OFDM Transmission with Non-binary LDPC Coding in Wireless Networks

Grzegorz Dziwoki, Marcin Kucharczyk, and Wojciech Sulek

Silesian University of Technology, Institute of Electronics
{grzegorz.dziwoki,marcin.kucharczyk,wojciech.sulek}@polsl.pl

Abstract. High-quality information exchange between upper layers of the communication network (e.g. TCP, IP layers) requires reliable connection of communicating devices on the physical layer. Any non-corrected errors at this level force the upper layers to perform proper action to recover transmitted information. It reduces data throughput and increases delay to unacceptable level for some services. Among physical media, wireless one is the most hostile environment, due to its unpredictable behavior. In that case, OFDM (Orthogonal Frequency Division Multiplex) modulation and LDPC (Low Density Parity Check) error correction codes appear the best choice to provide high transmission quality on the physical layer. This paper presents the results of the authors' simulation of a LDPC-coded OFDM system with particular emphasis on codes over high order Galois fields (non-binary) which are not commercialized yet.

Keywords: wireless network, physical layer, OFDM modulation, LDPC code.

1 Introduction

LDPC-coded OFDM modulation becomes a popular transmission scheme on the physical layer of diverse communication networks. OFDM modulation has an established position among many existing commercial standards of wireless networks. The LDPC coding is often an option in the current specifications, although it seems to be a strong candidate as main coding method in the future releases of them. LDPC-coded OFDM transmission is successfully adopted for instance in IEEE WiFi [1], IEEE WiMAX [2], and it is also considered in the future releases of a 3GPP LTE (Long Term Evolution) specification called the LTE-Advanced.

Wireless environment, convenient from consumers' perspective, poses a serious obstacle to high-rate reliable transmission. Multipath propagation and mobility cause that the transmission channel is frequency selective and dynamic. The OFDM modulation, owing to the intrinsic subchannel orthogonality, copes with the selectivity by enabling a simple implementation of channel equalization, where each subchannel is equalized separately [3]. Additionally, MIMO (Multiple Input Multiple Output) transmission [4], different subchannel modulation and coding rules improve the system performance. Recently popular binary LDPC

A. Kwiecień, P. Gaj, and P. Stera (Eds.): CN 2013, CCIS 370, pp. 222–231, 2013.

error correction coding is an appealing technique for data protection, which outperforms other one [5]. But, when length of the codeword is short to moderate, the classical binary LDPC code should be replaced with LDPC one over high order finite fields (non-binary) in order for the performance preservation [6].

Numerous scientific reports explore properties of different LDPC and OFDM combinations. For example, optimization of the binary LDPC codes for OFDM systems is investigated in [7,8], iterative channel estimation supported by the LDPC decoding (turbo equalization) is considered in [9,10], while non-binary LDPC-coded OFDM system for underwater acoustic communication is analyzed in [11].

This paper presents a simulation analysis of a general OFDM system with non-binary LDPC coding. Various configurations of the OFDM and the LDPC parameters settings are considered to evaluate system performance. The code rates were $R = 0.5$ as suggested in [11], but different system parameters including the LDPC code generation method were tested. A purpose was to further explore the influence of the relationship between the OFDM and LDPC settings on transmission quality, especially in case of constant time duration of the OFDM symbol and without restriction on relation between the field order p and the constellation size 2^s.

2 Transmission System

Evaluation of the system performance for various LDPC coding and OFDM parameters is a continuation of research presented in [12] where the focus was put on the binary codes only. The remaining elements of the transmission system like preambles, headers, training sequences, packet payload construction, etc. were omitted during simulations. There was assumed perfect synchronization and its corresponding blocks were omitted in the system model too. The kind of the transmitted information was not relevant. Concentrating on the LDPC and OFDM processing only, there was proposed a general model of the transmission system which is presented in Fig. 1. Additional elements of the model act as converters (mappers). They are responsible for matching the signal's structures between the different processing stages.

