Adaptive Synchronization for Stochastic Markovian Jump Neural Networks with Mode-Dependent Delays

Cheng-De Zheng*, Xixi Lv, and Zhanshan Wang

School of Science, Dalian Jiaotong University, Dalian, 116028, P.R. China
School of Information Science and Engineering, Northeastern University, Shenyang, 110004, P.R. China

Abstract. This paper studies the adaptive synchronization problem for a kind of stochastic Markovian jump neural networks with mode-dependent and unbounded distributed delays. By virtue of the Lyapunov stability theory and the stochastic analysis technique, a generalized LaSalle-type invariance principle for stochastic Markovian differential delay equations is utilized to investigate the globally almost surely asymptotical stability of the error dynamical system in the mean-square sense.

Keywords: Adaptive synchronization, stochastic perturbation, mode-dependent delay, Markovian jump.

1 Introduction

Since the pioneering work [5] of Pecora and Carroll in 1990, control and synchronization of chaotic systems have become an important topic during the past decades. There exist many benefits of having synchronization or chaos synchronization in some engineering applications, such as secure communication, chaos generators design, chemical reactions, biological systems, information science, and so on. Many excellent papers and monographs on synchronization of chaotic systems with or without time delays have been published [2,6]. Variety of alternative schemes for ensuring the synchronization have been proposed, such as adaptive design control, feedback control, complete synchronization control, impulsive control, anti-synchronization control, and projective synchronization control. Because of the finite switching speed of amplifiers and the inherent communication time of neurons, time delays are frequently encountered in various engineering, biological, and economic systems. It has been revealed that time delay may cause periodic oscillations, bifurcation and chaotic attractors and

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so on. Thus synchronization of delayed chaotic neural networks has become an important research area.

Motivated by the preceding discussions, our objective in this paper is to study the adaptive synchronization problem for a kind of stochastic Markovian jump neural networks with mode-dependent and unbounded distributed delays. By employing the Lyapunov stability theory, by virtue of stochastic analysis, a generalized LaSalle-type invariance principle for stochastic Markovian differential delay equations is utilized to investigate the globally almost surely asymptotical stability of the error dynamical system in the mean-square sense.

Notation. Throughout this paper, let W^T, W^{-1} denote the transpose and the inverse of a square matrix W, respectively. Let $W > 0 (< 0)$ denote a positive (negative) definite symmetric matrix, I denotes the identity matrix of appropriate dimension, the symbol "*" denotes a block that is readily inferred by symmetry. The shorthand $\text{col}\{M_1, M_2, ..., M_k\}$ denotes a column matrix with the matrices $M_1, M_2, ..., M_k$. $\text{diag}\{\cdot\}$ stands for a diagonal or block-diagonal matrix. For $\tau > 0, \mathcal{C}\big([-\tau, 0]; \mathbb{R}^n\big)$ denotes the family of continuous functions ϕ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $||\phi|| = \sup_{-\tau \le s \le 0} |\phi(s)|$. Moreover, let $(\Omega, \mathbb{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathbb{F}_t\}_{t \ge 0}$ satisfying the usual conditions and $\mathbb{E}\{\cdot\}$ representing the mathematical expectation. Denote by $\mathcal{C}^p_{\mathbb{F}_0}\big([-\tau, 0]; \mathbb{R}^n\big)$ the family of all bounded, $\mathbb{F}_0$-measurable, $\mathcal{C}\big([-\tau, 0]; \mathbb{R}^n\big)$-valued random variables $\xi = \{\xi(s) : -\tau \le s \le 0\}$ such that $\sup_{-\tau \le s \le 0} \mathbb{E}|\xi(s)|^p < \infty$. $|| \cdot ||$ stands for the Euclidean norm; Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 Problem Description and Preliminaries

We consider the following neural networks with mixed time delays

$$
\begin{aligned}
\mathrm{d}x(t) =& \Big[- C(\eta(t))x(t) + A(\eta(t))\hat{f}(x(t)) \\
& + B(\eta(t))\hat{f}(x(t - \tau(t, \eta(t))) + D(\eta(t)) \int_{-\infty}^{t} K(t-s)\hat{f}(x(s))\mathrm{d}s + J\Big]\mathrm{d}t, \qquad (1) \\
x(t) =& \varphi_1(t), \qquad t \in (-\infty, 0],
\end{aligned}
$$

