respect to the uniform power scheme if and only if

$$d_{ij} \geq ql_j \text{ and } d_{ji} \geq ql_i.$$

Similarly, for the linear power scheme the q-independence is equivalent to the following:

$$d_{ij} \geq ql_i \text{ and } d_{ji} \geq ql_j.$$

We say that a set of links is *nearly equilength* [6] if the lengths of any pair of links in the set differ by a factor of at most two. The following theorem together with Lemma 1 shows that q-independence is essentially equivalent to q^{α}-signal property for a given nearly equilength set of links.

Theorem 2. *[6] Suppose that $\alpha > m$. Let L be a q-independent set of nearly equilength links for a parameter $q > 2$. Then L is a $\Omega(q^{\alpha})$-signal set when the powers are uniform.*

Remark. Note that this theorem is true not only for the uniform power scheme but also for the linear power scheme and mean power scheme because the power levels of the nodes in these cases differ just by factors of order 2^{α} from some uniform power.

Recall that if $q \in O(1)$ then, using Theorem 1, a $\Omega(q^{\alpha})$-signal set can be transformed into a constant number of feasible subsets. So when the link-lengths are "almost equal" the optimal schedule length and the chromatic number of $G_q(L)$ differ by at most a constant factor; hence, in this case $G_q(L)$ is a good approximation for the SINR model. But what happens when the link-lengths are not close?

Theorem 3. *Suppose that $\alpha > m$. Let T denote the optimal schedule length of the set L in the SINR model, assuming that uniform, linear or mean power scheme is used. Then $T = O(\log \Delta\chi(G_q(L)))$ where $\Delta = max_{i,j\in L} l_i/l_j$ denotes the ratio of the lengths of the longest and the shortest links.*

Proof. We prove the claim only for the uniform power scheme, because the two other cases can be proven similarly (in virtue of the remark after Theorem 2). It is enough to show that each q-independent subset of L can be scheduled in $O(\log \Delta)$ subsets. Let $S \subseteq L$ be such a subset. S can be split into $\log \Delta$ subsets each of which is a nearly equilength set. Indeed, if l_1 is the length of the shortest link, then

$$S = \bigcup_{t=1}^{\log \Delta} \{i \in L : 2^{t-1} \leq l_i/l_1 < 2^t\}.$$

According to Theorem 2, each of these subsets can be scheduled into a constant number of feasible subsets, which gives $O(\log \Delta)$ subsets in total. □

Note that in general the parameter Δ does not depend on the number of links, so it can theoretically be arbitrarily large with respect to n. On the other hand, the length of any schedule does not exceed n, the number of links. Next we show that the upper bound from Theorem 3 is tight for the uniform and linear power schemes. We show that there are examples of networks for which $\log \Delta = n$ and $T = \Theta(\log \Delta \chi(G_q(L)))$ when these power schemes are in use. These examples are the q-independent variants of exponential networks from [13].

Theorem 4. *Suppose that either the uniform power scheme or the linear power scheme is used, and let $q \in O(1)$. Then for each $n > 0$ there is a set of n links L on the line, such that $OPT(L) = \Theta(\log \Delta \chi(G_q(L)))$ and $\log \Delta = \Theta(n)$, where $OPT(L)$ is the minimal schedule length for L.*

Proof. Let us consider the following simple linear network. There are n links $\{1, 2, \ldots, n\}$ sequentially aligned on a straight line in the increasing order of numbers going from left to right. We set $l_i = 2^i$ for $i = 1, 2, \ldots, n$, i.e. $l_1 = 2$ is the length of the shortest link and $l_n = 2^n$ is the length of the longest one, hence $\log \Delta = \log l_n/l_1 = n - 1$. Each link has its sender on the left side and the receiver on the right side. For each $i > 1$ we define $d_{i,i-1} = ql_i$. Now we can calculate all the other distances. For each $i > 1$ we have:

$$d_{i1} = \sum_{t=2}^{i-1} (d_{t,t-1} + l_t) + d_{i,i-1} = \sum_{t=2}^{i-1} l_t + q \sum_{t=2}^{i} l_t = \\ = (q+1) \sum_{t=2}^{i-1} l_t + ql_i = (2q+1)l_i - 4(q+1),$$

where we used the fact that $l_i = 2^i$. Now if $i > j$ then $d_{ij} = d_{i1} - d_{j1} - l_j = (2q+1)(l_i - l_j) - l_j$ and $d_{ji} = d_{i1} - d_{j1} + l_i = (2q+1)(l_i - l_j) + l_i$. It is easy to check that this set of links is q-independent with respect to both uniform and linear power schemes.

