Hybrid State Variables - Fuzzy Logic Modelling of Nonlinear Objects

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Abstract. In this paper a new hybrid method for modelling of nonlinear dynamic systems is proposed. It uses fuzzy logic system together with state variables technique to obtain the local linear approximation performed continuously for successive operating points. This approach provides good accuracy and allows the use of very convenient and well-known method from linear control theory to analyse the obtained model.

1 Introduction

Models of various physical phenomena are often used in practice. This is because the possession of the real object model allows to build precise control system, failure prediction and knowledge extraction from the modelled object [17]. Unfortunately, often only simplified models of the analysed objects are available which have too low precision and therefore they are not very useful in practice. The simplified models are often linear and they usually do not include all the phenomena. As a result, these models are accurate enough in certain (i.e. linear) operating points only. However, they are very useful, considering the fact of the possibility of use the well-known methods of the control theory, which refer to the linear models.

While the objects in the real world are usually nonlinear it would be very useful to have an improved (i.e. more precise) linear model which will be able to model accurately enough the nonlinear phenomenon. In the literature this issue is widely investigated. For example in [3, 12, 13] there is proposed method for modelling of dynamic systems using the theory of the state variables with use the method of sector-nonlinearity. The modelling is based on identification of the sectors which are the basis for the local linear approximation of a nonlinear object. Other authors ([21]) propose the use of models of the plants which have a known structure and parameters of the linear part of the plant and a static nonlinearity that is not known. The proposed isolated nonlinearities allows to obtain the accurate model of the plant, based on the initially known simplified linear model. However this method transforms the linear model into the nonlinear one and lost the advantage of the linear modelling.

In this paper we propose a new method of the nonlinear modelling in which the linear model is improved by the way which allows to increase their accuracy

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and maintaining the advantage of the linear model. In contrast to the sector nonlinearities method [3, 12, 13] our method uses fuzzy rules to modelling variability of individual (selected) coefficients of the matrix (from the used algebraic equations and a state variables theory) instead of using them to modelling of nonlinearity of object sectors as a whole. Thus, the output of fuzzy rules used in the model will be referenced more to the variability of some model parameters than to indicate the sectors of its nonlinearity. This implies undeniably benefit which includes possibility of interpretation of dependence between values of defined coefficients (which are functions of the model parameter/s) and points of work.

This paper is organized into six sections. In the next section an idea of the proposed modelling method is presented. In Section 3 we describe intelligent system for nonlinear modelling. Section 4 shows the evolutionary generation of the models of nonlinear dynamic systems and Section 5 presents experimental results. Conclusions are drawn in Section 6.

2 Idea of the Proposed Method

Let's consider the nonlinear dynamic system described by the linear algebraic equation and based on the the state variables technique ([15]), i.e.

$$\mathbf{x}(k+1) = (\mathbf{I} + \mathbf{A} \cdot T) \cdot \mathbf{x}(k) + \mathbf{B} \cdot T \cdot \mathbf{u}(k), \tag{1}$$

$$\mathbf{y} = \mathbf{C} \cdot \mathbf{x}, \tag{2}$$

where $\mathbf{A}$,$\mathbf{B}$ and $\mathbf{C}$ are system input and output matrices, $\mathbf{I}$ is the identity matrix with appropriate size, $\mathbf{x}, \mathbf{u}, \mathbf{y}$ are vectors of state variables, input and output signals respectively. This model presents a local linear approximation of the nonlinear object in an arbitrary chosen operating point. It refers to continuous objects noted in discrete form with time step T, connected with the current time t by the dependency $t = kT$, where $k = 1, 2, ...$ Modelling with use of the dynamic phenomena description as state variables and fuzzy rules is based on the observable canonical form of the state equations [15].

The significant improvement (in the sense of increasing the accuracy) of such model is possible when the local linear approximation will be performed continuously for each current operating point ([17]). More precisely, the system matrix in linear model will be corrected by adding a correction matrix $\mathbf{P_A}$ in such a way to increase the model accuracy, i.e.:

$$\mathbf{x}(k+1) = (\mathbf{I} + (\mathbf{A} + \mathbf{P_A}(k)) \cdot T) \cdot \mathbf{x}(k) + \mathbf{B} \cdot T \cdot \mathbf{u}(k), \tag{3}$$

where $\mathbf{P_A}(k)$ is the corrections matrix for system matrix $\mathbf{A}$. Despite the fact that operating point changes over time during the process, a local re-determination of coefficients matrix in any new point is possible. For the discretization with the suitable short time step T that solution is enough accurate, even if the first order approximation is used.

3 Intelligent System for Nonlinear Modelling

In proposed method the coefficients of the correction matrix $\mathbf{P}_{\mathbf{A}}(k)$ are generated by multi-input, single-output fuzzy system [1, 2, 5–11, 14, 18–20, 22]. This idea is graphically depicted on fig. 1.