where $x(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \in \mathbb{R}^n$ denotes the state of the i-th neuron at time t, the positive diagonal matrix $C(\eta(t))$ is the self-feedback term, $A(\eta(t)), B(\eta(t)), D(\eta(t)) \in \mathbb{R}^{n \times n}$ are the interconnection matrices representing the weight coefficients of the neurons. $\hat{f}(x(t)) = [\hat{f}_1(x_1(t)), \hat{f}_2(x_2(t)), ..., \hat{f}_n(x_n(t))]^T \in \mathbb{R}^n$ denotes the neural activation function, the bounded function $\tau(t, \eta(t))$ represents unknown time-varying delay with $0 \le \tau(t, \eta(t)) \le \bar{\tau}(\eta(t)) \le \bar{\tau}, \dot{\tau}(t, \eta(t)) \le \tau_d(\eta(t)) \le \tau_d$, where $\bar{\tau}(\eta(t)), \bar{\tau}$ are positive scalars. $J = [J_1, J_2, ..., J_n]^T$ is an external input, $\varphi_1(t)$ is a real-valued initial vector function that is continuous on the interval $(-\infty, 0]$. $K(t-s) = \mathrm{diag}\{k_1(t-s), k_2(t-s), ..., k_n(t-s)\}$ denotes the delay kernel. It is assumed that $k_i(\cdot)$ is a real value non-negative continuous function defined in $[0, \infty)$ satisfying

$$\int_0^{\infty} k_i(s)\mathrm{d}s = 1,\ i = 1, 2, ..., n.$$

$\{\eta(t), t \geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in finite set $\mathcal{N} = \{1, 2, ..., N\}$ with given probability space $(\Omega, \mathbb{F}, \mathbb{P})$ and the initial model η_0. Let $\Pi = [\pi_{ij}]_{N\times N}$ denote the transition rate matrix with transition probability:

$$\mathbb{P}(\eta(t+\delta) = j|\eta(t) = i) = \begin{cases} \pi_{ij}\delta + o(\delta), & i \neq j, \\ 1 + \pi_{ii}\delta + o(\delta), & i = j, \end{cases}$$

where $\delta > 0, \lim_{\delta\to 0^+} \frac{o(\delta)}{\delta} = 0$ and π_{ij} is the transition rate from mode i to mode j satisfying $\pi_{ij} \geq 0$ for $i \neq j$ with

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}, \qquad i, j \in \mathcal{N}.$$

For convenience, each possible value of $\eta(t)$ is denoted by $i (i \in \mathcal{N})$ in the sequel. Then we have

$$C_i = C(\eta(t)),\quad A_i = A(\eta(t)),\quad B_i = B(\eta(t)),\quad D_i = D(\eta(t)),\quad \tau_i(t) = \tau(t, \eta(t)).$$

Throughout this paper, the following assumption is made on the neuron activation functions:

Assumption 1. Each neural activation function $\hat{f}_j(\cdot)$ is bounded and there exist real constants σ_j^-, σ_j^+ such that

$$\sigma_j^- \leq \frac{\hat{f}_j(\xi) - \hat{f}_j(\zeta)}{\xi - \zeta} \leq \sigma_j^+, \qquad \forall\ \xi, \zeta \in \mathbb{R},\ \xi \neq \zeta,\quad j = 1, 2, ..., n.$$

For notational simplicity, we denote
$\Sigma_1 = \mathrm{diag}\{\sigma_1^-, \sigma_2^-, ..., \sigma_n^-\},\ \Sigma_2 = \mathrm{diag}\{\sigma_1^+, \sigma_2^+, ..., \sigma_n^+\}$.

In order to observe the synchronization behavior of system (1), we construct the response system as follows

$$\begin{aligned} \mathrm{d}y(t) =& \Big[- C_i y(t) + A_i \hat{f}(y(t)) + B_i \hat{f}(y(t-\tau_i(t))) + D_i \int_{-\infty}^{t} K(t-s)\hat{f}(y(s))\mathrm{d}s \\ & + J + u(t)\Big]\mathrm{d}t + \upsilon_i(t, y(t) - x(t), y(t-\tau_i(t)) - x(t-\tau_i(t)))\mathrm{d}\omega(t), \\ y(t) =& \varphi_2(t), \qquad\qquad t \in (-\infty, 0], \end{aligned} \tag{2}$$

where $u(t)$ is an appropriate control input that will be designed in order to obtain a certain control objective. $\omega(t)$ is a one-dimensional Brown motion defined on a complete probability space $(\Omega, \mathbb{F}, \mathbb{P})$ with a natural filtration $\{\mathbb{F}_t\}_{t\geq 0}$, and $\upsilon_i : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the noise intensity vector. This type of stochastic perturbation can be regarded as a result from the occurrence of external random fluctuation and other probabilistic causes.

