

Artificial Fish Swarm Optimization Algorithm Based on Mixed Crossover Strategy^{*}

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Abstract. The nonlinear constrained optimization problems have been widely used in many fields, such as engineering optimization and artificial intelligence. According to the deficiency of artificial fish swarm algorithm (AFSA), that the artificial fishes walk around aimlessly and randomly or gather in non-global optimal points, a hybrid algorithm-artificial fish swarm optimization algorithm based on mixed crossover strategy is presented. By improving the artificial fish's behaviors, the genetic operation of mixed crossover strategy is used as a local search strategy of AFSA. So the efficiency of local convergence of AFSA is improved, and the algorithm's running efficiency and solution quality are improved obviously. Based on test verification for typical functions, it is shown that the hybrid algorithm has some better performance such as fast convergence and high precision.

1 Introduction

A great deal of the nonlinear constrained optimization problems have been often met in many fields, such as engineering optimization and artificial intelligence. Some of these problems belong to the NP-hard problem, and it is the normal way the constrained problem is transformed into the unconstrained problem. Intelligent algorithm has developed by analyzing biological evolutionary theory in recent years, such as genetic algorithm, artificial fish swarm algorithm and so on, which has been widely used in the optimization field because of its unique optimization mechanism, generalization and flexibility [1-2].

Artificial fish swarm algorithm (ASFA) is an optimizing method based on animal autonomy, which is proposed by simulating fish behavior, and it is a concrete application of swarm intelligence theory. The main feature of ASFA is that it is not need to learn special information of problems, but need to compare the advantages and disadvantages of problems, and the global optimal is finally emerged out by local optimizing behavior of every artificial fish individual [3]. The basic ASFA has characteristics of seizing search direction and avoiding the problem of the local optimal to some extent, but when the artificial fishes walk around aimlessly and randomly or

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gather in non-global optimal points, or when the region of optimizing is large or tends to be flat, the convergence to the global optimum is slowed down and searching performance is deteriorated and even it can falls into the local minimum. The algorithm has generally fast convergence speed in initial of optimizing, but that is often relatively slow in late of optimizing. The solution attained is the satisfactory solution, which has the low precision [3-4].

In order to escape more easily the local optimal for the basic AFSA, and to improve searching efficiency and precision, the genetic operation of mixed crossover strategy and the Gaussian mutation operation are introduced into the basic ASFA in the article. The artificial fishes can effectively escape from the local optimal through rational allocation of parameters, and the diversity of artificial fish can be kept. Meanwhile local fields can be further searched and the searching efficiency is accelerated, so that the swarm converges quickly to the global optimum and the solution of high precision is attained.

2 Treatment of Nonlinear Constrained Optimization Problems

The nonlinear constrained optimization problem is mainly considered in the article, which is described as following [5]:

$$\begin{aligned} \min_{x \in D \subset [L, U]} f(x) \\ \text{st. } g_i(x) \leq 0, \quad i = 1, 2, \dots, m \end{aligned} \quad (1)$$

where $[L, U]$ is n dimension vector field of the space domain R^n and $[L, U] = \{x = (x_1, x_2, \dots, x_n) \mid l_i \leq x_i \leq u_i, i = 1, 2, \dots, n\}$. The set $D = \{x \mid x \in [L, U], g_i(x) \leq 0, i = 1, 2, \dots, m\}$ is the feasible region of solution. If there exists in $x^* \in D$ and makes $f(x^*) \leq f(x)$ that is tenable to any $x \in D$, then x^* is called as the global optimal solution and $f(x^*)$ is called as the global optimal.

For nonlinear constrained optimization problems as formula (1), it is transformed into multi-objective optimization problem which has only two objectives [5].

$$\min\{f_1(x), f_2(x)\} \quad (2)$$

In which $f_1(x)$ is the objective function of the original problem (1) and $f_2(x) = \max(0, g_i(x), i = 1, 2, \dots, m)$. Obviously, the minimization of the first objective function means to find x^* which can make the objective function of the original problem to reach the minimum, while the second objective function is to select the maximum between zero and the constrained function value that the violation of the constraints is maximum in all constraints. So as long as $f_2(x) = 0$, all constraints of the original problem must be less than zero, that is the minimization process of $f_2(x)$, in essence, is to try to find out the point x^* , which meets all constraints. Therefore, the simultaneous minimizing of two objectives is to find the point meets all constraints and makes $f_1(x)$ to reach the minimum, namely the optimal solution of the original nonlinear constrained optimization problems. The relation of the model (1) and the model (2) is as follow [6]:

Theorem 1. The necessary and sufficient condition x^* is the optimal solution: x^* is efficient solution of model (2), that is

$$x^* \in D, \underline{\exists} f_1(x^*) = \min_{x \in D} f_1(x), x \in D \quad (3)$$

Proof 1. See reference [6].