Data bits submitted for transmission are grouped in blocks of p bits each at first and then the K consecutive p-bit blocks comprise a single word. The word is encoded using a GF(2^p) LDPC encoder with rate $R = K/N$. The obtained codewords of length N are matched by the word-to-symbol converter to the IFFT size L and the subchannel constellation size 2^s of the OFDM modulator. After IFFT processing, the OFDM waveform with a cyclic prefix is transmitted over a dispersive channel. The length of the cyclic-prefix is at least as long as the length of the channel impulse response. Complying with this requirement, the frequency subchannels can be equalized separately using one-tap equalizer in case of static or slow fading environment. Next, the equalized OFDM symbols are soft detected, and the obtained a posteriori probabilities are used as the soft decision metrics in the LDPC decoder. The receiver's converter located between

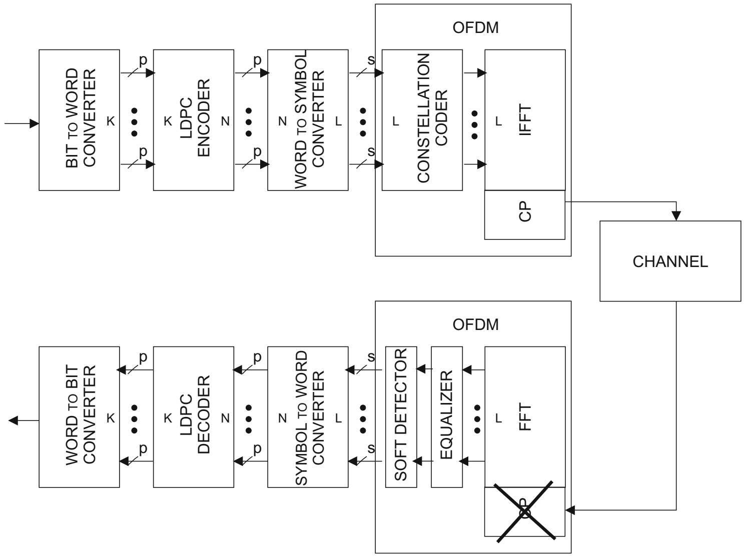

Fig. 1. LDPC-coded OFDM transmission system

the OFDM and LDPC processing units performs far more complicated tasks than its transmitter counterpart. In addition to forming the proper structure of the output signal, it must recalculate the input probabilities according to input properties of the LDPC decoder.

The key units of the transmission system are discussed a bit more in the next subsections.

2.1 OFDM Modulation

An orthogonal frequency division multiplexing (OFDM) modulation is a kind of multicarrier modulation with strong resistance to interchannel (ICI) and intersymbol (ISI) interferences caused by multipath propagation over a wireless channel. The OFDM modulation uses a lot of orthogonal narrowband subchannels with slow signalization in each of them. The subchannels can be considered as a set of AWGN channels with different gains and consequently different SNR (signal-to-noise) ratios. Overall transmission speed depends on the FFT length, the sizes of the QAM constellations in the subchannels and coding rate of the error correction code.

The OFDM symbol of length L is created in frequency domain and then it undergoes IFFT processing. A cyclic-prefix of length G (removed at the receiver side), being the exact copy of the tail of the transformed OFDM symbol, is inserted at the beginning of the symbol itself before it is transmitted. The properly chosen length of the cyclic-prefix ensures time separation of consecutive OFDM symbols at the receiver side. Reliable communication with the OFDM modulation requires good synchronization and channel equalization. The latter is mainly performed in frequency domain after the FFT transformation.

2.2 LDPC Coding

LDPC codes are linear block codes defined over the Galois field GF(q) with restriction to fields of the size being power of two ($q = 2^p$). In the case of the well known binary codes the field size is 2 (thus $p = 1$), whereas for the non-binary codes $p > 1$.