Let $e(t) = y(t) - x(t)$ be the error state, it yields the synchronization error dynamical systems as follows:

$$\begin{aligned} \mathrm{d}e(t) =& \Big[- C_i e(t) + A_i f(e(t)) + B_i f(e(t-\tau_i(t))) \\ &+ D_i \int_{-\infty}^{t} K(t-s) f(e(s)) \mathrm{d}s + Z(t)e(t) \Big] \mathrm{d}t + \upsilon_i(t, e(t), e(t-\tau_i(t))) \mathrm{d}\omega(t), \\ \doteq& \rho_i(t)\mathrm{d}t + \upsilon_i(t)\mathrm{d}\omega(t), \qquad (3) \\ e(t) =& \varphi(t), \qquad t \in (-\infty, 0], \end{aligned}$$

where $f(e(t)) = \hat{f}(y(t)) - \hat{f}(x(t)),\ \varphi(t) = \varphi_2(t) - \varphi_1(t)$. From Assumption 1, it is easy to derive that

$$f_j(0) = 0, \quad \sigma_j^- \leq \frac{f_j(s)}{s} \leq \sigma_j^+, \qquad \forall\ s \in \mathbb{R},\ s \neq 0, \quad j = 1, 2, ..., n. \qquad (4)$$

Furthermore, we make the following assumption:

Assumption 2. The noise intensity vector is assumed to be of the form:

$$\upsilon_i(t) = E_i e(t) + F_i e(t - \tau_i(t)), \qquad (5)$$

where E_i, F_i are known real matrices.

Instead of the usual linear feedback, in this paper, we consider the following feedback controller:

$$u(t) = Z(t)e(t), \qquad (6)$$

where the feedback strength $Z(t) = \mathrm{diag}\{z_1(t), z_2(t), ..., z_n(t)\}$ is updated by the following law:

$$\dot{z}_j(t) = -\gamma_j e_j^2(t), \qquad (7)$$

where $\gamma_j > 0$ is an arbitrary constant, $j = 1, 2, ..., n$.

3 Main Result

As well known, Itô's formula plays important role in the stability analysis of stochastic Markovian systems and we cite some related results here [1]. Consider a general stochastic Markovian delay system

$$\mathrm{d}z(t) = f(t, z(t), z(t-\kappa), \eta(t))\mathrm{d}t + g(t, z(t), z(t-\kappa), \eta(t))\mathrm{d}\omega(t), \qquad (8)$$

on $t \geq t_0$ with initial value $z(t_0) = z_0 \in \mathbb{R}^n$, where $\kappa > 0$ is time delay, $f : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{N} \to \mathbb{R}^n$ and $g : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{N} \to \mathbb{R}^{n+m}$. Let $\mathcal{C}^{2,1}\big(\mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{N}, \mathbb{R}^+\big)$ denote the family of all nonnegative functions $V(t, z, v, \eta(t))$ on $\mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{N}$ which are continuously twice differentiable in z, v and once differentiable in t. Let $\pounds$ be the weak infinitesimal generator of the random process $\{z(t), \eta(t)\}_{t \geq t_0}$ along the system (8) (see [3]), i.e.

$$\begin{aligned}\pounds V(t, z_t, v_t, i) := \lim_{\delta \to 0^+} \big[\mathbb{E}\big\{V(t+\delta, z_{t+\delta}, v_{t+\delta}, \eta(t+\delta))\big| z_t, v_t, \eta(t) = i\big\} \\ - V(t, z_t, v_t, \eta(t) = i)\big], \quad (9)\end{aligned}$$

then, by the Dynkin's formula, one can get

$$\mathbb{E}V(t, z(t), v(t), i) = \mathbb{E}V(t_0, z(t_0), v(t_0), i) + \mathbb{E}\int_{t_0}^{t} \pounds V(s, z(s), v(s), i)\mathrm{d}s.$$

Similar to Lemma 1 of [4], we can obtain a generalized LaSalle-type invariance principle for stochastic Markovian differential delay equations (8) stated as follows.

