Observer-Based Adaptive Neural Networks Control of Nonlinear Pure Feedback Systems with Hysteresis

Yongming Li[1,2,*], Shaocheng Tong[1], and Tieshan Li[2]

[1] College of Science, Liaoning University of Technology, Jinzhou, Liaoning, 121001, P.R. China
[2] Navigation College, Dalian Maritime University, Dalian, Liaoning, 116026, P.R. China
l_y_m_2004@163.com, jztsc@sohu.com, tieshanli@126.com

Abstract. In this paper, the problem of adaptive neural output feedback control is investigated for a class of uncertain nonlinear pure feedback systems with unknown backlash-like hysteresis. In the design, RBF neural networks are used to approximate the nonlinear functions of systems, and a neural state observer is designed to estimate the unmeasured states. By utilizing the neural state observer, and combining the backstepping technique with adaptive control design, an observer-based adaptive neural output feedback control approach is developed. It is proved that the proposed control approach can guarantee that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded (SUUB), and both observer error and tracking error can converge to a small neighborhood of the origin.

Keywords: Output feedback, adaptive neural control, backlash-like hysteresis, backstepping design, stability analysis.

1 Introduction

In the past decades, the control of nonlinear systems preceded by hysteresis has been a challenging and yet rewarding problem. The main reason is that hysteresis can be encountered in a wide range of physical systems and devices [1]. On the other hand, since the hysteresis nonlinearity is non-differentiable, the system performance is often severely deteriorated and usually exhibits undesirable inaccuracies or oscillations and even instability [2].

Recently, in order to control uncertain nonlinear systems with unknown backlash-like hysteresis, many adaptive controllers have been developed by backstepping technique. For example, [3-4] proposed adaptive state feedback control designs for a class of uncertain nonlinear systems with unknown backlash-like hysteresis, while, [5] proposed an adaptive fuzzy output feedback controller for

* Corresponding author.

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a class of uncertain nonlinear systems preceded by unknown backlash-like hysteresis. However, the backstepping control methods in [3]-[5] all focus on the uncertain nonlinear systems in strict-feedback form, there are few results available in the literature on the nonlinear systems in pure-feedback form. As stated in [6], a nonlinear pure-feedback system has no affine appearance of the state variables to be used as virtual controls and the actual control just like a strict-feedback nonlinear systems, which makes the backstepping control design and the stability of the closed-loop system are more difficult and challenging. Motivated by the above observations, an adaptive neural output-feedback control approach is presented for a class of uncertain nonlinear pure feedback systems, preceded by unknown backlash-like hysteresis and without the measurements of the states.

2 Problem Formulations and Preliminaries

Consider a class of SISO n-th order nonlinear systems in the following form:

$$\begin{cases} \dot{x}_1 = F_1(\underline{x}_2) \\ \dot{x}_2 = F_2(\underline{x}_3) \\ \quad\vdots \\ \dot{x}_{n-1} = F_{n-1}(\underline{x}_n) \\ \dot{x}_n = F_n(\underline{x}_n) + \phi(v) \\ y = x_1 \end{cases} \tag{1}$$

where $\underline{x}_i = [x_1, \cdots, x_i]^{\mathrm{T}} \in R^i$, $(i = 1, \cdots, n)$ is the state vector of the system, and $y \in R$ is the output, respectively. $F_i(\cdot)$ is an unknown smooth nonlinear function; $v \in R$ is the control input and $\phi(v)$ denotes hysteresis type of nonlinearity. This paper assumes that the states of the system (1) are unknown and only the output y is available for measurement.