The (N, K) LDPC code with a source vector length K and a code vector length N is defined by a low density parity check matrix $\mathbf{H}_{M\times N}$ with GF(2^p) entries, where $M = N - K$ is the number of parity checks. Note that the information vectors are over GF(2^p), therefore the source block length is $K \cdot p$ bits and the code block length is $N \cdot p$ bits. We denote the entries of the parity check matrix as $h_{mn};\ m = 1, \ldots, M;\ n = 1, \ldots, N$. The column weight d_c of $\mathbf{H}$ is the average number of non-zero entries in the columns. The row weight has the similar meaning in relation to the rows of the parity check matrix.

A row vector $\mathbf{c}$ (in GF(2^p)) of length N is a valid codeword if it satisfies the parity check equation:

$$\mathbf{H}\mathbf{c}^T = \mathbf{0}_{M\times 1} \ , \tag{1}$$

where the operations are performed in the Galois field arithmetic. Equation (1) can be partitioned into M checks associated with M rows of $\mathbf{H}$.

The parity check matrix $\mathbf{H}$ may be randomly constructed. But, in this paper, the codes are created with the PEG algorithm, which is preferred in the case of relative short-length block codes. The detailed description of the algorithm can be found in [13].

The goal of the decoder is to find the most probable originally transmitted vector $\mathbf{c}$ that satisfies (1), taking into account the received channel values. In the soft decision decoding system, the values initializing the decoder are likelihoods. Considering the iterative decoding algorithm, a convenient representation of the parity check matrix is the Tanner graph, which is a bipartite graph with variable nodes (VNs) and check nodes (CNs). The edges in the graph are associated with positions of the non-zero entries in the parity check matrix. The classical formulation of generalized belief propagation (BP) decoding algorithm assumes two major calculation steps performed in every iteration and in the each node: check node processing for calculation of probabilities associated with given check equation and variable node processing with tentative decoding. A brief description of the decoding procedure can be found in [6].

2.3 Wireless Channel

Radio channel is probably the most hostile environment. The transmitted signals are distorted by two major overlapping phenomenons other than background noise. They are shortly explained below:

multipath propagation causes frequency selectivity of the channel, i.e. the channel characteristic varies in frequency and received signal may experience deep fading;

mobility causes time variation of the received signal power. It is the result of relative movement of transmitter, receiver, and even the whole surrounding environment.

Because communication systems work in various frequency bands, the respective channel models may have different number of taps of the impulse response and different statistical properties. For simulation purposes the channels are usually normalized according to carrier frequency and sampling period.

A model of frequency selective wireless channel for terrestrial propagation in an urban area was considered in the investigations. It is based on COST 207 typical urban propagation profile (TU6) with parameters given in Table 1. Among many possible instances of randomly generated channel characteristic, the chosen one, presented in Fig. 2 was used in further simulations of the system model.

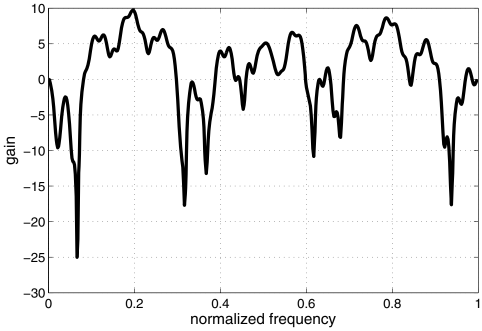

Fig. 2. The frequency response of the wireless channel (randomly generated according to COST 207 TU6 propagation profile)

Table 1. TU6 profile

Tap number	Delay [μs]	Power [dB]
1	0	-3
2	0.2	0
3	0.5	-2
4	1.6	-6
5	2.3	-8
6	5	-10

3 Numerical Experiments

3.1 Assumptions

The simulations was performed according to the following assumptions:

- the parameters of the LDPC codes are presented in Table 2. All codes have rate $R = 0.5$ and are generated according to PEG algorithm. The maximum number of iteration of the BP decoding algorithm amounts to 40;

Table 2. LDPC codes

No.	N	K	p	column weight d_c
1	128	64	2	2.5
2	128	64	4	2.5
3	128	64	6	2
4	192	96	4	2.5
5	256	128	1	3
6	256	128	3	2.5
7	384	192	2	2.5
8	512	256	1	3
9	768	384	1	3