Lemma 1. Assume that system (8) exists a unique solution $z(t, \xi)$ on $t > 0$ for any given initial data $\{z(\theta) : -\kappa \leq \theta \leq 0\} = \xi \in \mathcal{C}^p_{\mathbb{F}_0}\big([-\tau, 0]; \mathbb{R}^n\big)$, moreover, both $f(t, z, v, \eta(t))$ and $g(t, z, v, \eta(t))$ are locally bounded in (z, v) and uniformly bounded in t. If there are a function $V \in \mathcal{C}^{2,1}\big(\mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{N}, \mathbb{R}^+\big), \chi \in L^1(\mathbb{R}^+, \mathbb{R}^+)$ and $\psi_1, \psi_2 \in \mathcal{C}\big(\mathbb{R}^n, \mathbb{R}^+\big)$ such that

$$\begin{aligned}&\mathbb{E}\pounds V(t, z, v, \eta(t)) \leq \chi(t) - \psi_1(z) + \psi_2(v), \quad (t, z, v, \eta(t)) \in \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{N}, \\ &\psi_1(z) \geq \psi_2(z), \quad \forall z \neq 0, \\ &\lim_{||z|| \to \infty} \inf_{0 \leq t < \infty} V(t, z, v, \eta(t)) = \infty.\end{aligned}$$

Then

$$\lim_{t \to \infty} z(t, \xi) = 0 \quad a.s.$$

for every $\xi \in \mathcal{C}^p_{\mathbb{F}_0}\big([-\tau, 0]; \mathbb{R}^n\big)$.

In order to get the main result, we propose the following lemma:

Lemma 2. For each $i \in \mathcal{N}$, we have the following equalities

$$\begin{aligned}&\pounds\left\{\int_{t-\tau(t,\eta(t))}^{t} x(s)^T Q(\eta(t)) x(s)\mathrm{d}s\right\} \\ =& x(t)^T Q_i x(t) - (1 - \dot{\tau}_i(t)) x(t - \tau_i(t))^T Q_i x(t - \tau_i(t)) \\ &+ \sum_{j=1}^{N} \pi_{ij} \left\{\int_{t-\tau_i(t)}^{t} x(s)^T Q_j x(s)\mathrm{d}s + \tau_j(t) x(t - \tau_i(t))^T Q_i x(t - \tau_i(t))\right\}, \quad (10)\end{aligned}$$

$$\begin{aligned}\pounds&\left\{\int_{t-\tau_i(t)}^{t}\int_{\theta}^{t}x(s)^TRx(s)\mathrm{d}s\mathrm{d}\theta\right\}\\=&\tau_i(t)x(t)^TRx(t)-(1-\dot{\tau}_i(t))\int_{t-\tau_i(t)}^{t}x(s)^TRx(s)\mathrm{d}s\\&+\sum_{j=1}^{N}\pi_{ij}\tau_j(t)\int_{t-\tau_i(t)}^{t}x(s)^TRx(s)\mathrm{d}s.\end{aligned}\tag{11}$$

Now, we begin to state our main result.

Theorem 1. *Consider the system (3) satisfying Assumption 1, the drive system (1) and the response system (2) can be synchronized for any* $0\le\tau_i(t)\le\bar{\tau}_i\le\bar{\tau},\quad\dot{\tau}_i(t)\le\tau_{di}<1$, *if there exist symmetric definite positive matrices* $Q_{1i},Q_{3i},Q_{4i},R_1,R_3,R_4,X,H$, *diagonal positive matrices* P_i,S_i,U_i,W *and positive number* α *and any real matrices* Q_{2i},R_2 *satisfying the following inequalities*

$$\mathcal{Q}_i\equiv\begin{bmatrix}Q_{1i}&Q_{2i}\\ *&Q_{3i}\end{bmatrix}>0,\tag{12}$$

$$\mathcal{R}\equiv\begin{bmatrix}R_1&R_2\\ *&R_3\end{bmatrix}>0,\tag{13}$$

$$\sum_{j=1}^{N}(\pi_{ij}\mathcal{Q}_j+\pi'_{ij}\bar{\tau}_j\mathcal{R})\le(1-\tau_{di})\mathcal{R},\tag{14}$$

