Spectrum Access Game for Cognitive Radio Networks with Incomplete Information

Jerzy Martyna

Institute of Computer Science, Jagiellonian University,
ul. Prof. S. Lojasiewicza 6, 30-348 Cracow, Poland

Abstract. In this paper, the competitive interactions of radio devices dynamically accessing the radio spectrum in the cognitive radio network are studied. The dynamic spectrum access is modelled by a game with incomplete information. The notion of incomplete information means that some players do not completely know the structure of the game. This paper provides a spectrum auction to address the problem of radio channel allocation for cognitive radio networks. The VCG auction to maximise the auctioneer's revenue or maximise social welfare in the spectrum auction is also examined. A dynamic programming algorithm is then applied to solve the spectrum auction problem. Some simulation results are provided.

Keywords: cognitive radio networks, game theory, wireless communication.

1 Introduction

Cognitive radio network is believed to be an effective solution to enhance overall spectrum efficiency. The devices with cognitive radio networks are able to switch between bandwidthss to adapt to varying channel qualities, network congestion, interference, service requirements, etc. [1–3]. Keeping in mind that the Federal Communications Commission (FCC) Spectrum Policy Task Force has published a report [4] in 2002, in which it thoroughly investigates the underutilisation in the radio spectrum. Recent measurements by the FCC show that 70 % of the allocated spectrum in the United States is not utilized. Cognitive radio networks are envisioned to be able to exploit all holes in the spectrum, by means of knowledge of the environment and cognition capability, to adapt their radio parameters accordingly.

Dynamic spectrum access is based on software-defined radio technology, which is proposed to enhance the adaptability and flexibility of wireless transmission. To realize dynamic spectrum access, spectrum management, together with an appropriate model, is required. The spectrum management optimizes to fully utilise spectrum bands. The service provider wishing to increase his profit by increasing the revenue with limited spectrum bands can do so by allowing secondary users (SUs) to access the unused spectrum bands of primary users (PUs). Because SUs cannot provide possible interference under some minimum service constraint to PUs, cognitive radio devices have some constraints to utilize the

A. Kwiecień, P. Gaj, and P. Stera (Eds.): CN 2013, CCIS 370, pp. 232–239, 2013.

spectrum bands of PUs. Game theory is a fundamental technology for spectrum management in these networks.

Researchers have been drawn to explore the dynamic spectrum access system in the papers [5, 6]. Traditionally, dynamic spectrum assignment is proposed in [7] as spectrum broker. Distributed spectrum allocation approaches [8, 9] have been studied to enable efficient spectrum sharing only based on local observations. F. Fu et al. [10] focuses on developing solutions for wireless secondary users to successfully compete with others in limited and time-varying spectrum opportunities based on the auction mechanism. A class of auctions, i.e. the multiunit sealed-bid auction (i.e., the Vickrey auction [11]), is suitable to execute in a deterministic time with an acceptable signalling effort in comparison to the English auction [12].

The Vickrey auction has many weaknesses. For example, the it does not allow for price discovery, meaning that the it does not allow for discovery of the market price if the buyers are unsure of their own valuations. Moreover, sellers may use skill bids to increase profit. A way to omit these weakness has been proposed by E. H. Clarke [13] and T. Groves [14]: the auction mechanism referred to as the VCG auction. However, the VCG auction is vulnerable to bidder collusion and to shill bidding with respect to the buyers. The VCG auction isn't necessarily maximise seller revenues; seller revenues may even be zero.

Recently, in a paper by T. Wysocki and A. Jamalipour [15], the portfolio theory has been applied as a spectrum management tool for QoS management and pricing in cognitive radio networks. This approach incorporates the variance of investment returns, a key measure of economic welfare, into pricing and trading strategies. In order to assess opportunistic spectrum access scenarios in cognitive radios, two oligopoly game models are reformulated by L. C. Cremene and D. Dumitrescu [16] in terms of Cournot and Stackelberg games. A new optimal auction mechanism to determine the assigned frequency bands and prices in the cognitive radio, called the generalized Branco's mechanism, was proposed by Sung Hyun Chun [17]. Nevertheless, these papers have been not devoted to the regulator rights allocation for the primary and the secondary networks.