3 Artificial Fish Swarm Optimization Algorithm Based on Mixed Crossover Strategy

3.1 Artificial Fish Swarm Algorithm

Artificial fish swarm algorithm is a class of stochastic optimization algorithm based on swarm intelligence, and its mathematical model is described as follow [7]: supposed in a objective searching space of n-dimension, there has N artificial fishes which composed of a swarm, and the state of every artificial fish can be expressed as the vector $X = (x_1, x_2, \dots, x_n)$, in which $x_i (i = 1, 2, \dots, n)$ is the variable of optimization. The food concentration of the current location of artificial fish is expressed as $y = f(x)$, in which y is the objective function. The distances between individuals of artificial fish are expressed as $d_{ij} = \|X_i - X_j\|$. The feeling range of artificial fish and the crowded degree factor are expressed respectively as visual and σ . The step artificial fish moves and the maximum tentative number artificial fish preys each time are expressed respectively as step and *try_number*.

In the process of iteration for each time, artificial fishes are renewed by preying behavior, gathering behavior, following behavior and so on, so the optimizing is realized. The concrete behaviors are described as follows [3]:

①preying behavior: Supposed the current state of artificial fish is X_i , and a state X_j is selected randomly within its visual range (namely $d_{ij} \leq \text{Visual}$). If the food concentration of this state is more than that of the current state, then the artificial fish forwards a step to the direction. Conversely, a state X_j is reselected randomly, and judges whether it meets the forward conditions. After repeated *try_number* times, if the forward conditions do not still be met, then artificial fish moves randomly a step.

②gathering behavior: Supposed the current state of artificial fish is X_i , the number of partners within its visual range is n_f . If n_f/N is less than δ , which shows that the central position of partners is not too crowded, meanwhile, the food concentration of the central position is more than that of the current state, then the artificial fish forwards a step to the central position, otherwise, preying behavior is carried.

③following behavior: Supposed the current state of artificial fish is X_i , an optimal partner within its visual range is X_{max} . If the number of partners within its visual range is n_f and meets $n_f/N < \delta$, meanwhile, the food concentration of X_{max} position is more than that of the current state, then artificial fish forwards a step to X_{max} position, otherwise, preying behavior is carried.

④stochastic behavior: Supposed the current state of artificial fish is X_i , and a state X_j is selected randomly within its visual range, in order to enlarge search range.

⑤moving strategy: The current environment of artificial fish do be evaluated, that is gathering behavior and following behavior are simulated and executed, then the behavior that has high the food concentration value is executed, and the default behavior way is preying behavior.

⑥constraint behavior: In the process of optimizing, the corresponding constraint conditions need to be added, in order to adjust the situation that the solved solution can be not feasible solution because of some operations, such as gathering behavior, stochastic behavior and so on.

Finally, artificial fishes aggregate around the local optimal, and around the optimal fields of the better value can aggregate generally more artificial fishes.

3.2 Genetic Operation of Mixed Crossover Strategy

Mixed strategies are defined as follows [8]: In a normal game $G = \{S_1, \dots, S_n; u_1, \dots, u_n\}$ with n players, set the strategy space S_i for player i is $S_i = \{s_{i1}, \dots, s_{ik}\}$. The player i selects a strategy randomly in the available k strategies based on a probability distribution $p_i = (p_{i1}, \dots, p_{ik})$. The strategies for every player obtained by this way are called mixed strategies, where $j = 1, \dots, k$, $0 \leq p_{ij} \leq 1$ and $p_{i1} + \dots + p_{ik} = 1$. Meanwhile, the original strategy is called pure strategy.

Crossover operator is the most important genetic operator in GA, so the research on crossover operators reflects the research progress on GA. A new crossover operator, mixed crossover strategy, is proposed by mixing the four different crossover operators, including one-point, two-point, uniform and uniform two-point crossover [9]. Every crossover operator is called as pure crossover strategy in mixed crossover strategy, and has its own probability distribution for crossover. The probability distribution of every crossover strategy is adjusted by strengthen or weaken in current population so that the mixed strategy is adjusted. This algorithm implements the choice of crossover strategy automatically. The performance of the algorithm becomes more stable and effective.

The crossover strategy is described as follows [9]:

1) Initialization:

Using pure crossover strategy h , $h \in \{1, 2, 3, 4\}$, which expressed as the crossover strategy of one-point, two-point, uniform, and uniform two point crossover strategy, respectively. The probability distribution of mixed strategy vector ρ is initialized.

