$(\in, \in \vee q_{(\lambda,\mu)})$-Fuzzy Completely Semiprime Ideals of Semigroups

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Abstract We introduce a new kind of generalized fuzzy completely ideal of a semigroup called $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals. These generalized fuzzy completely semiprime ideals are characterized.

Keywords Fuzzy algebra · Fuzzy points · $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals · Level subsets · Completely semiprime ideals

1 Introduction

Fuzzy semigroup theory plays a prominent role in mathematics with ranging applications in many disciplines such as control engineering, information sciences, fuzzy coding theory, fuzzy finite state machines, fuzzy automata, fuzzy languages.

Using the notion of a fuzzy set introduced by Zadeh [1] in 1965, which laid the foundation of fuzzy set theory, Rosenfeld [2] inspired the fuzzification of algebraic structures and introduced the notion of fuzzy subgroups. Since then fuzzy algebra came into being. Bhakat and Das gave the concepts of fuzzy subgroups by using the "belongs to" relation ($\in$) and "quasi-coincident with" relation (q) between a fuzzy point and a fuzzy set, and introduced the concept of a $(\in, \in \vee q)$-fuzzy subgroup [3–6]. It is worth to point out that the ideal of quasi-coincident of a fuzzy point with a fuzzy set, which is mentioned in [7], played a vital role to generate some different types of fuzzy subgroups. In particular, $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup, which provides sufficient motivation to researchers to review various concepts and results from the realm of

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abstract algebra in the broader framework of fuzzy setting. Zhan [8], Jun et al. [9] introduced the notion of $(\in, \in \vee q)$-fuzzy interior ideals of a semigroup. Davvaz [10–13] defined $(\in, \in \vee q)$-fuzzy subnear-rings and characterized H_ν-fuzzy submodules, R-fuzzy semigroups using the relation $(\in, \in \vee q)$.

Later, the definition of a generalized fuzzy subgroup was introduced by Yuan [13]. Based on it, Liao [14] expanded common "quasi-coincident with" relationship to generalized "quasi-coincident with" relationship, which is the generalization of Rosenfeld's fuzzy algebra and Bhakat and Das's fuzzy algebra. And a series results were gotten by using generalized "quasi-coincident with" relationship [15–18]. When $\lambda = 0$ and $\mu = 1$ we get common fuzzy algebra by Rosenfeld and When $\lambda = 0$ and $\mu = 0.5$ we get the $(\in, \in \vee q)$-fuzzy algebra defined by Bhakat and Das and when $\lambda = 0$ and $\mu = 0.5$ we get the $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy algebra.

The concept of a fuzzy ideal in semigroups was developed by Kuroki. He studied fuzzy ideals, fuzzy bi-ideals and fuzzy semiprime ideals in semigroups [19–21]. Fuzzy ideals, generated by fuzzy sets in semigroups, are considered by Mo and Wang [22]. After that Bhakat and Das [23] investigated fuzzy subrings and several types of ideals, including fuzzy prime ideals and $(\in, \in \vee q)$-fuzzy prime ideals. Jun et al.[24–26] studied L-fuzzy ideals in semigroups, fuzzy h-ideals in hemirings,fuzzy ideals in inclines. Besides, Bahushri [27] did some research on c-prime fuzzy ideals in nearrings. It is now natural to investigate similar type of generalization of the existing fuzzy subsystems of some algebraic structures. Our aim in this paper is to introduce and study $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals, and obtain some properties: an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy ideal is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal, if and only if $A_t (\neq \emptyset)$ is a completely semiprime ideal, $\forall t \in (\lambda, \mu)$. This showed that $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals are generalizations of the existing concepts of two types of fuzzy ideals.

2 Preliminaries

Throughout the paper we always consider S as a semigroup.

A mapping from A to $[0, 1]$ is said to be a fuzzy subset of S.

A fuzzy subset A of S of the form $A(y) = \begin{cases} \lambda(\neq 0), & y = x \\ 0, & y \neq x \end{cases}$ is said to be a fuzzy point support x and value λ is denoted by x_t.