According to [2], the control input v and the hysteresis type of nonlinearity $\phi(v)$ in system (1) can be described by

$$\frac{d\phi}{dt} = \alpha \left|\frac{dv}{dt}\right| (cv - \phi) + B_1 \frac{dv}{dt} \tag{2}$$

where a, c and B_1 are constants, satisfying $c > B_1$. Let

$$\begin{cases} F_i(\underline{x}_{i+1}) = f_i(\underline{x}_i, x_{i+1}) + x_{i+1}, i = 1, 2, \ldots, n-2 \\ F_{n-1}(\underline{x}_n) = f_{n-1}(\underline{x}_{n-1}, \frac{1}{c}x_n) + \frac{1}{c}x_n \\ F_n(\underline{x}_n) = f_n(\underline{x}_{n-1}, \frac{1}{c}x_n) \end{cases}$$

The nonlinear pure-feedback system (1) is equivalent to the following system

$$\begin{cases} \dot{x}_i = f_i(\underline{x}_i, x_{i+1}) + x_{i+1}, i = 1, 2, \ldots, n-2 \\ \dot{x}_{n-1} = f_{n-1}(\underline{x}_{n-1}, \frac{1}{c}x_n) + \frac{1}{c}x_n \\ \dot{x}_n = f_n(\underline{x}_{n-1}, \frac{1}{c}x_n) + \varphi(v) \\ y = x_1 \end{cases} \tag{3}$$

Based on the analysis in [2], (2) can be solved explicitly as

$$\begin{aligned} &\phi(v) = cv(t) + d_1(v), \\ &d_1(v) = [\phi_0 - cv_0]e^{-\alpha(v-v_0)\mathrm{sgn}\dot{v}} + e^{-\alpha v \mathrm{sgn}\dot{v}} \int_{v_0}^{v} [B_1 - c]e^{\alpha\eta\mathrm{sgn}\dot{v}} d\eta \end{aligned} \tag{4}$$

where $v(0) = v_0$ and $\phi(v_0) = \phi_0$.

Based on above solution it is shown in [2] that $d_1(v)$ is bounded. Thus using (4), (3) can be reformulated as

$$\begin{cases} \dot{x}_i = f_i(\underline{x}_i, x_{i+1}) + x_{i+1}, i = 1, 2, \ldots, n-2 \\ \dot{x}_{n-1} = f_{n-1}(\underline{x}_{n-1}, \frac{1}{c}x_n) + \frac{1}{c}x_n \\ \dot{x}_n = f_n(\underline{x}_{n-1}, \frac{1}{c}x_n) + cv(t) + d_1(v) \\ y = x_1 \end{cases} \tag{5}$$

Define

$$\begin{cases} \chi_i = x_i, i = 1, 2, \ldots, n-1 \\ \chi_n = \frac{1}{c}x_n \end{cases} \tag{6}$$

From (5) and (6), we have

$$\begin{cases} \dot{\chi}_i = f_i(\underline{\chi}_i, \chi_{i+1}) + \chi_{i+1}, i = 1, 2, \ldots, n-1 \\ \dot{\chi}_n = \frac{1}{c}f_n(\underline{\chi}_n) + v(t) + \frac{1}{c}d_1(v) \\ y = \chi_1 \end{cases} \tag{7}$$

Assumption 1. There exists a known constant L_i such that

$$\left| f_i(\underline{\chi}_i) - f_i(\underline{\hat{\chi}}_i) \right| \le L_i \left\| \underline{\chi}_i - \underline{\hat{\chi}}_i \right\|, \; i = 1, 2, \ldots, n$$

where $\underline{\hat{\chi}}_i = [\hat{\chi}_1, \hat{\chi}_2, \ldots, \hat{\chi}_i]^{\mathrm{T}}$ is the estimate of $\underline{\chi}_i = [\chi_1, \chi_2, \ldots, \chi_i]^{\mathrm{T}}$.

Control Objective. Our control objective is to design an adaptive neural networks output control controller such that all the signals involved in the closed-loop system are bounded, the observer error is as small as the desired, and the output y tracks the reference signal $y_r(t)$ within a neighborhood of zero.