Suppose that the uniform power scheme is used. Consider any *feasible* subset of links S of size k with the links $i_1 < i_2 < \cdots < i_k$. For the longest link i_k we have:

$$A(S, i_k) = \sum_{t=1}^{k-1} \frac{l_{i_k}^\alpha}{d_{i_t i_k}^\alpha} = \sum_{t=1}^{k-1} \frac{l_{i_k}^\alpha}{((2q+1)(l_{i_k} - l_{i_t}) + l_{i_k})^\alpha} \geq \frac{k-1}{(2q+2)^\alpha}.$$

We should have also that $A(S, i_k) \leq 1/\beta$ which allows us bound the number of links in S:

$$k \leq \frac{(2q+2)^\alpha}{\beta} + 1.$$

Hence, one cannot schedule the given set of links into less than $\frac{\beta n}{(2q+2)^\alpha + \beta} \in \Omega(n)$ feasible subsets.

For the case of the linear power scheme a similar argument works if we consider $A(S, i_1)$. □

These results show that for the uniform and the linear power schemes the difference between the schedules in the SINR model and the Conflict-Graph model depends on the structure of the network with the factor $O(\log \Delta)$. In the next section we show that for the mean power scheme a better bound can be achieved.

4 The Mean Power Scheme

The following results have been proven in [7]. The proofs are presented here for reader's convenience, as they reflect the main ideas connecting Conflict-Graph and SINR models. As we saw in the previous section, the results for approximating SINR with conflict graphs can be unsatisfactory when using the uniform or the linear power scheme. It turns out that when one uses the mean power scheme, a better approximation is achieved. In this section we assume that the nodes are placed on a doubling metric space of dimension $m < \alpha$.

Theorem 5. *[7] Suppose that $\alpha > m$ and that the mean power scheme is used and let S be a 3-independent set of links. Then S can be scheduled into $O(\log n)$ subsets.*

It follows from Theorem 5 that the gap between schedule lengths in the Conflict-Graph model and the SINR model is more scalable for the mean power scheme than for the other two power schemes.

Corollary 1. *Suppose that $\alpha > m$. Let T denote the optimal schedule length of the set of links L in the SINR model assuming that the mean power scheme is used. Then $T = O(\min\{\log \Delta, \log n\}\chi(G_2(L)))$, where $n = |L|$.*

The proof of Theorem 5 is based on the following crucial lemma.

For each set of links S and each link $i \in S$, let $\gamma(S, i)$ denote the number of links $j \in S$ such that $l_j \geq n^2 l_i$, and either $A(i, j) \geq 1/(2n)$ or $A(j, i) \geq 1/(2n)$, where $n = |S|$.

Lemma 2. *[7] Suppose that the mean power scheme is used for a 2-independent set of links S. There is a constant $N = N(C, m)$ such that for each link $i \in S$, $\gamma(S, i) \leq N$.*

Let us postpone the proof of this lemma and turn to the proof of Theorem 5.

Proof (Proof of Theorem 5). It suffices to show that each 3-independent subset of L can be scheduled in $O(\log n)$ subsets. Let S be a 3-independent subset. Let us assume $\beta = 1$ for simplicity of expressions. S is scheduled in three stages.
Stage 1. First we split S into $O(\log n)$ subsets that have certain desired properties. Let l_1 be the length of the shortest link. Then $S = \cup_{t=1}^{\log \Delta} Q_t$, where

$$Q_t = \{i \in S : 2^{t-1} \leq l_i/l_1 < 2^t\}.$$

By rearranging the terms we can write

$$S = \bigcup_{t=1}^{2\log(2n)} B_t \text{ with } B_t = \bigcup_{k=0}^{\infty} Q_{t+2k\cdot\log(2n)},$$

i.e. the set B_t (for $t = 1, 2, \ldots 2\log(2n)$) is formed by taking each $2\log(2n)$-th set, starting from Q_t. It is not hard to check that for any two links $i, j \in B_t$ with $l_i \geq l_j$, either $l_i \leq 2l_j$ or $l_i > 2n^2 l_j$. Each set B_t is scheduled separately and the union of those schedules is taken.

