Adaptive NN Control for a Class of Strict-Feedback Discrete-Time Nonlinear Systems with Input Saturation

Xin Wang[1], Tieshan Li[1,*], Liyou Fang[1], and Bin Lin[2]

[1] Navigational College, Dalian Maritime University, Dalian 116026, China
[2] Department of Information Science and Technology,
Dalian Maritime University, Dalian 116026, China
luyunliunanting@yeah.net, tieshanli@126.com, binlin@dlmu.edu.cn

Abstract. In this paper, an adaptive neural network (NN) control scheme is proposed for a class of strict-feedback discrete-time nonlinear systems with input saturation. which is designed via backstepping technology and the approximation property of the HONNs, aimed to solve the the input saturation constraint and system uncertainty in many practical applications. The closed-loop system is proven to be uniformly ultimately bounded (UUB). At last, a simulation example is given to illustrate the effectiveness of the proposed algorithm.

Keywords: input saturation, discrete-time, adaptive control, backstepping, high-order neural networks (HONNs).

1 Introduction

In recently years, the research on the neural network control of various nonlinear uncertain systems has advanced significantly. In the literature of adaptive neural network control, neural networks (NNs) are primarily used as on-line approximators for the unknown nonlinearities due to their inherent approximation capabilities. By using the idea of backstepping design [1], several adaptive neural-networks control [2-6] have been presented for some classes of uncertain nonlinear strict-feedback systems.

However, the above mentioned methods are limited to the continuous-time domain, they are not directly applicabled to discrete-time systems due to the noncausal problem in the controller design procedures. Recently, the adaptive control via the universal approximators for uncertain discrete-time nonlinear systems has obtained many results. For example, the approach proposed in [7] was given to achieve the tracking control of a class of unknown nonlinear dynamic systems using a discrete-time NN controller. Subsequently, several elegant adaptive control schemes were studied in [8-13] for discrete-time nonlinear systems based on the approximation property of the

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neural network. For instance, both the state and ouput feedback adaptive neural network controllers were presented for a class of discrete-time nonlinear systems in the strict-feedback form [8]. A novel approach in designing neural network based adaptive controllers for a class of nonlinear discrete-time systems was presented in [9].

For practical perspective, the input saturation may cause serious influence on system stability and performance[14]. Therefore, the effects of input saturation cannot be ignored in the controller design [15]. Recently, to solve the input saturation constraint, some adaptive continuous nonlinear systems with input saturtion has been addressed in [16-18]. But there are still little works for the discrete-time nonlinear system [19].

In this paper, adaptive neural network controller via backstepping is presented for a class of discrete-time nonlinear system with input saturation. During the controller design process, the HONNs are used to approximate the unknown nonlinear function in the system. With an aided design system of the input saturation, the input saturation constraint of the discrete-time nonlinear system is solved in the controller design. By using the lyapunov analysis method, the closed-loop systems are proven to be UUB, and the tracking error converges to a small neigborhood of the origin. At last, the simulation results show the effectiveness of the proposed method.

2 Problem Formulation and Preliminaries

2.1 Problem Formulation

Consider the following single-input single-output (SISO) discrete-time nonlinear system in strict-feedback form in [7]:

$$\begin{cases} x_i(k+1) = f_i(\bar{x}_i(k)) + g_i(\bar{x}_i(k))x_{i+1}(k) \quad i = 1,2\ldots n-1 \\ x_n(k+1) = f_n(\bar{x}_n(k)) + g_n(\bar{x}_n(k))u(k) \\ y_k = x_1(k) \end{cases} \tag{1}$$

Where $\bar{x}_i(k) = [x_1(k), x_2(k), \ldots, x_i(k)]^T \in R^i, i = 1,2\ldots n$, $u(k) \in R$ and $y_k \in R$ are the state variables, system input and output respectively; $f_i(\bar{x}_i(k))$ and $g_i(\bar{x}_i(k)), i = 1,2,\ldots,n$ are unknow smooth functions.

The control objective is to design an adaptive NN controller for system (1) such that: (i) all the signals in the closed-loop system are UUB and (ii) the system output follows the desired reference signal $y_d(k)$. The desired reference signal $y_d(k) \in \Omega_y, \forall k > 0$ is smooth and known, where $\Omega_y := \{\chi | \chi = x_1\}$.