Rewrite (7) as

$$\begin{cases} \dot{\chi}_1 = f_1(\hat{\chi}_1, \hat{\chi}_{2,f}) + \chi_2 + \Delta f_1 \\ \dot{\chi}_2 = f_2(\underline{\hat{\chi}}_2, \hat{\chi}_{3,f}) + \chi_3 + \Delta f_2 \\ \quad\vdots \\ \dot{\chi}_{n-1} = f_{n-1}(\underline{\hat{\chi}}_{n-1}, \hat{\chi}_{n,f}) + \chi_n + \Delta f_{n-1} \\ \dot{\chi}_n = \frac{1}{c} f_n(\underline{\hat{\chi}}_n) + v(t) + \frac{1}{c} d_1(v) + \frac{1}{c}\Delta f_n \\ y = \chi_1 \end{cases} \tag{8}$$

where $\Delta f_i = f_i(\underline{\chi}_i, \chi_{i+1}) - f_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f})$, $i = 1, 2, \ldots, n-1$, $\Delta f_n = f_n(\underline{\chi}_n) - f_n(\underline{\hat{\chi}}_n)$; $\underline{\hat{\chi}}_i$ is the estimates of $\underline{\chi}_i$, $\hat{\chi}_{i,f}$ is the filtered signal defined by [6]

$$\hat{\chi}_{i,f} = H_L(s)\hat{\chi}_i \tag{9}$$

where $H_L(s)$ is a Butterworth low-pass filter (LPF), the corresponding filter parameters of Butterworth filters with the cutoff frequency $\omega_C = 1\text{rad/s}$ for different values of n.

RBF neural networks are universal approximators, i.e., they can approximate any smooth function on a compact space, thus we can assume that the nonlinear terms in (8) can be approximated as

$$\hat{f}_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f} \,|\theta_i) = \theta_i^{\mathrm{T}} \varphi_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}),\ 1 \le i \le n \tag{10}$$

where $\underline{\hat{\chi}}_{n+1,f} = 0$. The optimal parameter vector θ_i^* is defined as

$$\theta_i^* = \arg\min_{\theta_i \in \Omega_i} [\sup_{(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}) \in U_i} \left| \hat{f}_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f} \,|\theta_i) - f_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}) \right|]$$

where Ω_i and U_i are compact regions for θ_i and $(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f})$, respectively. Also the minimum approximation error ε_i is defined as

$$\varepsilon_i = f_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}) - \hat{f}_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f} \,|\theta_i^*),\ \varepsilon_n = \frac{1}{c} f_n(\underline{\hat{\chi}}_n) - \hat{f}_n(\underline{\hat{\chi}}_n \,|\theta_n^*) \tag{11}$$

Assumption 2. There exist unknown constants ε_i^* and τ_i ($\tau_n = 0$), such that $|\varepsilon_i| \le \varepsilon_i^*$ and $|\hat{\chi}_{i+1} - \hat{\chi}_{i+1,f}| \le \tau_i$, $i = 1, 2, \ldots, n$.

By (10) and (11), System (8) can be rewritten as

$$\begin{cases} \dot{\chi}_1 = \theta_1^{*\mathrm{T}} \varphi_1(\hat{\chi}_1, \hat{\chi}_{2,f}) + \varepsilon_1(\hat{\chi}_1, \hat{\chi}_{2,f}) + \chi_2 + \Delta f_1 \\ \dot{\chi}_2 = \theta_2^{*\mathrm{T}} \varphi_2(\underline{\hat{\chi}}_2, \hat{\chi}_{3,f}) + \varepsilon_2(\underline{\hat{\chi}}_2, \hat{\chi}_{3,f}) + \chi_3 + \Delta f_2 \\ \quad\vdots \\ \dot{\chi}_{n-1} = \theta_{n-1}^{*\mathrm{T}} \varphi_{n-1}(\underline{\hat{\chi}}_{n-1}, \hat{\chi}_{n,f}) + \varepsilon_{n-1}(\underline{\hat{\chi}}_{n-1}, \hat{\chi}_{n,f}) + \chi_n + \Delta f_{n-1} \\ \dot{\chi}_n = \theta_n^{*\mathrm{T}} \varphi_n(\underline{\hat{\chi}}_n) + \varepsilon_n(\underline{\hat{\chi}}_n) + v(t) + \frac{1}{c} d_1(v) + \frac{1}{c}\Delta f_n \\ y = \chi_1 \end{cases} \tag{12}$$