$$\sum_{j=1}^{N}\pi_{ij}Q_{4j}\le R_4,\tag{15}$$

$$\Omega_i=\begin{bmatrix}\Omega_{11i}&E_i^T(P_i+\bar{\tau}H)G_i&H&\Omega_{14i}&P_iB_i&P_iD_i&H\\ *&\Omega_{22i}&0&0&\Omega_{25i}&0&0\\ *&*&-Q_{4i}-H&0&0&0&H\\ *&*&*&\Omega_{44i}&0&0&0\\ *&*&*&*&\Omega_{55i}&0&0\\ *&*&*&*&*&-W&0\\ *&*&*&*&*&*&-H\end{bmatrix}<0,\tag{16}$$

where

$$\begin{aligned}\Omega_{11i}=&-P_iC_i-C_iP_i-2\alpha P_i+E_i^T(P_i+\bar{\tau}H)E_i-H+X\\&+\sum_{j=1}^{N}\pi_{ij}P_j-\Sigma_1\Sigma_2S_i+Q_{1i}+\bar{\tau}_iR_1+Q_{4i}+\bar{\tau}R_4,\\ \Omega_{14i}=&P_iA_i+\frac{1}{2}(\Sigma_1+\Sigma_2)S_i+Q_{2i}+\bar{\tau}_iR_2,\\ \Omega_{22i}=&G_i^T(P_i+\bar{\tau}H)G_i-\Sigma_1\Sigma_2U_i-X-(1-\tau_{di})Q_{1i}+\sum_{j=1}^{N}\pi'_{ij}\bar{\tau}_jQ_{1i},\end{aligned}$$

$$\Omega_{25i} = \frac{1}{2}(\Sigma_1 + \Sigma_2)U_i - (1-\tau_{di})Q_{2i} + \sum_{j=1}^{N} \pi'_{ij}\bar{\tau}_j Q_{2i},$$

$$\Omega_{44i} = W - S_i + Q_{3i} + \bar{\tau}_i R_3, \quad \Omega_{55i} = -U_i - (1-\tau_{di})Q_{3i} + \sum_{j=1}^{N} \pi'_{ij}\bar{\tau}_j Q_{3i},$$

with $\pi'_{ij} = \max\{\pi_{ij}, 0\}$.

Proof. Define the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t,e_t,i) =& e(t)^T P_i e(t) + \sum_{j=1}^{n} w_j \int_0^{\infty} k_j(s) \int_{t-s}^{t} f_j^2(e_j(\theta))\mathrm{d}\theta \mathrm{d}s \\ &+ \int_{t-\tau_i(t)}^{t} \xi(s)^T \mathcal{Q}_i \xi(s)\mathrm{d}s + \int_{t-\tau_i(t)}^{t}\int_{\theta}^{t} \xi(s)^T \mathcal{R}\xi(s)\mathrm{d}s\mathrm{d}\theta \\ &+ \int_{t-\bar{\tau}}^{t} e(s)^T Q_{4i} e(s)\mathrm{d}s + \int_{t-\bar{\tau}}^{t}\int_{\theta}^{t} e(s)^T R_4 e(s)\mathrm{d}s\mathrm{d}\theta \\ &+ \int_{t-\bar{\tau}}^{t}\int_{\theta}^{t} \upsilon_i(s)^T H \upsilon_i(s)\mathrm{d}s\mathrm{d}\theta + \sum_{j=1}^{n} \frac{p_{ji}}{\gamma_j}(z_j(t)+\alpha)^2, \end{aligned}$$

with $\xi(s) = \mathrm{col}\{x(s), f(x(s))\}$, $W = \mathrm{diag}\{w_1, w_2, ..., w_n\}$, and α being a large positive constant which can be determined arbitrary.

Based on Lemma 2, by using the well-known Itô's differential formula, calculating the weak infinitesimal generator along the trajectory of (3) results in