2) Crossover operation based on mixed strategy

(1) In every generation g , selects a crossover strategy h according to mixed strategies vector ρ , and uses crossover strategy h to cross to individuals, then generates offspring.

(2) Calculate offspring fitness value, and rank according to fitness.

(3) The method of adjustment of mixed strategy in offspring generation is as follow:

If the number of offspring individuals, which fitness value greater than the parents fitness, is more than half of the number of parent after crossover then strengthens the crossover strategy, that is

if $\forall l \neq h$ then

$$\begin{aligned} \rho_1^{(k+1)} &= \rho_1^{(k)} - \rho_1^{(k)} \times \gamma \\ \text{else } \rho_h^{(k+1)} &= \rho_h^{(k)} + (1 - \rho_h^{(k)}) \times \gamma \end{aligned}$$

Else weakens the crossover strategy, that is

$$\begin{aligned} \text{if } \forall l \neq h \text{ then } \rho_1^{(k+1)} &= \rho_1^{(k)} + \frac{1}{(\text{no_strategy} - 1)} \times \rho_1^{(k)} \times \gamma \\ \text{else } \rho_h^{(k+1)} &= \rho_h^{(k)} - \rho_h^{(k)} \times \gamma \end{aligned}$$

where $\gamma \in (0,1)$ uses to adjust the probability distribution of mixed strategies, “no_strategy” is the number of mixed pure crossover strategy, where $\gamma = 1/2$.

3.3 Artificial Fish Swarm Optimization Algorithm Based on Mixed Crossover Strategy (MCSG-AFSA)

Artificial fish swarm optimization algorithm based on mixed crossover strategy (MCSG-AFSA) is proposed by introducing the genetic operation of mixed crossover strategy in basic ASFA. The algorithm can make effectively artificial fishes to get rid of the limitation of the local optimal, and the local field is further searched, so the searching efficiency is accelerated. Meanwhile the diversity of population is increased by introducing mutation operator [10], so the swarm converges quickly to the global optimal and the solution of high precision is attained finally.

The algorithm flow of MCSG-AFSA algorithm as follow [1]:

(1) Initialization:

The iteration number of the initial bulletin board *Beststep* is equal to 0 when the state of the optimal artificial fish does continuously not change or change little, and set the initial iteration number *Num* is equal to 0 and the maximal threshold of the times that the optimal do not continuously change *maxbest* is equal to 5. *n* artificial fishes are generated randomly in feasible region of control variable, so the initial fish swarm is formed.

(2) The initialization of genetic parameters:

The initial probability of mixed strategy vector is set at 0.25, and the crossover probability and the maximum times of running are set.

(3) The initial value of the bulletin board assignment:

The formula (1) is selected to calculate function value *y* of the current state of individual artificial fish for the initial fish swarm. Then the size of *y* is compared, the minimum of *y* is selected into the bulletin board, and the fish is assigned to the bulletin board.

(4) Behavior selection:

Every artificial fish simulates following behavior and gathering behavior, and the behavior that *y* value is less is executed actually after behavior selected (the default behavior is preying behavior).

(5) Renew the bulletin board:

After every behavior for every artificial fish, *y* of its own and that of the bulletin board are tested. If the former is better than the latter, then *y* of the bulletin board is

substituted by that of its own and *Beststep* is set as 0. Otherwise, the behavior of artificial fish that the minimum y is calculated by formula (2) is executed actually, and the default behavior is preying behavior. Then the step (5) is executed.

(6) The condition judgment that mixed crossover strategy and mutation operator of genetic algorithm introduced:

Judge whether the value of *Beststep* reaches *Maxbest*. If the value of *Beststep* reaches *Maxbest*, then mixed crossover strategy and mutation algorithm of the step (7) are executed, otherwise, the algorithm is transformed to execute the step (8).

(7) Mixed crossover strategy and mutation operation:

All artificial fishes in swarm execute operations as follows, except the optimal individual of the bulletin board.

① mixed crossover operation:

According to the crossover probability P_c , the corresponding individuals are selected from artificial fish swarm, and execute the genetic operation of mixed crossover strategy in 3.2 section, so function values y of new individuals are calculated out and every y is compared with the optimal of the bulletin board. If the current y is better than the optimal of the bulletin board, so y of the bulletin board is substituted by the current y .

② mutation operation:

The corresponding individuals are selected randomly from artificial fishes swarm according to the mutation probability P_m , and execute Gaussian mutation. Then function values of new individuals are calculated out and every y is compared with the optimal of the bulletin board. If the current y is better than the optimal of the bulletin board, so y of the bulletin board is substituted by the current y .