3 Neural State Observer Design

Note that the states $x_2, x_3, \ldots, x_{n-1}$ and x_n in system (1) are not available for feedback, therefore, a state observer should be established to estimate the states, and then neural networks adaptive output feedback control scheme is investigated.

In this paper, a neural state observer is designed for (12) as follows

$$\begin{cases} \dot{\hat{\chi}}_1 = \hat{\chi}_2 + \theta_1^{\mathrm{T}} \varphi_1(\underline{\hat{\chi}}_1, \hat{\chi}_{2,f}) + k_1(y - \hat{\chi}_1) \\ \dot{\hat{\chi}}_2 = \hat{\chi}_3 + \theta_2^{\mathrm{T}} \varphi_2(\underline{\hat{\chi}}_2, \hat{\chi}_{3,f}) + k_2(y - \hat{\chi}_1) \\ \quad \vdots \\ \dot{\hat{\chi}}_{n-1} = \hat{\chi}_n + \theta_{n-1}^{\mathrm{T}} \varphi_{n-1}(\underline{\hat{\chi}}_{n-1}, \hat{\chi}_{n,f}) + k_{n-1}(y - \hat{\chi}_1) \\ \dot{\hat{\chi}}_n = v(t) + \theta_n^{\mathrm{T}} \varphi_n(\underline{\hat{\chi}}_n) + k_n(y - \hat{\chi}_1) \\ \hat{y} = \hat{\chi}_1 \end{cases} \tag{13}$$

Rewriting (13) in the following form:

$$\begin{cases} \dot{\hat{\chi}} = A\hat{\chi} + Ky + \bar{F} + E_n v(t) \\ \hat{y} = E_1^{\mathrm{T}} \hat{\chi} \end{cases} \tag{14}$$

where $\hat{\chi} = [\hat{\chi}_1, \cdots, \hat{\chi}_n]^{\mathrm{T}}$, $A = \begin{bmatrix} -k_1 & \\ \vdots & I_{n-1} \\ -k_n & \cdots 0 \end{bmatrix}$, $\bar{F} = [\theta_1^{\mathrm{T}} \varphi_1(\hat{\chi}_1, \hat{\chi}_{2,f}), \cdots, \theta_n^{\mathrm{T}} \varphi_n (\underline{\hat{\chi}}_n)]^{\mathrm{T}}$, $K = [k_1, \cdots, k_n]^{\mathrm{T}}$, $E_1^{\mathrm{T}} = [1, 0, \cdots, 0]$ and $E_n^{\mathrm{T}} = [0, \cdots, 0, 1]$.

The coefficient k_i is chosen such that the polynomial $p(s) = s^n + k_1 s^{n-1} + \cdots + k_{n-1}s + k_n$ is a Hurwitz. Thus, given a $Q^{\mathrm{T}} = Q > 0$, there exists a positive definite matrix $P^{\mathrm{T}} = P > 0$ such that

$$A^{\mathrm{T}} P + PA = -Q \tag{15}$$

Let $e = \chi - \hat{\chi} = [e_1, \cdots, e_n]^{\mathrm{T}}$ be observer error, then from (11)-(12), we have the observer errors equation