Assumption 2. The signal of $g_i(\bar{x}_i(k)), i = 1,2,\ldots,n$ are known and there exist co-nstants $\underline{g}_i > 0$ and $\bar{g}_i > 0$ such that $\underline{g}_i \le |g_i(\bar{x}_i(k))| \le \bar{g}_i$, $\forall \bar{x}_n(k) \in \Omega$.

2.2 Input Saturation Constraint

Considering the input saturation constraints u satisfies $-u_{\min} \le u \le u_{\max}$, where $u_{\min}$ and $u_{\max}$ are the known lower limit and up limit of input constraints. Thus,

$$u = sat(v) = \begin{cases} u_{\max}, & if \quad v > u_{\max} \\ v, & if \quad -u_{\min} \le v \le u_{\max} \\ -u_{\min}, if & v < -u_{\min} \end{cases} \tag{2}$$

where v is the designed controller input of the system .

Then , to be convenient to consider the affect of the input saturation constraint, the aided design system is considered as follows:

$$e(k+1) = \begin{cases} -c_1 e(k) - \dfrac{f(\cdot)}{e^2} \cdot e + (u - v), & |e| \ge \theta \\ 0, & |e| < \theta \end{cases} \tag{3}$$

where e is a state variable of the aided design system, θ is a small positive design parameters, $c_1 > 0$ also a design paramaters. and $f(\cdot) = f(\eta_n, \Delta u) = |\eta_n \cdot \Delta u| + \frac{1}{2}\Delta u^2$ (η_n is a error variable in control design) is a aided design function.

3 Adaptive NN Control Design with Input Saturation Constraint

Consider the strict-feedback SISO nonlinear discrete-time system described in (1). Since assumption 1 is only valid on the compact set Ω, it is necessary to guarantee the system's states remaining in Ω for all time. We will design an adaptive control $u(k)$ for system (1) which makes system output y_k follow the desired reference signal $y_d(k)$, and simultaneously guarantees $\bar{x}_n(k) \in \Omega, \forall k > 0$ under the condition that $\bar{x}_n(0) \in \Omega$.

The strict-feedback form (1) is transformed to n-step head function as follows:

$$\begin{aligned} &x_1(k+n) = F_1(\bar{x}_n(k)) + G_1(\bar{x}_n(k)) x_2(k+n-1), \\ &\quad \vdots \\ &x_{n-1}(k+2) = F_{n-1}(\bar{x}_n(k)) + G_{n-1}(\bar{x}_n(k)) x_n(k+1) \\ &x_n(k+1) = f_n(\bar{x}_n(k)) + g_n(\bar{x}_n(k)) u(k) \\ &y_k = x_1(k). \end{aligned} \tag{4}$$

Form the definition of $G_i(\overline{x}_n(k))$ in each step, it is clear that the value of $G_i(\overline{x}_n(k))$ is the same as $G_i(\overline{x}_i(k))$, therefore $G_i(\overline{x}_n(k))$ satisfy:

$$\underline{g_i} \le G_i(\overline{x}_n(k)) \le \overline{g}_i, \forall \overline{x}_n(k) \in \Omega$$

Now we can construct the controller for (4) via backstepping method without the problem of causality contradiction. For convenience of analysis and discussion, for $i = 1,2,\ldots,n-1$ let:

$$F_i(k) = F_i(\overline{x}_n(k)), G_i = G_i(\overline{x}_n(k)), f_n(k) = f_n(\overline{x}_n(k)), g_n(k) = g_n(\overline{x}_n(k))$$

Before further going, let $k_i = k - n + i, i = 1,2\ldots n-1$

Step 1: For $\eta_1(k) = x_1(k) - y_d(k)$, its n th difference is given by

$$\eta_1(k+n) = F_1(k) + G_1(k)x_2(k+n-1) - y_d(k+n) \tag{5}$$

Considering $x_2(k+n-1)$ as a fictitious control, if we choose

$$x_2(k+n-1) = x_{2d}^*(k) = -\frac{1}{G_1(k)}[F_1(k) - y_d(k+n)] \tag{6}$$

It is obvious that $\eta_1(k+n) = 0$. Since $F_1(k)$ and $G_1(k)$ are unknown, they are not available for constructing a fictitious control $x_{2d}^*(k)$. However, $F_1(k)$ and $G_1(k)$ are function of system state $\overline{x}_n(k)$, therefore we can use HONN to approximate $x_{2d}^*(k)$ as follow:

$$x_{2d}^*(k) = W_1^{*T} S_1(z_1(k)) + \varepsilon_z(z_1(k)),\ z_1(k) = [\overline{x}_n^T(k), y_d(k+n)]^T \tag{7}$$