③ *Beststep* is set as 0.

(8) The terminal condition judgment:

Judge whether *Num* reaches the maximum iteration number *Maxnumber* or whether the optimal reaches within satisfactory error. If the both do not meet, then *Num* is set as *Num*+1, *Beststep* is set as *Beststep*+1, and the algorithm is transformed to execute the step (4). Otherwise, the algorithm is transformed to execute the step (9).

(9) The algorithm terminal and the optimal output, namely the state and function value of artificial fish in the bulletin board.

4 Optimization Test

Five group typical functions are selected to test verification in the paper, and the running result of the algorithm in the paper is compared with that of other algorithms.

Based on test verification for typical functions, it is shown that the hybrid algorithm has some better performance such as fast convergence and high precision, as shown in table1.

Table 1. The Comparison the running result of test functions between MCSG-AFSA and other algorithms

Test function	F1	F2	F3	F4	F5
The optimal	0.25000	-6961.814	13.59084	1.00000	3.791340
MCSG-AFSA	0.25000	-6961.814	13.59084	1.00000	3.791340
Other algorithms [see reference1]	0.25000	-6961.814	13.59085	0.95825	3.791340
The average result of reference[1]	0.25004	-6961.352	13.59393	0.99999	3.791297
The average result of MCSG-AFSA	0.25001	-6961.669	13.59065	0.99999	3.791331

The results of table show that MCSG-AFSA is better than other algorithms obviously, and embodies the certain superiority. MCSG-AFSA is programmed to realize on Matlab7.0 language platform. Test functions F1-F5 are shown as follows:

$$\begin{aligned}
 \text{F1: } & \min f(x) = 100(x_2 - x_1)^2 + (1 - x_1)^2 \\
 & \text{s.t. } \begin{cases} g_1(x) = -x_1 - x_2^2 \leq 0 \\ g_2(x) = -x_1^2 - x_2 \leq 0 \\ -0.5 \leq x_1 \leq 0.5, -1 \leq x_2 \leq 1 \end{cases} \\
 \text{F2: } & \min f(x) = (x_1 - 10)^3 + (x_2 - 20)^3 \\
 & \text{s.t. } \begin{cases} g_1(x) = 100 - (x_1 - 5)^2 - (x_2 - 5)^2 \leq 0 \\ g_2(x) = -82.81 + (x_1 - 6)^2 + (x_2 - 5)^2 \leq 0 \\ 13 \leq x_1 \leq 100, 0 \leq x_2 \leq 100 \end{cases} \\
 \text{F3: } & \min f(x) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2 \\
 & \text{s.t. } \begin{cases} g_1(x) = 4.84 - x_1^2 - (x_2 - 2.5)^2 \leq 0 \\ g_2(x) = -4.84 + (x_1 - 0.05)^2 + (x_2 - 2.5)^2 \leq 0 \\ 0 \leq x_1 \leq 6, 0 \leq x_2 \leq 6 \end{cases} \\
 \text{F4: } & \min f(x) = \sin^3(2\pi x_1) \sin^3(2\pi x_2) \\
 & \text{s.t. } \begin{cases} g_1(x) = x_1^2 + x_2 + 1 \leq 0 \\ g_2(x) = 1 - x_1 + (x_2 - 4)^2 \leq 0 \\ 0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10 \end{cases} \\
 \text{F5: } & \min f(x) = x_1^2 + x_2^2 \\
 & \text{s.t. } \begin{cases} g_1(x) = x_1 + x_2 - 1 \leq 0 \\ g_2(x) = x_1 + x_2^2 - 1 \leq 0 \\ g_3(x) = x_1^2 + x_2^2 - 9 \leq 0 \end{cases}
 \end{aligned}$$

5 Conclusion

According to the deficiency of AFSA, that the artificial fishes walk around aimlessly and randomly or gather in non-global optimal points, artificial fish swarm optimization algorithm based on mixed crossover strategy is presented, in order to solve the nonlinear constrained optimization problems. The genetic operation of mixed crossover strategy and the Gaussian mutation operation are used in the algorithm, which can jump out the local optimization and avoid the limitation of early maturity when the optimal that the algorithm solves does continuously not change or change little. Based on test verification for typical functions, it is shown that new algorithm has some better performance such as fast convergence and high precision, and can solve very well this kind of optimization problems.

However, in order to improve the precision and the convergence speed of this kind of problems, AFSA will be fused with other algorithms, and some concepts will be introduced into AFSA such as multi-population, synergetic algorithm and so on, and how to use these to solve actual problems, so these will to be done in the next step research work.

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