Stage 2. Take B_1 w.l.o.g. Recall that B_1 is a union of non-intersecting sets Q_t, $t \in I$, where I is some set of indices. Moreover, each of these sets Q_t is nearly equilength and 3-independent by construction. By Theorem 2 (and the remark after it), Q_t is an $\Omega(2^\alpha)$-signal set and, by Theorem 1, it can be transformed into a 2-signal schedule $\{Q_t^1, Q_t^2, \ldots\}$ consisting of $O(1)$ subsets. Since $B_1 = \bigcup_{t\in I} Q_t$, by rearranging the terms we also have that

$$B_1 = \bigcup_k S_k \text{ with } S_k = \bigcup_{t\in I} Q_t^k,$$

i.e. S_k is the union of k-th subsets (in an arbitrary ordering) of schedules for all Q_t. Hence the number of different S_k is in $O(1)$. In the following stage each S_k is scheduled separately.

Stage 3. Take S_1 w.l.o.g. Recall that S_1 is 3-independent; hence Lemma 2 holds. Let N be the constant from Lemma 2. In order to schedule S_1, take $N+1$ subsets $R_1, R_2 \ldots, R_{N+1}$ (initially empty) in a fixed order. Consider the elements of S_1 in an increasing order of link-lengths and add each next link i to the first subset R_t such that for each link $j \in R_t$ with $l_j \geq n^2 l_i$,

$$A(i,j) < 1/(2n) \text{ and } A(j,i) < 1/(2n).$$

Note that such R_t exists for each link i, in virtue of Lemma 2. It remains to prove that each set R_t is feasible. Take R_1 w.l.o.g. and consider some link $i \in R_1$. By construction in Stage 1, R_1 can be split into two subgroups as follows: $R_1^1 = \{j \in R_1 : l_j/l_i \in [1/2, 2]\}$, $R_1^2 = \{j \in R_1 : l_j/l_i \geq 2n^2 \text{ or } l_i/l_j \geq 2n^2\}$. The construction in Stage 2 guarantees that $A(R_1^1, i) \leq 1/2$. The choice of R_1 in Stage 3 makes sure that for each link $j \in R_1^2$, $A(j,i) < 1/(2n)$. Using additivity of $A(.,i)$ yields that $A(R_1^2, i) < 1/2$. Using additivity of $A(.,i)$ once again yields $A(R_1, i) < 1$. It is easy to check that the union of all schedules computed consists of $O(\log n)$ subsets. This completes the proof. □

In order to prove Lemma 2, we need another technical lemma which encapsulates the properties of doubling metric spaces that will be used in the proof.

Lemma 3. *Let $\{p_0, p_1, p_2, \ldots, p_k\}$ be a set of points in a metric space of doubling dimension m and let c_1, c_2, c_3 and $\{b_0, b_1, b_2, \ldots, b_k\}$ be positive real numbers, such that*

a) $b_s \geq c_1 b_0$ for $s = 1, 2, \ldots, k$,

b) $d(p_0, p_s) \leq c_2 b_0 b_s$ *for* $s = 1, 2, \ldots, k$ *and*
c) $d(p_s, p_t) \geq c_3 b_s b_t$ *for* $s, t = 1, 2, \ldots, k, t \neq s$.
Then $k \leq C \left(\left(\frac{2c_2}{c_1 c_3} \right)^2 + 1 \right)^m + 1$.