We make the following key contributions. We have investigated the spectrum access game for cognitive radio networks with incomplete information. Spectrum auctions are suitable for selling the rights to the primary and secondary users on the radio channels. Additionally, we have proposed the VCG action that can be used to maximise the auctioneer's revenue or to maximise social welfare in the spectrum auction. Next, we considered the case in which the bids of a network are independent of the networks it shares a channel with, and provided an optimal dynamic programming algorithm for the access allocation problem. Using simulations, we provided some numerical results that the above algorithm performs optimally in a variety of scenarios.

The rest of this paper is structured as follows. Section 2 provides our system model. Section 3 presents auction framework and is followed by the formulation of the spectrum auction game and solves the auction game by use dynamic programming algorithm. Simulation results are presented in Sect. 4. Section 5 concludes the paper.

2 System Model

Let M be identical orthogonal channels in the cognitive radio network. We recall that a regulator conducts an auction to sell the rights to be the primary and secondary networks on the channels. N bidders participate in the auction, with each bidder being an independent network. Each network can evaluate utilities or valuations that are functions of the number of channels on which they get primary and secondary user rights, how many and which other networks they share these channels with, etc.

The assumption of incomplete information is here related to two different notions: imperfect information and imperfect channel state information (CSI). *Imperfect information* means that a player (device) does not know exactly what action other players take at that point in games. *Imperfect channel state information* means that a player has perfect CSI about its own channel, but it has imperfect CSI about any other device's channel.

Following the approach given by J. C. Harsanyi [18], the Bayesian game can be obtained by introducing some randomness in a strategic game. Suppose that each player knows its own the utility network function and does not know the network utility function of all the other players. In other words, each player knows that there exists a finite set of possible types $\mathcal{T}_\rangle$ for each player. The corresponding type for each player is a random variable that follows a probability distribution known by all the players.

The dynamic spectrum access problem can be modelled as a Bayesian game. In order to give insight into the Bayesian game, we provide the description of this game.

Definition 1 (Bayesian Game). *[18]*

A Bayesian game is completely described by the following set of parameters:

- *A set of $\mathcal{N}$ players, $\mathcal{N} = \{1, \ldots, N\}$.*
- *A finite set of T types of players $\mathcal{T} = \{1, \ldots, T\}$.*
- *A probability density function of the different types of players: $\{f(t) \in [0, 1] \mid \forall t \in \mathcal{T}\}$.*
- *A set of T finite sets of strategies: $\mathcal{S}_\infty, \mathcal{S}_\in, \ldots, \mathcal{S}_\mathcal{T}$ each one for for each type of player.*
- *A set of T utility function $u_i : \mathcal{T} \times \mathcal{S} \to \mathcal{R}_+$ for each type of player.*

The set of types $\mathcal{T}$ corresponds to all the possible probability distributions that can model the channel realisation of each player. It is defined as follows.

Definition 2 (Strategy Set). *The strategy set in the game is defined as $\mathcal{S} = \mathcal{S}_1 \times \ldots \times \mathcal{S}_N$, where S_i is the strategy set of player i and is given by*

$$\mathcal{S}_i \left\{ x_i = (x_{i,1}, \ldots, x_{i,N}) : \quad \forall n \in \{1, \ldots, N\}, \quad x_{i,n} > 0 \;\text{ and }\; \sum_{n=1}^{N} x_{i,n} \leq x_{\max} \right\}$$

where x_i is the valuation of a network i for a channel alocation k.

We assume that $u_i(k)$ be the i-th network utility from the channel allocation $k \in \mathcal{K}$. This value means that since network i will share channels with other networks in the allocation k, the actual utility that network i will derive from an allocation k after the networks start using the allocated channels. In other words, the term network utility $u_i(k)$ should be understood to mean valuation of network i for the channel allocation k.