Definition 2.1 *Let A be a fuzzy subset of S, for all $t, \lambda, \mu \in [0, 1]$ and $\lambda < \mu$, a fuzzy point x_t is called belonging to A if $A(x) \geq t$, denoted by $x_t \in A$; A fuzzy point x_t is said to be generalized quasi-coincident with A if $t > \lambda$ and $A(x) + t > 2\mu$, denoted by $x_t q_{(\lambda,\mu)} A$. If $x_t \in A$ or $x_t q_{(\lambda,\mu)} A$, then denoted by $x_t \in \vee q_{(\lambda,\mu)} A$.*

Definition 2.2 *[15] A fuzzy subset A of S is said to be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy subsemigroup if for all $x, y \in S, t_1, t_2 \in (\lambda, 1]$, $x_{t_1}, y_{t_2} \in A$ implies $(xy)_{t_1 \wedge t_2} \in \vee q_{(\lambda,\mu)} A$.*

Theorem 2.3 *[15] A fuzzy subset A of S is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy subsemigroup if and only if $A(xy) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu$, for all $x, y \in S$.*

Definition 2.4 *[15] A fuzzy subset A of S is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (right) ideal if (i) A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy subsemigroup of S;*
(ii) For all $x_t \in A$, $y \in S$, implies $(yx)_t \in \vee q_{(\lambda,\mu)} A$ ($(xy)_t \in \vee q_{(\lambda,\mu)} A$).
If A is both an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left ideal and an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy right ideal, then A is said to be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy ideal.

Theorem 2.5 *[15] A fuzzy subset A of S is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy ideal if and only if for all $t \in (\lambda, \mu]$, the non-empty set A_t is an ideal.*

Theorem 2.6 *[15] A fuzzy subset A of S is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy left (right) ideal if and only if for all $x, y \in S$, (i) $A(xy) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu$; (ii)$A(xy) \vee \lambda \geq A(y) \wedge \mu (A(x) \wedge \mu)$.*

Definition 2.7 *[15] An ideal I of S is said to be a completely semiprime ideal, if for all $x \in S$, $x^2 \in I$ implies $x \in I$.*

Based on [23], in a semigroup we have the following definitions and theorems:

Definition 2.8 *A fuzzy ideal A of S is said to be a fuzzy completely semiprime ideal,if for all $x \in S$, $t \in (0, 1]$, $(x^2)_t \in A$ implies $x_t \in A$.*

Theorem 2.9 *A fuzzy ideal A of S is a fuzzy completely semiprime ideal, if and only if $A(x^2) = A(x)$, for all $x \in S$.*

Definition 2.10 *An $(\in, \in \vee q)$-fuzzy ideal A of S is said to be an $(\in, \in \vee q)$-fuzzy completely semiprime ideal, if for all $x \in S$, $t \in (0, 1]$, $(x^2)_t \in A$ implies $x_t \in \vee q A$.*

Theorem 2.11 *An $(\in, \in \vee q)$-fuzzy ideal of S is an $(\in, \in \vee q)$-fuzzy completely semiprime ideal if and only if $A(x) \geq A(x^2) \wedge 0.5$, for all $x \in S$.*

Lemma 2.12 *Let $\{H_t | t \in I \subset [0, 1]\}$ be a family of completely semiprime ideals of S such that for all $s, t \in I$, $t < s$, $H_s \subset H_t$. Then $\cup_{t \in I} H_t$, $\cap_{t \in I} H_t$ are completely semiprime ideals of S.*

3 $(\in, \in \vee q_{(\lambda,\mu)})$-Fuzzy Completely Semiprime Ideals

In this section, we give the new definition of an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of semigroups. Then some equivalent descriptions and properties of it are discussed.

Definition 3.1 *An* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy ideal* A *of* S *is said to be an* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy completely semiprime ideal, if for all* $x \in S, t \in (\lambda, 1]$, $(x^2)_t \in A$ *implies* $x_t \in \vee q_{(\lambda,\mu)} A$.