Proof. For each pair of indices $s, t \in \{1, 2, \ldots, k\}$, the triangle inequality implies that

$$d(p_s, p_t) \leq d(p_s, p_0) + d(p_t, p_0).$$

Combining this with inequalities b) and c) results in the following expression

$$c_2 b_0 b_s + c_2 b_0 b_t \geq c_3 b_s b_t. \tag{3}$$

Assume w.l.o.g. that $b_s \leq b_t$. Then it follows from (3) that $b_0 \geq \frac{c_3}{2c_2} b_s$. If we fix $t = \arg\max_t b_t$ and apply the previous argument to all pairs s, t with $s \in \{1, 2, \ldots, k\} \backslash \{t\}$, we find that $b_0 \geq \frac{c_3}{2c_2} b_s$ *for all indices* $s \in \{1, 2, \ldots, k\} \backslash \{t\}$. Suppose w.l.o.g. that those indices are $1, 2, \ldots, k-1$. Plugging these inequalities in b) and using the assumption a) we find that the following inequalities hold for all indices $s, t \in \{1, 2, \ldots, k-1\}$ with $i \neq j$:

$$d(p_s, p_t) \geq c_1^2 c_3 b_0^2 \text{ and } d(p_0, p_s) \leq \frac{2c_2^2}{c_3} b_0^2.$$

The first inequality asserts that the balls of radius $c_1^2 c_3 b_0^2 / 2$ with centers at points p_s for different $s \in \{1, 2, \ldots, k-1\}$ don't intersect. The second inequality asserts that those balls are contained in the bigger ball of radius $(2c_2^2/c_3 + c_1^2 c_3/2) b_0^2$ with the center at p_0. Then the property of the metric space mentioned before the lemma implies the following upper bound on the number of points: $k - 1 \leq C \left(\left(\frac{2c_2}{c_1 c_3} \right)^2 + 1 \right)^m$, which completes the proof. □

The proof of Lemma 2 follows.

Proof (Proof of Lemma 2). Suppose that the mean power scheme is used for a q-independent set of links S with $q \geq 2$. Let us fix some link and assign it the number 0. Assume w.l.o.g. that $R = \{1, 2, \ldots, k\}$ is the subset of links $j \in S$ such that

$$l_j \geq n^2 l_0, \tag{4}$$

and either $A(0, j) \geq 1/(2n)$ or $A(j, 0) \geq 1/(2n)$, the last statement being equivalent to the following:

$$\min \{d_{0j}, d_{j0}\} \leq (2n)^{1/\alpha} \sqrt{l_0 l_j}. \tag{5}$$

We need to show that $\gamma(S, 0) = |R| \leq N$ for a constant N. q-independence implies that $A(i, j) \leq q^{-\alpha}$ and $A(j, i) \leq q^{-\alpha}$ for all $i, j \in R$, hence

$$d_{ij} \geq q\sqrt{l_i l_j} \text{ and } d_{ji} \geq q\sqrt{l_i l_j}. \tag{6}$$

Let us assume w.l.o.g. that $l_i \leq l_j$. Applying the triangle inequality gives $d(s_j, s_i) \geq d_{ji} - l_i \geq q\sqrt{l_i l_j} - l_i$, and since $l_i \leq l_j$,

$$d(s_j, s_i) \geq (q-1)\sqrt{l_i l_j}. \tag{7}$$

A similar argument with the inequality $d(r_j, r_i) \geq d_{ij} - l_i$ gives

$$d(r_j, r_i) \geq (q-1)\sqrt{l_i l_j}. \tag{8}$$

Let us choose a node p_0 (the sender or the receiver of the link 0) and a set of nodes P differently, depending on the following two cases:

Case 1. There is a subset $R_1 \subseteq R$ with $|R_1| \geq |R|/2$, such that $d_{0j} \leq (2n)^{1/\alpha}\sqrt{l_0 l_j}$ for all $j \in R_1$. In this case we take p_0 to be the sender node of the link 0, i.e. s_0, and P to be the set of receiver nodes of the links in R_1, i.e. $P = \{r_j | j \in R_1\}$.

Case 2. If the first case does not hold, then according to the pigeonhole principle (applied to (5)) there is a subset $R_2 \subseteq R$ with $|R_2| \geq |R|/2$, such that $d_{j0} \leq (2n)^{1/\alpha}\sqrt{l_0 l_j}$ for all $j \in R_2$. In this case we take p_0 to be the receiver node of the link 0, i.e. r_0, and P to be the set of sender nodes of the links in R_2, i.e. $P = \{s_j | j \in R_2\}$.