Letting $\hat{W}_1$ be the estimate of W_1^*, consider the direct adaptive fictitious control:

$$x_2(k+n-1) = x_{2f}(k) = \hat{W}_1^T(k) S_1(z_1(k)) \tag{8}$$

and the adaptive law as

$$\hat{W}_1(k+1) = (1 - \Gamma_1\sigma_1)\hat{W}_1(k_1) + \Gamma_1 S_1(z_1(k_1))\eta_1(k+1) \tag{9}$$

substituting (6), (7) and (8) into (5) yields

$$\eta_1(k+n) = G_1(k)[\tilde{W}_1(k) S_1(z_1(k)) - \varepsilon_{z1}] \tag{10}$$

Choose the lyapunov function candidate

$$V_1(k) = \frac{1}{\overline{g}_1}\eta_1^2(k) + \sum_{j=0}^{n-1} \tilde{W}_1^T(k_1+j)\Gamma_1^{-1}\tilde{W}_1(k_1+j),\ k_1 = k - n + 1 \tag{11}$$

Noting the fact that $\tilde{W}_1^T(k)S_1(z_1(k)) = \eta_1(k+1)/G_1(k_1) + \varepsilon_{z1}$, the first difference of (11) is

$$\Delta V_1 = \frac{1}{\bar{g}_1}[\eta_1^2(k+1) - \eta_1^2(k)] + \tilde{W}_1^T(k+1)\Gamma_1^{-1}\tilde{W}_1(k+1) - \tilde{W}_1^T(k_1)\Gamma_1^{-1}\tilde{W}_1(k_1)$$

Step i : Simulated the procedure in step 1, for $\eta_i(k) = x_i(k) - x_{if}(k_{i-1})$, we can get the following direct adaptive fictitious controller and its adaptive law

$$\begin{aligned} & x_{i+1}(k+n-i) = x_{(i+1)f}(k) = \hat{W}_i^T S_i(z_i(k)) \\ & \hat{W}_i(k+1) = (1 - \Gamma_i\sigma_i)\hat{W}_i(k_i) - \Gamma_i S_i(z_i(k_i))\eta_i(k+1) \\ & z_i(k) = [\bar{x}_n^T(k), x_{if}(k)]^T \in \Omega_{zi} \subset R^{n+1} \end{aligned} \tag{12}$$

Then we can obtain

$$\eta_i(k+n-i+1) = G_i(k)[\tilde{W}_i^T(k)S_i(z_i(k)) - \varepsilon_{zi}] \tag{13}$$

Choose the lyapunov function candidate as follows

$$V_i(k) = \sum_{j=1}^{i-1} V_j(k)_1 + \frac{1}{\bar{g}_i}\eta_i^2(k) + \sum_{j=0}^{n-i}\tilde{W}_i^T(k_i+j)\Gamma_i^{-1}\tilde{W}_i(k_i+j),\ k_i = k-n+i.$$

Step n**:** For $\eta_n(k) = x_n(k) - x_f(k-1)$. its first difference is

$$\eta_n(k+1) = x_n(k+1) - x_{nf}(k) = f_n(k) + g_n(k)u(k) - x_{nf}(k) \tag{14}$$

To deal with the affect of the input saturation constraint in this step, we employ the aided design system in (3). Now, consider the direct adaptive fictitious controller as

$$u(k) = \hat{W}_n^T S_n(z_n(k)) + e(k) \tag{15}$$

and the adaptive law as follows:

$$\hat{W}_n(k+1) = (1 - \Gamma_n\sigma_n)\hat{W}_n(k\ ) - \Gamma_n S_n(z_n(k\ ))\eta_n(k+1) \tag{16}$$

Substituting (15) and (16) into (14) , we can obtain

$$\eta_n(k+1) = g_n(k)[\tilde{W}_n^T(k)S_n(z_n(k)) + e(k) - \varepsilon_{zn}] \tag{17}$$