$$\dot{e} = Ae + \varepsilon + d + \Delta f + \tilde{\Theta} \tag{16}$$

where $\Delta f = [\Delta f_1, \cdots, 1/c\Delta f_n]^{\mathrm{T}}$, $\varepsilon = [\varepsilon_1, \cdots, \varepsilon_n]^{\mathrm{T}}$ and $d = [0, \cdots, 0, 1/cd_1(v)]^{\mathrm{T}}$; $\tilde{\Theta} = [\tilde{\theta}_1^{\mathrm{T}} \varphi_1(\underline{\hat{\chi}}_1, \hat{\chi}_{2,f}), \cdots, \tilde{\theta}_n^{\mathrm{T}} \varphi_n(\underline{\hat{\chi}}_n)]^{\mathrm{T}}$ and $\tilde{\theta}_i = \theta_i^* - \theta_i$ $(i = 1, \ldots, n)$.

Consider the following Lyapunov candidate V_0 for (16) as

$$V_0 = e^{\mathrm{T}} P e \tag{17}$$

The time derivative of V_0 along the solutions of (16) is

$$\dot{V}_0 = -e^{\mathrm{T}} Q e + 2e^{\mathrm{T}} P(\varepsilon + d + \Delta f + \tilde{\Theta}) \tag{18}$$

Using Young's inequality, Assumptions 1 and 1, we have

$$2e^{\mathrm{T}}P\varepsilon + 2e^{\mathrm{T}}Pd \le 2\|e\|^2 + \|P\|^2\|\varepsilon^*\|^2 + \|P\|^2\|d_1^*\|^2 \tag{19}$$

$$2e^{\mathrm{T}}P\tilde{\Theta} \le \|e\|^2 + \|P\|^2 \sum_{l=1}^{n} \tilde{\theta}_l^T \tilde{\theta}_l \tag{20}$$

$$2e^{\mathrm{T}}P\Delta f \le \|e\|^2 + \|P\|^2 (\sum_{i=1}^{n} L_i^2 \|e\|^2 + \sum_{i=1}^{n-1} L_i^2 \tau_i^2) \le r_0\|e\|^2 + M_0 \tag{21}$$

where $r_0 = 1 + \|P\|^2 \sum\limits_{i=1}^{n} L_i^2$, $M_0 = \|P\|^2 \sum\limits_{i=1}^{n-1} L_i^2 \tau_i^2$, $\varepsilon^* = [\varepsilon_1^*, \cdots, \varepsilon_n^*]^{\mathrm{T}}$ and $|d_1| \le d_1^*$. with d_1^* is a positive unknown constant. Substituting the equations (19)-(21) into (18), we obtain

$$\dot{V}_0 \le -e^{\mathrm{T}}Qe + (r_0+3)\|e\|^2 + \|P\|^2\|\varepsilon^*\|^2 + \|P\|^2\|d_1^*\|^2 + M_0 + \|P\|^2 \sum_{l=1}^{n} \tilde{\theta}_l^{\mathrm{T}} \tilde{\theta}_l \tag{22}$$

4 Adaptive Neural Control Design and Stability Analysis

The n-step adaptive neural networks backstepping output feedback control design is based on the following change of coordinates:

$$\begin{aligned} z_1 &= y - y_r \\ z_i &= \hat{\chi}_i - \alpha_{i-1}, \quad i = 2, \cdots, n \end{aligned} \tag{23}$$

where α_{i-1} is called the intermediate control function, which will be given later.

Step 1: Consider the following Lyapunov function candidate:

$$V_1 = V_0 + \frac{1}{2}z_1^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1 + \frac{1}{2\bar{\gamma}_1}\tilde{\varepsilon}_1^2 \tag{24}$$

where $\gamma_1 > 0$ and $\bar{\gamma}_1 > 0$ are design constants. $\hat{\varepsilon}_1$ is the estimate of ε_1^* and $\tilde{\varepsilon}_1 = \varepsilon_1^* - \hat{\varepsilon}_1$.