Theorem 3.2 *An* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy ideal of* S *is an* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy completely semiprime ideal if and only if* $A(x) \vee \lambda \geq A(x^2) \wedge \mu$*, for all* $x \in S$.

Proof $\Rightarrow$ Let A be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S.Assume that there exists x_0 such that $A(x_0) \vee \lambda < A(x_0^2) \wedge \mu$. Choose t to satisfy $A(x_0) \vee \lambda < t < A(x_0^2) \wedge \mu$. Then we have $A(x_0^2) > t, \lambda < t < \mu$, $A(x_0) < t$ and $A(x_0) + t < 2\mu$. So $(x_0^2)_t \in A$. But $(x_0)_t \overline{\in \vee q_{(\lambda,\mu)}} A$,a contradiction.
$\Leftarrow$ For all $x \in S, t \in (\lambda, 1]$ and $(x^2)_t \in A$, then $A(x^2) \geq t$ and $\lambda < t$. So $A(x) \vee \lambda \geq A(x^2) \wedge \mu \geq t \wedge \mu$. Since $\lambda < \mu$, then $A(x) \geq t \wedge \mu$.
If $t \geq \mu$, then $A(x) \geq \mu$, we have $A(x) + t > \mu + \mu = 2\mu$, so $x_t q_{(\lambda,\mu)} A$.
If $t < \mu$, then $A(x) \geq t$. So $x_t \in A$.

Hence, $x_t \in \vee q_{(\lambda,\mu)} A$. That is to say, A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal.

Theorem 3.3 *A non-empty subset* S_1 *of* S *is a completely semiprime ideal if and only if* χ_{S_1} *is an* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy completely semiprime ideal of* S.

Proof $\Rightarrow$ Let S_1 be a completely semiprime ideal, then χ_{S_1} is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy ideal of S. If $(x^2)_t \in \chi_{S_1}$, then $\chi_{S_1}(x^2) \geq t > 0$, so $x^2 \in S_1$. Since S_1 is a completely semiprime ideal, we have $x \in S_1$, then $x_t \in \vee q_{(\lambda,\mu)} \chi_{S_1}$. Thus χ_{S_1} is an $(\in, \in \vee q_{(\lambda,\mu)})-$ fuzzy completely semiprime ideal.
$\Leftarrow$ Let χ_{S_1} be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal, then S_1 is an ideal of S. Now $x^2 \in S_1$, then $\chi_{S_1}(x^2) = 1$. Since χ_{S_1} is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal, by the Theorem 3.2, we have $\chi_{S_1}(x) \vee \lambda \geq \chi_{S_1}(x^2) \wedge \mu$ $= \mu$ Then $\chi_{S_1}(x) \geq \mu > 0$. So $\chi_{S_1}(x) = 1$,i.e.,$x \in S_1$. Therefore, S_1 is a completely semiprime ideal.

Remark When $\lambda = 0, \mu = 1$, we can obtain the corresponding results in the sense of Rosenfeld; When $\lambda = 0, \mu = 0.5$, we can get the corresponding results in the sense of Bhakat and Das.

Theorem 3.4 *An* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy ideal* A *of* S *is an* $(\in, \in \vee q_{(\lambda,\mu)})$*-fuzzy completely semiprime ideal if and only if non-empty set* A_t *is a completely semiprime ideal for all* $t \in (\lambda, \mu]$.

Proof $\Rightarrow$ Let A be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S, then the non-empty set A_t is an ideal, for all $t \in (\lambda, \mu]$. Let $x^2 \in A_t$, then $A(x^2) \geq t$. By Theorem3.2,we have $A(x) \vee \lambda \geq A(x^2) \wedge \mu \geq t \wedge \mu = t$. So $x \in A_t$. Hence A_t is a completely semiprime ideal.
$\Leftarrow$ Let A_t be a completely semiprime ideal of S, by Theorem2.5, we have that A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy ideal. Suppose that A is not an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal. By Theorem3.2, there exists x_0 such that $A(x_0) \vee \lambda < A(x_0^2) \wedge \mu$. Choose t such that $A(x_0) \vee \lambda < t < A(x_0^2) \wedge \mu$. Then we have $A(x_0^2) > t$, $A(x_0) <$

$t, \lambda < t < \mu$ and $x_0^2 \in A_t$. Since A_t is completely semiprime, we have $x_0 \in A_t$, a contradiction.