$$\begin{aligned} \pounds V_i =& 2e(t)^T P_i \rho_i(t) + \mathrm{trace}\left[\upsilon_i(t)^T P_i \upsilon_i(t)\right] + \sum_{j=1}^{N} \pi_{ij} e(t)^T P_j e(t) \\ &+ 2\sum_{j=1}^{n} \frac{p_{ji}}{\gamma_j}(z_j(t)+\alpha)\dot{z}_j(t) + \sum_{j=1}^{n} w_j \int_0^{\infty} k_j(s)\left[f_j^2(e_j(t)) - f_j^2(e_j(t-s))\right]\mathrm{d}s \\ &+ \xi(t)^T \mathcal{Q}_i \xi(t) - (1-\dot{\tau}_i(t))\xi(t-\tau_i(t))^T \mathcal{Q}_i \xi(t-\tau_i(t)) \\ &+ \sum_{j=1}^{N} \pi_{ij} \left\{ \int_{t-\tau_i(t)}^{t} \xi(s)^T \mathcal{Q}_j \xi(s)\mathrm{d}s + \tau_j(t)\xi(t-\tau_i(t))^T \mathcal{Q}_i \xi(t-\tau_i(t)) \right\} \\ &+ \tau_i(t)\xi(t)^T \mathcal{R}\xi(t) - (1-\dot{\tau}_i(t)) \int_{t-\tau_i(t)}^{t} \xi(s)^T \mathcal{R}\xi(s)\mathrm{d}s \\ &+ \sum_{j=1}^{N} \pi_{ij}\tau_j(t) \int_{t-\tau_i(t)}^{t} \xi(s)^T \mathcal{R}\xi(s)\mathrm{d}s + e(t)^T Q_{4i} e(t) \\ &- e(t-\bar{\tau})^T Q_{4i} e(t-\bar{\tau}) + \sum_{j=1}^{N} \pi_{ij} \int_{t-\bar{\tau}}^{t} e(s)^T Q_{4j} e(s)\mathrm{d}s + \bar{\tau} e(t)^T R_4 e(t) \\ &- \int_{t-\bar{\tau}}^{t} e(s)^T R_4 e(s)\mathrm{d}s + \bar{\tau}\upsilon_i(t)^T H \upsilon_i(t) - \int_{t-\bar{\tau}}^{t} \upsilon_i(s)^T H \upsilon_i(s)\mathrm{d}s. \end{aligned}$$

According to Assumption 1, by the Leibniz-Newton formula, we get

$$\mathbb{E}\pounds V_i \leq \zeta_i(t)^T \Omega_i \zeta_i(t) - e(t)^T X e(t) + e(t-\tau_i(t))^T X e(t-\tau_i(t)),$$

where

$$\zeta_i(t) = \mathrm{col}\Big\{ e(t), e(t-\tau_i(t)), e(t-\bar{\tau}), f(e(t)), \\ f(e(t-\tau_i(t))), \int_{-\infty}^{t} K(t-s) f(e(s))\mathrm{d}s, \int_{t-\bar{\tau}}^{t} \rho_i(s)\mathrm{d}s \Big\}.$$

The constant α plays an important role in making the matrix Ω_i negative definite. In fact, it can be chosen so big that the matrix Ω_i is negative definite.

From Eq. (16), we have

$$\begin{aligned}\mathbb{E}\pounds V_i &\leq -e(t)^T (X + \lambda_i I) e(t) + e(t-\tau_i(t))^T (X - \lambda_i I) e(t-\tau_i(t)) \\ &\doteq -\psi_1(e(t)) + \psi_2(e(t-\tau_i(t))),\end{aligned}$$

where λ_i denote the largest eigenvalue of the matrix Ω_i. Obviously, $\psi_1(e(t)) > \psi_2(e(t))$ for any $e(t) \neq 0$. Therefore, applying Lemma 1, we can conclude that the two coupled delayed neural networks (1) and (2) can be synchronized for almost every initial data.

4 Conclusion

In this paper, an adaptive feedback controller is proposed for the complete synchronization of stochastic Markovian jump neural networks with mode-dependent and unbounded distributed delays. A generalized LaSalle-type invariance principle for stochastic Markovian differential delay equations is employed to investigate the globally almost surely asymptotical stability of the error dynamical system, that is to say, the complete synchronization can be almost surely achieved.

References

1. Arnold, L.: Stochastic Differential Equations: Theory and Applications. Wiley, New York (1972)
2. Liu, Z., Lv, S., Zhong, S., Ye, M.: pth moment exponential synchronization analysis for a class of stochastic neural networks with mixed delays. Commun. Nonlinear Sci. Numer. Simulat. 15, 1899–1909 (2010)
3. Mao, X.: Exponential stability of stochastic delay interval systems with Markovian switching. IEEE Trans. Autom. Contr. 47(10), 1604–1612 (2002)
4. Mao, X.: A note on the LaSalle-type theorems for stochastic differential delay equations. J. Math. Anal. Appl. 268, 125–142 (2002)
5. Pecora, L., Carroll, T.: Synchronization in chaotic systems. Phys. Rev. Lett. 64, 821–824 (1990)
6. Zhang, Q., Lu, J.: Chaos synchronization of a new chaotic system via nonlinear control. Chaos, Solitons, Fractals 37, 175–179 (2008)