The valuation of a network i for a channel allocation $k \in \mathcal{K}$ depends on the number of channels on which network i has primary and secondary rights in the allocation k and is expressed by $x_i(k)$. The network utility is given by

$$u_i(k, \tau_i, x_i) = x_i(k) - \tau_i \tag{1}$$

where τ_i is the payment that network i makes to the auctioneer. We assume that the auctioneer determines the channel allocation and each network i makes the payment τ_i to the auctioneer. The social welfare of allocation k is given by

$$w(k) = \sum_{i=1}^{N} u_i(k, \tau_i, x_i) \ . \tag{2}$$

From a noncooperative point of view the goal of each network as the player is to selfishly maximise its own utility function.

It could be stated that this kind of game can be solved by use a Nash equilibrium (NE). Under the assumption of incomplete information there exists a unique NE at which all players use the available channels. Therefore, we can give the following definition.

Definition 3 (Nash Equilibrium). *The NE for the game with incomplete information is the vector* $x^* = \{x_1^*, \ldots, x_N^*\}$ *where*

$$\forall i \in \mathcal{N}, \quad x_i^* = \frac{x_{\max}}{N} \mathbf{1}_N \ .$$

It means that under the assumption of incomplete information, there is a unique NE at which all the networks uniformly spread all the data between all the available channels.

3 Spectrum Access Game for Cognitive Radio Networks with Incomplete Information

A possible objective for the game should be to achieve efficiency that maximises "social welfare". It follows from the revelation principle [19] that to maximise social welfare, it is sufficient to consider mechanisms in which the payments are chosen that for each bidder i. However, truth-telling is a weakly dominant strategy. This mechanism is called *incentive compatible*.

The Vickrey-Clarke-Groves mechanism [20] is the only known general incentive compatible mechanism that can be used to maximise social welfare. Under this mechanism for the given valuation function $z_i(.)$ of the bidders, the channel allocation k_i^* is chosen as follows. Let k_{-i}^* be the channel allocation that would have maximised the social welfare if network i did not participate in the auction. That is, k_{-i}^* satisfies the declared valuation function, namely

$$\sum_{j=1, j \neq i}^{N} z_j(k_{-i}^*) \geq \sum_{j=1, j \neq i} z_j(k) \quad \forall k \in \mathcal{K} \ . \tag{3}$$

By use the VCG auction the payment made by network i is given by

$$\tau_i = \sum_{j=1, j \neq i} z_j(k_{-i}^*) - \sum_{j=1, j \neq i}^{N} z_j(k^*) \ . \tag{4}$$

To implement the VCG auction, the channel allocations k^* and and k_{-i}^*, $i = 1, \ldots, N$ must be determined. To find the allocation, k^* and k_{-i}^* can be used the algorithm given in Sect. 3.1 for the channel allocation problem and for the bidders $\{1, \ldots, N\}$.

3.1 An Algorithm in a VCG Combinatorial Auction for Spectrum Allocation

In this section, we present an algorithm for solving the problem of access allocation in the CR network. The algorithm is polynomial-time when the number of possible parameters of sets of the secondary network on a fixed-bounded channel. In our study, we generalise an approach given in [21], namely:

(1) all objects in a combinatorial auction are indivisible; (2) the allocation in the auction has to be feasible, e.g. auctioneer's revenue must be maximised. The algorithm works as follows:

Let M be the channel number, such that $M_1 + \cdots + M_n = M$. The maximum possible renevue from all participating networks is given by $T(j_0, j_1, \ldots, j_n, i)$. Let j_0 be primary parts and j_1 secondary parts of type $t, t = 1, \ldots, n$, that must be allocated to the networks $1, \ldots, i$ in the auction. In other words, let $K(j_0, \ldots, j_n, t)$ be the set of allocations $k_i = \{n_{0,t} : v = 1, \ldots, i; t == 0, \ldots, n\}$ satisfying the following conditions:

$$\sum_{v=1}^{i} n_{v,0} = j_0, \quad 0 \leq n_{v,0} \leq M, \quad v = 1, \ldots, i \tag{5}$$

$$\sum_{v=1}^{i} n_{v,t} = j_t, \quad t = 1, \ldots, n; \quad 0 \leq n_{v,t} \leq M_t, \quad v = 1, \ldots, i \ . \tag{6}$$