Therefore A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal.

Corollary 3.5 A fuzzy set A of S is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime, ideal of S if and only if non-empty set A_t is a completely semiprime ideal, for all $t \in (\lambda, \mu]$.

Theorem 3.6 *Let I be any completely semiprime ideal of S. There exists an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal A of S such that $A_t = I$ for some $t \in (\lambda, \mu]$.*

Proof If we define a fuzzy set in S by

$$A(x) = \begin{cases} t, \ if \quad x \in I \\ 0, \ otherwise \end{cases} \text{for some } t \in (\lambda, \mu].$$

Then it follows that $A_t = I$.

For given $r \in (\lambda, \mu]$, we have

$$A_r = \begin{cases} A_t(= I), \ if \quad r \leq t \\ \emptyset, \qquad\quad if \quad t < r < \mu \end{cases}$$

Since I itself is a completely semiprime ideal of S, it follows that every non-empty level subset A_r of S is a completely semiprime ideal of S. By Corollary3.5, A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S, which satisfies the conditions of the Theorem.

Theorem 3.7 *Let A be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S such that $A(x) \leq \mu$ for all $x \in S$. Then A is a fuzzy completely semiprime ideal of S.*

Proof Let $x \in S$, $t \in (\lambda, 1]$ and $(x^2)_t \in A$. It follows that $x_t \in \vee q_{(\lambda,\mu)} A$ from Definition3.1.

By known conditions, we have $t \leq A(x^2) \leq \mu$ and $t < \mu$. Thus $A(x) + t \leq \mu + \mu = 2\mu$, *i.e.*, $x_t \overline{q_{(\lambda,\mu)}} A$. Hence $x_t \in A$. Therefore A is a fuzzy completely semiprime ideal of S.

Theorem 3.8 *Let A be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy ideal of S and B be an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S. Then $A \cap B$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of A_μ.*

Proof Let $x \in A_\mu$ and $(x^2)_t \in A \cap B$,then $(A \cap B)(x^2) \geq t$. So $A(x^2) \geq t$ and $B(x^2) \geq t$. Thus $(x^2)_t \in A$ and $(x^2)_t \in B$. Since B is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S, we have $x_t \in \vee q_{(\lambda,\mu)} B$, so $x_t \in B$ or $x_t q_{(\lambda,\mu)} B$.

Assume $x_t \in B$ which implies $B(x) \geq t$. If $t \leq \mu$, then $A(x) \geq \mu \geq t$. So $(A \cap B)(x) = A(x) \wedge B(x) \geq t$. Therefore $x_t \in (A \cap B)$ and $x_t \in \vee q_{(\lambda,\mu)} (A \cap B)$; If $t > \mu$, then $A(x) + t > \mu + \mu = 2\mu$ and $B(x) + t \geq t + t > 2\mu$, then $(A \cap B)(x) + t = (A(x) \wedge B(x)) + t > 2\mu$. So $x_t q_{(\lambda,\mu)} (A \cap B)$.

Assume $x_t q_{(\lambda,\mu)} B$ which implies $B(x) + t > 2\mu$. If $t \leq \mu$, then $B(x) > 2\mu - t \geq \mu \geq t$ and $A(x) \geq \mu \geq t$, so $x_t \in A$ and $x_t \in B$. Thus $x_t \in A \cap B$ and $x_t \in \vee q_{(\lambda,\mu)}(A \cap B)$. If $t > \mu$, then $A(x) + t > 2\mu$ and $(A \cap B)(x) + t = (A(x) \wedge B(x)) + t > 2\mu$. So $x_t q_{(\lambda,\mu)}(A \cap B)$ and $x_t \in \vee q_{(\lambda,\mu)}(A \cap B)$.