In both cases $|P| \geq |R|/2$, so upper-bounding $|P|$ yields an upper-bound for $|R|$.

Consider the first case. Let $|P| = k$, and w.l.o.g. $P = \{r_1, r_2, \ldots, r_k\}$. By definition, $d(p_0, r_j) \leq (2n)^{1/\alpha}\sqrt{l_0 l_j}$ for $j = 1, 2, \ldots, k$. On the other hand, from (8) we have $d(r_i, r_j) \geq (q-1)\sqrt{l_i l_j}$ for $i, j = 1, 2, \ldots, k$ and $i \neq j$. Hence, by denoting $b_0 = \sqrt{l_0}$, $p_t = r_t$ and $b_t = \sqrt{l_t}$ for $t = 1, 2, \ldots, k$, we get

$$d(p_0, p_t) \leq (2n)^{1/\alpha} b_0 b_t$$

$$d(p_s, p_t) \geq (q-1) b_s b_t, \text{ for } s, t = 1, 2, \ldots, k, s \neq t,$$

so Lemma 3 applies to the set of points $\{p_0, p_1, \ldots, p_k\}$, with positive real numbers $b_0, b_1, \ldots, b_k$ as defined above and $c_1 = \sqrt{n^2} = n$ (because of (4)) $c_2 = (2n)^{1/\alpha}$, $c_3 = (q-1)$. This application of Lemma 3 gives the needed upper bound:

$$|P| = k \leq C\left(\left(\frac{2(2n)^{1/\alpha}}{(q-1)n}\right)^2 + 1\right)^m + 1 \in O(1),$$

where last relation is due to the assumption that $\alpha > 1$ and $q \geq 2$. Thus, if the first case holds, the lemma is proven. With almost the same steps the lemma can be proven for the second case, this time using (7). That will complete the proof. □

5 General Metric Spaces

It is known that some approximation results for scheduling in the SINR model hold true in general metric spaces [11]. When a non-Euclidean path-loss appears in practice, there is no apparent reason for it to obey metric space constraints. However, this research is valuable at least from the theoretical viewpoint. Hence, there is a natural question: could one transfer the approximation results to arbitrary metric spaces, without assuming "nice" metric properties such as the doubling property? The answer is negative and is very easy to prove. To see this, let us consider an abstract network of n *q-independent equilength* links using the mean power scheme (the latter is not important because the links have the same length), where $q \in O(1)$. We will define the distances between the nodes in such a way that the metric space constraints hold true, but the difference between the schedule lengths in the SINR model and the conflict-graph model is $\Theta(n)$.

Let us number the links $\{1, 2, \ldots, n\}$. For each link i we define $l_i = 1$. Let s_i and r_i denote the sender and the receiver node of the link i, respectively. The distances between the nodes are defined as follows:

1. sender to sender distances: $d(s_i, s_j) = q(l_i + l_j) = 2q$,
2. sender to receiver distances: $d(s_i, r_j) = d(s_i, s_j) + l_j = 2q + 1$,
3. receiver to receiver distances: $d(r_i, r_j) = d(s_i, s_j) + l_i + l_j = 2q + 2$.

It is straightforward to check that such distances define a metric. Moreover, the whole set of links in this metric is q-independent with respect to all three power schemes considered in this paper. Let us consider any subset of k links $\{i_1, i_2, \ldots, i_k\}$, where $k > 0$ and $i_1 < i_2 < \cdots < i_k$. Then we have:

$$A(S, i_1) = \sum_{t=2}^{k} \left(\frac{\sqrt{l_{i_1} l_{i_t}}}{d_{i_t i_1}} \right)^{\alpha} = \sum_{t=2}^{k} \frac{1}{(2q+1)^{\alpha}} = \frac{k-1}{(2q+1)^{\alpha}}.$$

It follows that any feasible subset of links must contain $O(1)$ links; hence, the optimal schedule length in the SINR model is $\Theta(n)$. Thus we proved the following.

Theorem 6. *For any $n > 0$ and $q \in O(1)$, there is a q-independent set of n equilength links on a metric space for which the optimal schedule length in the SINR model is $\Theta(n)$.*

This result implies that Theorem 2, Theorem 3 and Theorem 5 are very far from being true in general metric spaces. Thus, the conflict-graph model is appropriate to use only in metrics that can transform the independence of the links into SINR feasibility, to which an example is doubling metrics.