- time duration of the OFDM symbol is the same for all three considered cases. The number L of the subchannels is always equal to 128. The constellation sizes, assigned to the every subchannel, are 4-QAM ($s = 2$), 16-QAM ($s = 4$), 64-QAM ($s = 6$) respectively;
- the foregoing LDPC and OFDM parameters of the simulated transmission system may be combined in any way. It complements the commonly used approach presented in many science reports where constellation size, assigned to every OFDM subchannel, is often confined to two points ($s = 1$) (e.g. [8,10]) or follows the rule $s = p$ (e.g. [11]);
- the coherence time is sufficiently large to ensure a static channel conditions during transmission of a single OFDM symbol;
- synchronization and equalization are perfect. Referring to the equalization, the receiver knows the channel frequency characteristic. Although, MMSE (Minimum Mean Square Error) equalization algorithm with pilot sequences is usually used in practice, the foregoing assumption of perfect equalization is acceptable for the analyses herein, because the LDPC coding schemes and their combination with OFDM were the main concern of the research;
- WER (Word Error Rate) as a function of E_b/N_0 (bit energy to noise density) is used as a metric of system performance. The word *Word* in the name of the metric refers to a single codeword of the LDPC code.

3.2 Results

Scenario 1. The existing practical LDPC-coded OFDM systems use the binary LDPC codes. An exemplary results of the transmission performance for general model of the system are presented in Fig 3. There were considered all assumed instances of the 128-point OFDM modulations and three binary LDPC codes with the codewords length $N = \{256, 512, 768\}$. Apart from the subchannel constellation size of the OFDM modulation, the best system performance is obtained for the longest LDPC code ($N = 768$). Among the considered codes, there is also the case of the one-to-one relation at the word/symbol level. It means that one LDPC codeword is included exactly in one OFDM symbol. Despite this correspondence, the word-to-symbol converter is still necessary in the system due to the difference between p and s values. In the case of $s = 4$ and $s = 2$, the one LDPC codeword of length $N = 768$ is transmitted by 1.5 and 3 OFDM symbols respectively. Preservation of the exact relation between the LDPC and OFDM block lengths for these values of s requires use of shorter LDPC codes (here $N = 192$ and $N = 384$ respectively), that consequently has impact on the performance reduction.

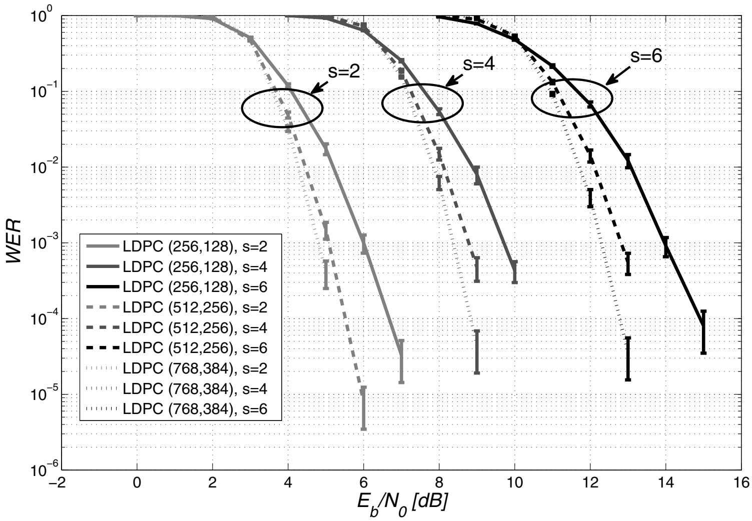

Fig. 3. WER for the binary LDPC-coded OFDM system

Scenario 2. Preservation of the direct relation between LDPC and OFDM signal structures at bit and word/symbol levels separately (i.e. $s = p$ and $N = L$) was the main assumption of the second analysis. The word-to-symbol converters are not required in case of this relationship. The system performance for this direct connection, in comparison with the best results for binary codes from Scenario 1, is presented in Fig. 4. Except for the absence of the word-to-symbol converters in the system model, another advantage is the system performance improvement. On the whole, the higher Galois field order, the better system performance. It is about 2 dB and 3 dB bit-energy reduction for $p = 4$ and $p = 6$ respectively for the investigated system. But, there is no improvement for $p = 2$. The reason is much shorter length of the non-binary LDPC code from binary point of view ($128 \cdot 2 = 256$ bits) in comparison with the binary one (768 bits). For example, the binary LDPC code with the same length in bits ($N = 256$) has slightly worse performance only (dotted curve in Fig. 4) than the non-binary one (light-gray dashed curve in Fig. 4).