Therefore,$A \cap B$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of A_μ.

Theorem 3.9 *Let $\{A_i \mid i \in I\}$ be a family of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals of S such that $A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$. Then $A = \cup_{i\in I} A_i$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S.*

Proof For all $x, y \in S$,

$$\begin{aligned} A(xy) \vee \lambda &= (\cup_{i\in I} A_i)(xy) \vee \lambda = (\vee_{i\in I} A_i(xy)) \vee \lambda \\ &= \vee_{i\in I} (A_i(xy) \vee \lambda) \\ &\geq \vee_{i\in I} (A_i(x) \wedge A_i(y) \wedge \mu) \\ &= (\vee_{i\in I} A_i(x)) \wedge (\vee_{i\in I} A_i(y)) \wedge \mu \\ &= (\cup_{i\in I} A_i(x)) \wedge (\cup_{i\in I} A_i(y)) \wedge \mu \\ &= A(x) \wedge A(y) \wedge \mu \end{aligned}$$

In the following we show that $\vee_{i\in I} (A_i(x) \wedge A_i(y) \wedge \mu)$=$(\vee_{i\in I} A_i(x)) \wedge (\vee_{i\in I} A_i(y)) \wedge \mu$ holds. It is clear that $\vee_{i\in I} (A_i(x) \wedge A_i(y) \wedge \mu) \leq (\vee_{i\in I} A_i(x)) \wedge (\vee_{i\in I} A_i(y)) \wedge \mu$. If possible, let $\vee_{i\in I} (A_i(x) \wedge A_i(y) \wedge \mu) \neq (\vee_{i\in I} A_i(x)) \wedge (\vee_{i\in I} A_i(y)) \wedge \mu$.

Then there exists t such that $\vee_{i\in I} (A_i(x) \wedge A_i(y) \wedge \mu) < t < (\vee_{i\in I} A_i(x)) \wedge (\vee_{i\in I} A_i(y)) \wedge \mu$. Since $A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$, there exists $k \in I$ such that $t < A_k(x) \wedge A_k(y) \wedge \mu$. On the other hand, $A_i(x) \wedge A_i(y) \wedge \mu < t$ for all $i \in I$, a contradiction. Hence, $\vee_{i\in I} (A_i(x) \wedge A_i(y) \wedge \mu) = (\vee_{i\in I} A_i(x)) \wedge (\vee_{i\in I} A_i(y)) \wedge \mu$.
$\forall x, y \in S, \quad A(xy) \vee \lambda = (\cup_{i\in I} A_i(xy)) \vee \lambda = \vee_{i\in I} (A_i(xy) \vee \lambda) \geq \vee_{i\in I}(A_i(x) \wedge \mu)$
$= (\vee_{i\in I} A_i(x)) \wedge \mu = A(x) \wedge \mu$.

Similarly prove $A(xy) \vee \lambda \geq A(y) \wedge \mu$,for all $x, y \in S$.
$A(x) \vee \lambda = (\cup_{i\in I} A_i(x)) \vee \lambda = \cup_{i\in I} (A_i(x) \vee \lambda) \geq \vee_{i\in I} \left(A_i(x^2) \wedge \mu\right) = \left(\vee_{i\in I} A_i(x^2)\right) \wedge \mu = A(x^2) \wedge \mu$.

By Theorem3.2, A is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S.