Choose intermediate control function α_1, adaptation functions θ_1 and $\hat{\varepsilon}_1$ as

$$\alpha_1 = -c_1 z_1 - z_1 - \theta_1^{\mathrm{T}} \varphi_1(\hat{\chi}_1, \hat{\chi}_{2,f}) + \dot{y}_r - \hat{\varepsilon}_1 \tanh(z_1/\kappa) \tag{25}$$

$$\dot{\theta}_1 = \gamma_1 \varphi_1(\hat{\chi}_1, \hat{\chi}_{2,f}) z_1 - \sigma\theta_1 \tag{26}$$

$$\dot{\hat{\varepsilon}}_1 = \bar{\gamma}_1 z_1 \tanh(z_1/\kappa) - \bar{\sigma}\hat{\varepsilon}_1 \tag{27}$$

where $c_1 > 0$, $\kappa > 0$, $\sigma > 0$ and $\bar{\sigma} > 0$ are design parameters. We have

$$\dot{V}_1 \le -r_1\|e\|^2 + \|P\|^2 \sum_{l=1}^{n} \tilde{\theta}_l^{\mathrm{T}} \tilde{\theta}_l - c_1 z_1^2 + z_1 z_2 + \frac{\sigma}{\gamma_1}\tilde{\theta}_1^{\mathrm{T}}\theta_1 + \frac{\bar{\sigma}}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + M_1 \tag{28}$$

where $r_1 = \lambda_{\min}(Q) - r_0 - \frac{7}{2} - L_1^2$ and $M_1 = \|P\|^2\|\varepsilon^*\|^2 + \|P\|^2\|d_1^*\|^2 + M_0 + \varepsilon_1^*\kappa' + (L_1\tau_1)^2$.

Step i **(**$i = 2, \cdots, n$**)**: The time derivative of χ_i is

$$\begin{aligned}\dot{z}_i &= \hat{\chi}_{i+1} + H_i - \tfrac{\partial\alpha_{i-1}}{\partial y}e_2 - \tfrac{\partial\alpha_{i-1}}{\partial y}(\tilde{\theta}_1^{\mathrm{T}}\varphi_1(\hat{\chi}_1, \hat{\chi}_{2,f}) + \varepsilon_1 + \Delta f_1) \\ &\quad + \tilde{\theta}_i^{\mathrm{T}}\varphi_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}) - \tilde{\theta}_i^{\mathrm{T}}\varphi_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f})\end{aligned} \tag{29}$$

where $\hat{\chi}_{n+1,f} = 0$ and $H_i = k_i e_1 + \theta_i^{\mathrm{T}}\varphi_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}) - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\chi}_k}\dot{\hat{\chi}}_k - \frac{\partial\alpha_{i-1}}{\partial\hat{\varepsilon}_1}\dot{\hat{\varepsilon}}_1 - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_k}\dot{\theta}_k - \sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)} - \frac{\partial\alpha_{i-1}}{\partial y}[\hat{\chi}_2 + \theta_1^{\mathrm{T}}\varphi_1(\hat{\chi}_1, \hat{\chi}_{2,f})]$.

Consider the following Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\gamma_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i \tag{30}$$

where $\gamma_i > 0$ is a design constant. We can obtain

$$\begin{aligned}\dot{V}_i &\le -r_i\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^{\mathrm{T}}\tilde{\theta}_l + \tfrac{i-1}{2}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1 + \tfrac{1}{2}\sum_{l=1}^{i}\tilde{\theta}_l^{\mathrm{T}}\tilde{\theta}_l - c_1 z_1^2 - \sum_{l=2}^{i-1}c_l z_l^2 \\ &\quad + \sum_{l=1}^{i-1}\tfrac{\sigma}{\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\theta_l + \tfrac{\bar{\sigma}}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + M_i + z_i[z_{i-1} + z_{i+1} + \alpha_i + H_i \\ &\quad + 2(\tfrac{\partial\alpha_{i-1}}{\partial y})^2 z_i)] + \tfrac{1}{\gamma_i}\tilde{\theta}_i^{\mathrm{T}}(\gamma_i z_i\varphi_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}) - \dot{\theta}_i)\end{aligned} \tag{31}$$

where $r_i = r_{i-1} - L_1^2 - \frac{1}{2}$ and $M_i = M_{i-1} + \frac{1}{2}\varepsilon_1^{*2} + L_1^2\tau_1^2$.