Then

$$T(j_0, j_1, \ldots, j_n, i) = \max \left\{ \sum_{v=1}^{i} z_v(n_{v,0}, n_{v.1}, \ldots, n_{v,n}), \ k_i \in K(j_0, j_1, \ldots, j_n, t) \right\} . \tag{7}$$

For finding the values of $T(j_0, j_1, \ldots, j_n, 1)$ is used the following equation:

$$T(j_0, j_1, \ldots, j_n, 1) = \begin{cases} z_1(j_0, j_1, \ldots, j_n) & \text{if } j_0 \leq M,\ j \leq M_t,\ t = 1, \ldots, n \\ -\infty & \text{otherwise} \end{cases} . \quad (8)$$

For the single network ($i \geq 1$) by use the Equation (8) is allocated all parts of channel to network 1. However, if $j_0 > M$, then $n_{1,0} > M$, which violates condition $0 \leq n_{v,0} \leq M, \quad v = 1, \ldots, i$. Analogously, if $j_1 > M_t$, then $n_{1,t} > M_t$ ($v = 1, \ldots, i$). Hence, if $j_0 > M$ or $j_t > M_t$ then $T(.)$ is set to $-\infty$.

For the two networks the following recursion is used:

$$T(j_0, j_1, \ldots, j_n, i) = \max \{T(j_0 - l_0, j_1 - l_1, \ldots, j_n - l_n, i-1) + z_i(l_0, l_1, \ldots, l_n)\} ,$$

$$\begin{aligned} &l_0 \in \{0, 1, \ldots, , \min(j_0, M)\} , \quad l_v \in \{0, 1, \ldots, \min(j_0, M)\} , \\ &l_v \in \{0, 1, \ldots, \min(j_v, M_v)\} , \quad v = 1, \ldots, n \ . \end{aligned} \quad (9)$$

In the Equation (9), if the primary parts, l_0, and the secondary parts l_1 of type v ($v = 1, \ldots, n$) are allocated to network i, then the pay $z_i(l_0, \ldots, l_n)$, and the maximum revenue from the networks $1, \ldots, i-1$ are allocated for the remaining parts, namely $T(j_0 - l_0, j_1 - l_1, \ldots, j_n - l_n, i-1)$. Thus, $l_v \leq \min(j_v, M_v)$ for $v = 1, \ldots, n$, and $l_0 \leq \min(j_0, M)$. Summarizing, the Equation (9) maximizes the renevue from networks $1, \ldots, i-1$ over all possible values of $l_0, \ldots, l_n$.

The Equation (9) is recursively repeated for all sets $M_1, \ldots, M_n$ such that $M_1 + \cdots + M_n = M$. Then, the optimal set $(M_1^*, \ldots, M_n^*)$ is found as follows:

$$(M_1^*, \ldots, M_n^*) = \arg \max_{M_1 + \cdots + M_n = M} T(M, m_1 M_1, \ldots, m_n M_n, N) \ . \quad (10)$$

It is obvious that the allocation with $M_1 = M_1^*, M_2 = M_2^*, \ldots, M_n = M_n^*$ maximises revenue over all channels.

4 Empirical Results

In this section, we report on computational results to prove that the given methodology solves the access allocation problem in cognitive radio networks.

In our simulation, we assumed that the number of secondary networks on channel can be selected from a possibly large set. Moreover, the set of secondary networks on each channel can be an arbitrary subset of the set of all secondary networks. We simulated the case in which the bid function of every network is different and is linearly approximated by the quadratic function.

We found the optimal revenue by using the dynamic programming algorithm given in previous section. The bid functions of each secondary network are approximated by the following functions respectively:

$$y_i(x) = A_1 \left(1 - \frac{e^{-a_1 x}}{x}\right), \quad i = 1, \ldots, N_1$$

$$y_i(x) = A_2 \left(1 - \frac{e^{-a_2 x}}{x}\right), \qquad i = N_1 + 1, \ldots, N$$

where A_1, A_2, a_1, a_2 are the parameters of networks, N is the number of networks N_1 is the number of primary networks. We studied the auction revenues of the used algorithm to find the channel allocation that maximises revenue. The revenue for the number of channels varied from 3 to 20. Next, we found the value of revenue that maximisea the auctioneer's revenue and allows us to compute all players' bids.