Theorem 3.10 *Let $\{A_i \mid i \in I\}$ be a family of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals of S such that $A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$. Then $A = \cap_{i\in I} A_i$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S.*

Proof For any $x, y \in S$,

$$\begin{aligned} A(xy) \vee \lambda &= (\cap_{i\in I} A_i)(xy) \vee \lambda = (\wedge_{i\in I} A_i(xy)) \vee \lambda \\ &= \wedge_{i\in I}(A_i(xy) \vee \lambda) \geq \wedge_{i\in I}(A_i(x) \wedge A_i(y) \wedge \mu) \\ &\geq (\wedge_{i\in I} A_i(x)) \wedge (\wedge_{i\in I} A_i(y)) \wedge \mu = A(x) \wedge A(y) \wedge \mu \\ A(xy) \vee \lambda &= (\cap_{i\in I} A_i)(xy) \vee \lambda = (\wedge_{i\in I} A_i(xy)) \vee \lambda = \wedge_{i\in I}(A_i(xy) \vee \lambda) \\ &\geq \wedge_{i\in I}(A_i(x) \wedge \mu) = (\wedge_{i\in I} A_i(x)) \wedge \mu = A(x) \wedge \mu \end{aligned}$$

Similarly prove $A(xy) \vee \lambda \geq A(y) \wedge \mu, \forall x, y \in S$.

$$\begin{aligned} A(x) \vee \lambda &= (\cap_{i\in I} A_i)(x) \vee \lambda \\ &= (\wedge_{i\in I} A_i)(x) \vee \lambda \\ &= \wedge_{i\in I}(A_i(x) \vee \lambda) \\ &\geq \wedge_{i\in I}\left(A_i(x^2) \wedge \mu\right) \end{aligned}$$

In the following we show that $(\wedge_{i\in I} A_i)(x) \vee \lambda = \wedge_{i\in I}(A_i(x) \vee \lambda)$ holds. It is clear that $\wedge_{i\in I}(A_i(x) \vee \lambda) \geq (\wedge_{i\in I} A_i(x)) \vee \lambda$.

If possible, let $\wedge_{i\in I}(A_i(x) \vee \lambda) > (\wedge_{i\in I} A_i(x)) \vee \lambda$. Then there exists t such that $\wedge_{i\in I}(A_i(x) \vee \lambda) > t > (\wedge_{i\in I} A_i(x)) \vee \lambda$. Since $A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$, there exists $k \in I$ such that $t > A_k(x) \vee \mu$. On the other hand, $A_i(x) \vee \mu > t$ for all $i \in I$, a contradiction.

Hence, $\wedge_{i\in I}(A_i(x) \vee \lambda) = (\wedge_{i\in I} A_i(x)) \vee \lambda$.

Here we finish the proof of the theorem.

Theorem 3.11 *Let S and S' be semigroups and $f : S \to S'$ be an onto homomorphism. Let A and B be $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals of S and S', respectively. Then*

(i) *$f(A)$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S';*
(ii) *$f^{-1}(B)$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S.*

Proof (i) For any $x' \in S$, then

$$\begin{aligned} f(A)(x'^2) \vee \lambda &= \vee\{A(z) \mid z \in S, f(z) = x'^2\} \vee \lambda \\ &\geq \vee\{A(x^2) \mid x \in S, f(x) = x'\} \vee \lambda \\ &= \vee\{A(x^2) \vee \lambda \mid x \in S, f(x) = x'\} \\ &\geq \vee\{A(x) \wedge \mu \mid x \in S, f(x) = x'\} \\ &= f(A)(x') \wedge \mu. \end{aligned}$$

Therefore $f(A)$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S'.

(ii) For all $x, y \in S$, $f^{-1}(B)(x^2) \vee \lambda = B(f(x^2)) \vee \lambda = B(f(x)^2) \vee \lambda \geq B(f(x)) \wedge \mu$. So $f^{-1}(B)$ is an $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of S.

4 Conclusion

In the study of fuzzy algebraic system, we notice that fuzzy ideals with special properties always play an important role. In this paper, we give the new definition of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideals of semigroups. Using inequalities, characteristic functions and level sets, we consider its equivalent descriptions. Apart from those, the properties of the union, intersection, homomorphic image and homomorphic preimage of $(\in, \in \vee q_{(\lambda,\mu)})$-fuzzy completely semiprime ideal of semigroups are investigated. Those results extend the corresponding theories of fuzzy completely semiprime ideals and enrich the study of fuzzy algebra. At present, although a series of work on this aspect have been done, there is much room for further study.

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