A consequence that we can draw from [11] (Theorem 4.4) is the following theorem.

Theorem 7. *In any metric space, for any set of a nearly-equilength links, the schedule length using the mean power scheme is at most $O(\log n)$ times more than the schedule length using the best possible power assignment.*

Combining this theorem with Theorem 6 we have the following corollary.

Corollary 2. *For any $n > 0$ and $q \in O(1)$, there is a q-independent set of n equilength links on a metric space for which the optimal schedule length in the SINR model is $\Theta(n/\log n)$, even when using the best possible power assignment.*

6 Conclusion

This paper presented a set of results trying to evaluate the asymptotic difference between the SINR schedules and Conflict-Graph based schedules in wireless networks. These results indicate that this gap is bounded in doubling metric spaces of small dimension such as the Euclidean plane. For the case of the uniform and linear power schemes the upper bound is $O(\log \Delta)$ and is sharp. For the mean power scheme the gap is in $O(\min\{\log n, \log \Delta)$, so the upper bound for the mean power scheme scales better with the number of links and the topology of the network. In the case of the mean power scheme no example of network meeting the upper bound is known to the author, so this could be a subject of a future work. At last, it was shown that in general metric spaces the difference between the schedules in the two models can be arbitrary.

References

1. Cardieri, P.: Modeling interference in wireless ad hoc networks. IEEE Commun. Surveys Tuts. 12(4), 551–572 (2010)
2. Grönkvist, J., Hansson, A.: Comparison between graph-based and interference-based STDMA scheduling. In: 2nd ACM International Symposium on Mobile Ad Hoc Networking & Computing, pp. 255–258. ACM (2001)
3. Moscibroda, T., Wattenhofer, R., Yves, W.: Protocol design beyond graph-based models. In: 5th ACM SIGCOMM Workshop on Hot Topics in Networks, Irvine, California, USA. ACM (2006)
4. Behzad, A., Rubin, I.: On the performance of graph-based scheduling algorithms for packet radio networks. In: IEEE Global Telecommunications Conference, pp. 3432–3436. IEEE (2003)
5. Iyer, A., Rosenberg, C., Karnik, A.: What is the right model for wireless channel interference? IEEE Trans. Wireless Commun. 8(5), 2662–2671 (2009)
6. Halldórsson, M.M.: Wireless scheduling with power control. In: Fiat, A., Sanders, P. (eds.) ESA 2009. LNCS, vol. 5757, pp. 361–372. Springer, Heidelberg (2009)
7. Tonoyan, T.: Algorithms for scheduling with power control in wireless networks. In: Marchetti-Spaccamela, A., Segal, M. (eds.) TAPAS 2011. LNCS, vol. 6595, pp. 252–263. Springer, Heidelberg (2011)
8. Rappaport, T.S.: Wireless Communications: Principles and Practice, 2nd edn. Prentice Hall (2002)
9. Tonoyan, T.: On the capacity of oblivious powers. In: Erlebach, T., Nikoletseas, S., Orponen, P. (eds.) ALGOSENSORS 2011. LNCS, vol. 7111, pp. 225–237. Springer, Heidelberg (2012)
10. Heinonen, J.: Lectures on Analysis on Metric Spaces, 1st edn. Springer (2000)
11. Halldórsson, M.M., Mitra, P.: Wireless capacity with oblivious power in general metrics. In: 22nd Annual ACM-SIAM Symposium on Discrete Algorithms, pp. 1538–1548. SIAM (2011)
12. Halldórsson, M.M., Wattenhofer, R.: Wireless communication is in APX. In: Albers, S., Marchetti-Spaccamela, A., Matias, Y., Nikoletseas, S., Thomas, W. (eds.) ICALP 2009, Part I. LNCS, vol. 5555, pp. 525–536. Springer, Heidelberg (2009)
13. Moscibroda, T., Wattenhofer, R.: The complexity of connectivity in wireless networks. In: 25th IEEE International Conference on Computer Communications, Barcelona, Spain, pp. 1–13. IEEE (2006)