5 Conclusion

In this paper, we have proposed a new methodology for spectrum allocation games with incomplete information. Although the given approach looks quite different from the traditional solutions, it is a sufficient for implementation. We demonstrated ways to apply the Bayesian game and the VCG auction mechanism to reach an effective solution to the problem. This approach is a step forward compared to recent approaches that only consider users having complete information, which may not be realistic in many practical applications.

References

1. Mitola, J.: Cognitive Radio an Integrated Agent Architecture for Software Defined Radio. Ph.D. Thesis, Royal Institute of Technology (KTH), Stockholm, Sweden (May 2000)
2. Haykin, S.: Cognitive Radio: Brain-empowered Wireless Communications. IEEE Journal on Selected Areas in Communications 23(2), 201–220 (2005)
3. Akyildiz, I.F., Lee, W.Y., Vuran, M.C., Mohanty, S.: Next Generation/dynamic Spectrum Access/cognitive Radio Wireless Networks: A Survey. Computer Networks 50(13), 2127–2159 (2006)
4. Spectrum Efficiency Working Group. Report of the Spectrum Efficiency Working Group. Technical Report, FCC, Washington, DC (November 2002)
5. Berger, R.J.: Open Spectrum: A Path to Ubiquitous Connectivity. FCC ACM Queue 1(3) (May 2003)
6. Peha, J.M.: Approaches to a Spectrum Sharing. IEEE Communication Magazine 43(2), 10–12 (2005)
7. Peng, C., Zheng, H., Zhao, B.Y.: Utilization and Fairness in Spectrum Assignment for Opportunistic Spectrum Access. In: ACM Mobile Networks and Applications (MONET) (May 2006)
8. Cao, L., Zheng, H.: Distributed Spectrum Allocation via Local Bargaining. In: Proc. IEEE/DySPAN (2005)
9. Etkin, R., Parekh, A., Tse, D.: Spectrum Sharing for Unlicensed Bands. In: Proc. IEEE DySPAN (2005)
10. Fu, F., van der Schaar, M.: Learning to Compete for Resource in Wireless Stochastic Game. IEEE Trans. on Vehicular Technology 58(4), 1904–1919 (2009)

11. Vickrey, W.: Counterspeculation, Auctions, and Competitive Scaled Tenders. The Journal of Finance 16(1), 8–37 (1961)
12. Kloeck, C., Jaekel, H., Jondral, F.K.: Dynamic and Local Combined Pricing, Allocation and Billing Systems with Cognitive Radios. In: Proc. 1st IEEE Int. Symp. DySPAN, vol. 2(1), pp. 73–81 (November 2005)
13. Clarke, E.: Multipart Pricing of Public Goods. Public Choice 11(1), 17–33 (1971)
14. Groves, T.: Incentives in Teams. Econometrica 41(4), 617–631 (1973)
15. Wysocki, T., Jamalipour, A.: A Spectrum Management in Cognitive Radio: Applications of Portfolio Theory in Wireless Communications. Wireless Communications, IEEE Comm. Soc. 18(4), 52–60 (2011)
16. Cremene, L.C., Dumitrescu, D.: Analysis of Cognitive Radio Scenes Based on Noncooperative Game Theoretical Modelling. IET Communications 6(13), 1876–1883 (2012)
17. Chun, S.H.: Secondary Spectrum Trading – Auction-based Framework for Spectrum Allocation and Profit Sharing. IEEE/ACM Trans. on Networking 21(1), 176–189 (2013)
18. Harsanyi, J.C.: Games with Incomplete Information Played by Bayesian Players, I–III. Management Science 50(12), 1804–1817 (2004)
19. Mas-Colell, A., Whinston, M., Green, J.: Microeconomic Theory. Oxford Univ. Press, London (1995)
20. Nisan, N., Ronen, A.: Computationally Feasible VCG Mechanisms. In: Proc. ACM Conf. Electron., pp. 242–252 (2000)
21. Tennenholtz, M.: Some Tractable Combinatorial Auctions. In: Proc. AAAI, pp. 98–103 (2000)