Choose the lyapunov function candidate

$$V_n(k) = \sum_{j=1}^{n-1} V_j(k) + \frac{1}{\bar{g}_n}\eta_n^2(k) + \tilde{W}_n^T(k)\Gamma_n^{-1}\tilde{W}_n(k). + \frac{1}{\bar{g}_n}e^2(k) \tag{18}$$

Noting the fact that: $\tilde{W}_n^T(k)S_n(z_n(k)) = \eta_n(k+1)/g_n(k) - e(k) + \varepsilon_{zn}$

The first difference of (18) with (16) and (17) is

$$\Delta V_n = \sum_{j=1}^{n-1}\Delta V_j + \frac{1}{\bar{g}_n}[\eta_n^2(k+1)-\eta_n^2(k)] + \tilde{W}_n^T(k+1)\Gamma_n^{-1}\tilde{W}_n(k+1)$$
$$-\tilde{W}_n^T(k)\Gamma_n^{-1}\tilde{W}_n(k) + \frac{1}{\bar{g}_n}[e^2(k+1)-e^2(k)]$$
$$\le \sum_{j=1}^{n-1}\Delta V_j - \frac{1}{\bar{g}_n}[\eta_n^2(k+1)-\eta_n^2(k)] + 2\varepsilon_{zn}\eta_n(k+1) + 2e(k)\eta_n(k+1)$$
$$-2\sigma_n\tilde{W}_n^T(k)\hat{W}_n(k) + S_n^T(z_n(k))\Gamma_n S_n(z_n(k))\eta_n^2(k+1)$$
$$+2\sigma_n\hat{W}_n^T(k)\Gamma_n S_n(z_n(k))\eta_n(k+1) + \sigma_n^2\hat{W}_n^T(k)\Gamma_n\hat{W}_n(k)$$
$$+\frac{1}{\bar{g}_n}[e^2(k+1)-e^2(k)]$$

Noting the fact that:

$$S_j^T(z_j(k_j))S_j(z_j(k_j)) < l_j,$$
$$S_j^T(z_j(k_j))\Gamma_j S_j(z_j(k_j)) \le \bar{\gamma}_j S_j^T(z_j(k_j))S_j(z_j(k_j)) \le \bar{\gamma}_j l_j$$
$$2\varepsilon_{zj}\eta_j(k+1) \le \frac{\bar{\gamma}_j\eta_j^2(k+1)}{\bar{g}_j} + \frac{\bar{g}_j\varepsilon_{zj}^2}{\bar{\gamma}_j}$$
$$2\sigma_j\hat{W}_j^T(k_j)\Gamma_j S_j(z_j(k_j))\eta_j(k+1) \le \frac{\bar{\gamma}_1 l_1\eta_1^2(k+1)}{\bar{g}_1} + \bar{g}_1\sigma_1^2\bar{\gamma}_1\left\|\hat{W}_1\right\|^2$$
$$2\tilde{W}_1^T(k_1)\hat{W}_1(k_1) = \left\|\tilde{W}_1(k_1)\right\|^2 + \left\|\hat{W}_1(k_1)\right\|^2 - \left\|W_1^*\right\|^2$$
$$e(k+1) = -[c_1 + f(\cdot)/e^2(k)]e(k) + \Delta u$$

And defining $c_2 = c_1 + f(\cdot)/e^2(k) > 0$, we can get

$$2e(k)\eta_n(k+1) \le \frac{1}{c_2}[\frac{\bar{\gamma}_n\eta_n^2(k+1)}{\bar{g}_n} + \frac{\bar{g}_n}{\bar{\gamma}_n}\Delta u^2] + \frac{\bar{\gamma}_n}{c_2\bar{g}_n}\eta_n^2(k+1) + \frac{\bar{g}_n}{c_2\bar{\gamma}_n}e^2(k+1)$$