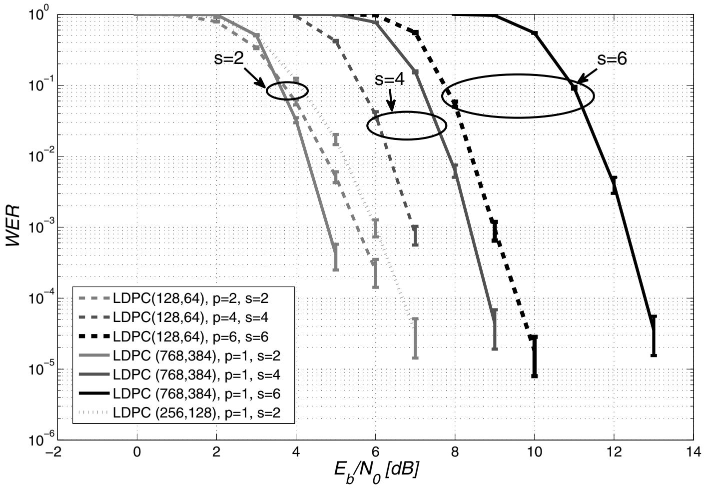

Fig. 4. WER for the nonbinary LDPC codes vs the best binary one

Scenario 3. The possibility of significant performance improvement was demonstrated in the previous scenario, when high Galois field orders were used in the code design. Unfortunately, on the other hand, an implementation complexity of the LDPC code also increases for high values of p. The third scenario explored the combination of different LDPC code parameters under assumptions $N \cdot p =$ const and the OFDM modulation with $L = 128$ and 64-QAM constellation ($s = 6$) in every subchannel. The purpose was to evaluate the differences in the system performance for various kinds of the LDPC codes, providing that the whole codeword is transmitted within a single OFDM symbol (in that case $N \cdot p = 768$).

The obtained results demonstrate a relatively good transmission quality for the code with $p = 3$ (Fig. 5). In comparison with the best one (for $p = 6$) it characterizes small reduction in performance only. Regarding practical implementation, there is a considerable improvement. The Belief Propagation decoding algorithm requires 2^p probabilities for each p-bit block of the codeword. The total amount of probabilities (for a single codeword) is 8192 for the best quality code among the considered ones. In case of the LDPC code, which is second in the performance rank ($p = 3$), it is 2048 probabilities only.