Choose α_i ($\alpha_n = v$), adaptation function $\dot{\theta}_i$ as

$$\alpha_i = -z_{i-1} - c_i z_i - 2(\frac{\partial\alpha_{i-1}}{\partial y})^2 z_i - H_i \tag{32}$$

$$\dot{\theta}_i = \gamma_i z_i\varphi_i(\underline{\hat{\chi}}_i, \hat{\chi}_{i+1,f}) - \sigma\theta_i \tag{33}$$

Substituting (32)-(33) into (31), we have

$$\begin{aligned}\dot{V}_i &\le -r_i\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^{\mathrm{T}}\tilde{\theta}_l + \tfrac{i-1}{2}\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1 + \tfrac{1}{2}\sum_{l=1}^{i}\tilde{\theta}_l^{\mathrm{T}}\tilde{\theta}_l - c_1 z_1^2 \\ &\quad - \sum_{l=2}^{i}c_l z_l^2 + \sum_{l=1}^{i}\tfrac{\sigma}{\gamma_l}\tilde{\theta}_l^{\mathrm{T}}\theta_l + \tfrac{\bar{\sigma}}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 + M_i + z_i z_{i+1}\end{aligned} \tag{34}$$

where $z_{n+1} = 0$. For $i = n$, we have

$$\begin{aligned}\dot{V}_n &\le -r_n\|e\|^2 - c_1\tfrac{z_1^2}{k_{b1}^2 - z_1^2} - \sum_{l=2}^{n}c_l z_l^2 - (\tfrac{\sigma}{2\gamma_1} - \tfrac{1}{2} - \|P\|^2 - \tfrac{n-1}{2})\tilde{\theta}_1^{\mathrm{T}}\tilde{\theta}_1 \\ &\quad - \sum_{l=2}^{n}(\tfrac{\sigma}{2\gamma_l} - \tfrac{1}{2} - \|P\|^2)\tilde{\theta}_l^{\mathrm{T}}\tilde{\theta}_l - \tfrac{\bar{\sigma}}{2\bar{\gamma}_1}\tilde{\varepsilon}_1^2 + \lambda\end{aligned} \tag{35}$$

where $\lambda = \sum_{l=1}^{n}\frac{\sigma}{2\gamma_l}\theta_l^{*\mathrm{T}}\theta_l^* + \frac{\bar{\sigma}}{2\bar{\gamma}_1}\varepsilon_1^{*2} + M_n$.

Let $c = \min\{2r_n/\lambda_{\min}(P), 2c_k, 2\gamma_1(\frac{\sigma}{2\gamma_1} - \frac{1}{2} - \|P\|^2 - \frac{n-1}{2}), 2\gamma_l(\frac{\sigma}{2\gamma_l} - \frac{1}{2} - \|P\|^2), \bar{\sigma}\}$, $k = 1, \ldots, n$, $l = 2, \ldots, n$. Then (35) becomes

$$\dot{V}_n \leq -cV_n + \lambda \tag{36}$$

By (36), it can be proved that all the signals in the closed-loop system are SUUB.

5 Conclusions

For a class of uncertain nonlinear pure feedback systems without the measurements of the states and with unknown backlash-like hysteresis, an adaptive neural output feedback control approach has been developed. The proposed control scheme mainly solved three problems. First, the proposed controlled system is feedback nonlinear system. Second, the proposed control scheme does not require that all the states of the system are measured directly. Third, the problem of unknown backlash-like hysteresis can be overcome. It is proved that the proposed control approach can guarantee that all the signals of the closed-loop system are SUUB, and both the observer and the tracking errors can be made as small as desired by appropriate choice of design parameters.

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