Then, we have

$$\Delta V_n \le \sum_{j=1}^{n-1}\Delta V_j - \frac{\rho_n}{c_2\bar{g}_n}\eta_n^2(k+1) - \frac{1}{\bar{g}_n}\eta_n^2(k) + \beta_n - \frac{1}{\bar{g}_n}e^2(k)$$
$$\sigma_n(1-\sigma_n\bar{\gamma}_n - \bar{g}_n\sigma_n\bar{\gamma}_n)\left\|\hat{W}_n(k)\right\|^2 + \frac{\bar{g}_n^2 + c_2\bar{\gamma}_n}{c_2\bar{\gamma}_n\bar{g}_n}e^2(k+1)$$

where $\rho_n = c_2 - c_2\bar{\gamma}_n - \bar{\gamma}_n l_n - \bar{g}_n\bar{\gamma}_n l_n - 2\bar{\gamma}_n$

$$\beta_n = \beta_{n-1} + \frac{\bar{g}_n \varepsilon_{zn}^2}{\bar{\gamma}_n} + \frac{\bar{g}_n \Delta u^2}{c_2 \bar{g}_n} + \sigma_n \left\| W_n^* \right\|^2$$

If we choose the design parameters as follows

$$\bar{\gamma}_n < \frac{c_2}{c_2 + l_n + \bar{g}_n l_n + 2}, \quad -\frac{\bar{g}_n^2}{c_2} \le \bar{\gamma}_n < 0, \quad \sigma_n < \frac{1}{(1+\bar{g}_n)\bar{\gamma}_n} \tag{19}$$

then $\Delta V_n \le 0$ once any one of the n errors satisfies $\left|\eta_j(k)\right| > \sqrt{\bar{g}_j \beta_n}$ and $j = 1,2,\ldots,n$, This demonstrates that the tracking error $\eta_1(k), \eta_2(k), \ldots, \eta_n(k)$ are bounded for all $k \ge 0$.

Based on the procedure above, we can conclude that $\bar{x}_n(k+1) \in \Omega$ and $u(k)$ are bounded if $\bar{x}_n(k) \in \Omega$. Finally, if we initialize $\bar{x}_n(0) \in \Omega$, and choose the design parameters as (20), there exists a k^*, such that all errors asymptotically converge to zero, and NN weight errors are all bounded. This implies that the closed-loop system is UUB. and $\bar{x}_n(k) \in \Omega, \hat{W}_i, i = 1,2...n$ will hold for all $k > 0$.

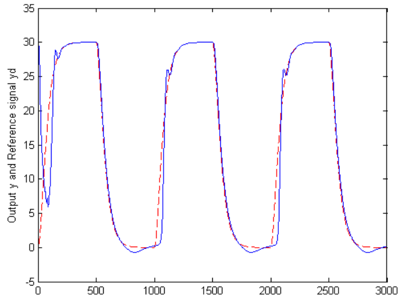
4 Simulation Studies

Now , we can apply adaptive NN control for a class of discrete-time nonlinear systems with input saturation to the ship heading control, the disderte-time ship heading control plant described by

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = -\dfrac{1}{T}(a_1 x_2(k) - a_2 x_2^3(k)) + \dfrac{K}{T} u(k) \\ y_k = x_1(k) \end{cases} \tag{20}$$

where a_1, a_2 denotes nonlinear coefficients of the ship, K, T are the parameter of rotary and trackability. It can be checked that Assumption 1 and 2 are satisfied. In ship heading control, the input saturation restrictions of the rudder angle is 35°.

The initial condition for ship heading control states is $x_0 = [30,0]^T$, and the HONN codes is $l_1 = 22, l_2 = 22$ .The simulation results are presented in Figs 1 and 2.



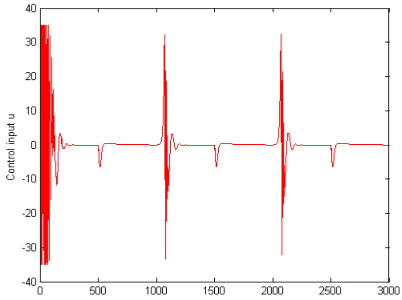


Fig. 1. The output y_k and the reference signal y_d **Fig 2.** The input of the controller u

5 Conclusion

By using the backstepping technique and the approximation property of the HONNs, as well as considering the input saturation, an adaptive control approach is proposed for a class of discrete-time nonlinear systems with input saturation. In this paper, the proposed controller solves the input saturation constraint and system uncertainty in practical applications, and all the signal of the resulting closed-loop system were guaranteed to be UUB, the tracking errors can be reduced to a small neighborhood of zero. The simulation example is proposed to show the performance of the presented scheme.

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