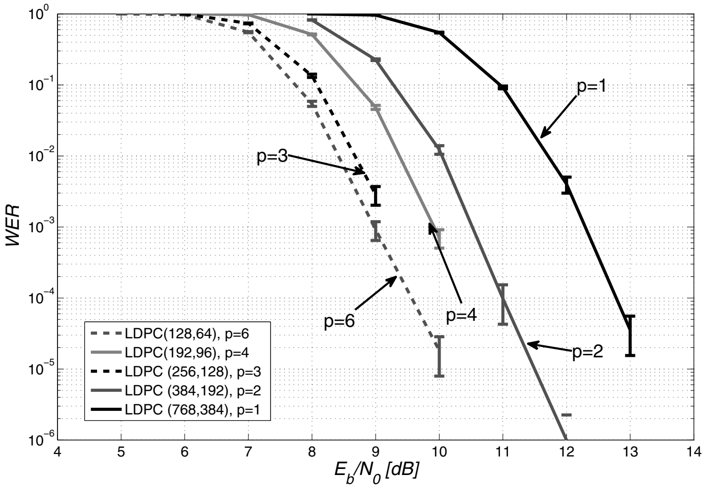

Fig. 5. WER for the LDPC codes comply with assumption $N \cdot p = \text{const}$

4 Conclusions

The numerical evaluation of the LDPC-coded OFDM systems for binary and non-binary LDPC codes was the main purpose of the paper. A commercial use of the non-binary LDPC codes is probably a question of time. The obtained results show their strong potential for improvement of the system performance. The improvement can be noticed especially for Galois field of orders higher than 4. The presented case study for the 128-point OFDM modulation and subchannel coding with 64-QAM constellation shows the attractive results for LDPC code over Galois field of order 2^3.

The certain disadvantage of the non-binary LDPC codes is more complex implementation than the binary one. But seeking for more efficient system solutions (e.g. in power consumption), a wider look at the whole system is necessary. Note that beside the LDPC coder/encoder and OFDM modulator/demodulator there are the converters (mapper/demapper) between them (except for the direct assignment), which consume resources too. A compromise is to find a combination of the LDPC and OFDM parameters to best fit implementation and performance issues.

A closer look at implementation issues is assumed in future work. It particularly concerns the non-binary LDPC decoder and its combination with the symbol-to-word converter.

Acknowledgement. This work was supported by the Polish National Science Centre under Grant number 4698/B/T02/2011/40.

References

1. IEEE 802.11-2012:IEEE Standard for Information technology-Telecommunications and information exchange between systems. Local and metropolitan area networks-Specific requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications (2012)
2. IEEE 802.16e-2006: IEEE Standard for Local and metropolitan area network. Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems (2006)
3. Wang, Z., Giannakis, G.: Wireless Multicarrier Communications: Where Fourier Meets Shannon. IEEE Signal Process. Mag. 47, 28–48 (2000)
4. Martyna, J.: The Least Squares SVM Approach for a Non-Linear Channel Prediction in the MIMO System. In: Kwiecień, A., Gaj, P., Stera, P. (eds.) CN 2011. CCIS, vol. 160, pp. 27–36. Springer, Heidelberg (2011)
5. MacKay, D.J.C.: Good Error-Correcting Codes Based on Very Sparse Matrices. IEEE Trans. Inf. Theory 45, 399–431 (1999)
6. Davey, M.C., MacKay, D.: Low-Density Parity Check Codes over GF(q). IEEE Commun. Lett. 2, 165–167 (1998)
7. Lu, B., Yue, G., Wang, X.: Performance Analysis and Design Optimization of LDPC-Coded MIMO OFDM Systems. IEEE Trans. Signal Process. 52, 348–361 (2004)
8. Serener, A., Natarajan, B., Gruenbacher, D.: Lowering the Error Floor of Optimized Short-Block-Length LDPC-Coded OFDM via Spreading. IEEE Trans. Veh. Technol. 57, 1646–1656 (2008)
9. Guan, W., Xiang, H.: Low-Complexity Channel Estimation Using Short-Point FFT/IFFT for an Iterative Receiver in an LDPC-Coded OFDM System. Wireless Personal Communications 64, 739–747 (2012)
10. Oh, M.K., Kwon, H., Park, D.J., Lee, Y.H.: Iterative Channel Estimation and LDPC Decoding with Encoded Pilots. IEEE Trans. Veh. Technol. 57, 273–285 (2008)
11. Huang, J., Zhou, S., Willett, P.: Nonbinary LDPC Coding for Multicarrier Underwater Acoustic Communication. IEEE J. Sel. Areas Commun. 26, 1684–1696 (2008)
12. Dziwoki, G., Kucharczyk, M., Sulek, W.: Transmission over UWB Channels with OFDM System using LDPC Coding. In: Proc. SPIE 7502, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments, pp. 75021Q–75021Q6 (2009)
13. Hu, X.Y., Eleftheriou, E., Arnold, D.M.: Regular and Irregular Progressive Edge-Growth Tanner Graphs. IEEE Trans. Inf. Theory 51, 386–398 (2005)