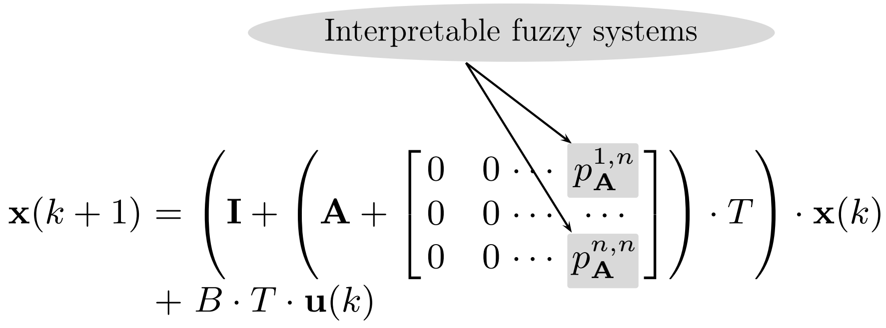

Fig. 1. The idea of the hybrid modelling method based on fuzzy logic system and modelling technique with the use of state variables

Each of the systems has a collection of N fuzzy IF − THEN rules in the form:

$$\mathcal{R}^r : \text{IF } x_1 \text{ is } A_1^r \text{ AND } \ldots \text{ AND } x_n \text{ is } A_n^r \text{ THEN } y \text{ is } B^r, \tag{4}$$

where $\mathbf{x} = [x_1, \ldots, x_n] \in \mathbf{X} \subset \mathbf{R}^n$ is a vector of input signals, $y \in \mathbf{Y} \subset \mathbf{R}$ is an output value, $A_1^r, A_2^r, \ldots, A_n^r$ and $B^r(y), r = 1, \ldots, N$ are fuzzy sets characterized by membership functions $\mu_{A_i^r}(x_i)$ and $\mu_{B^r}, i = 1, \ldots, n, r = 1, \ldots, N$.

Each fuzzy rule (4) determines fuzzy set $\overline{B}^r \subset R$ whose membership function is given by following formula

$$\mu_{\overline{B}^r}(y) = \mu_{\mathbf{A}^r \to B^r}(\overline{\mathbf{x}}, y) = T\left\{ \mathop{T^*}_{i=1}^{n} \left(\mu_{A_i^r}(x_i)\right), \mu_B^r(y) \right\}, \tag{5}$$

where T and T^* are t-norms operators (not necessarily the same) [19]. As a result of aggregation of the fuzzy sets $\overline{B}^r$ we obtain the fuzzy set B' with membership function given by

$$\mu_{B'}(y) = \mathop{S}_{r=1}^{N} \left\{\mu_{\overline{B}^r}(y)\right\}, \tag{6}$$

where S denotes t-conorm operator [19]. The defuzzification can be realized by the center of area method defined in the discrete form by following formula

$$\overline{y} = \frac{\sum_{r=1}^{N} \overline{y}_B^r \cdot \mu_{B'}(\overline{y}_B^r)}{\sum_{r=1}^{N} \mu_{B'}(\overline{y}_B^r)}, \tag{7}$$

where $\overline{y}_B^r$ are centers of the membership functions $\mu_{B^r}(y), r = 1, \ldots, N$.

4 Evolutionary Construct of the Matrix of the Corrections

In order to create interpretable model of the dynamic processes we use the evolutionary strategy (μ, λ) (see e.g. [4, 10]). The purpose of this is to obtain the parameters of systems described in the previous sections. In the process of evolution we assumed, that:

- In single chromosome $\mathbf{X}$ all parameters of fuzzy systems (7) are coded in following way:

$$\mathbf{X} = \begin{pmatrix} \overline{x}^A_{1,1,1}, \sigma^A_{1,1,1}, \cdots, \overline{x}^A_{1,1,n}, \sigma^A_{1,1,n}, \overline{y}^B_{1,1}, \sigma^B_{1,1}, \cdots, \\ \overline{x}^A_{1,N,1}, \sigma^A_{1,N,1}, \cdots, \overline{x}^A_{1,N,n}, \sigma^A_{1,N,n}, \overline{y}^B_{1,N}, \sigma^B_{1,N}, \cdots, \\ \overline{x}^A_{M,1,1}, \sigma^A_{M,1,1}, \cdots, \overline{x}^A_{M,1,n}, \sigma^A_{M,1,n}, \overline{y}^B_{M,1}, \sigma^B_{M,1}, \cdots, \\ \overline{x}^A_{M,N,1}, \sigma^A_{M,N,1}, \cdots, \overline{x}^A_{M,N,n}, \sigma^A_{M,N,n}, \overline{y}^B_{M,N}, \sigma^B_{M,N} \end{pmatrix}, \tag{8}$$

  where $\overline{x}^A_{m,r,i}$ and $\sigma^A_{m,r,i}$ are parameters of the input fuzzy sets A^r_i, $m = 1, \ldots, M$; $r = 1, \ldots, N$; $i = 1, \ldots, n$, and $\overline{x}^B_{m,r}$ and $\sigma^B_{m,r}$ are parameters of the output fuzzy sets B^r for m-th fuzzy system; M - number of nonzero elements of correction matrix $\mathbf{P_A}$.
- Fitness function is based on difference between output signals $\hat{x}_1$, $\hat{x}_2$ generated by the created model at step $k+1$ and corresponding reference x_1, x_2 values:

$$fitness(\mathbf{X}) = \sqrt{\frac{1}{2 \cdot K} \sum_{k=1}^{K} \begin{pmatrix} (x_1(k+1) - \hat{x}_1(k+1))^2 + \\ (x_2(k+1) - \hat{x}_2(k+1))^2 \end{pmatrix}}, \tag{9}$$

  where K is a number of reference values. Starting values for the model are reference values at step 1. In practical implementation, the actual reference values will be obtained by the non-invasive observation, for example by processing a data packets, which are sent in the real-time in the Ethernet network (see e.g. [16]).
- Genes in chromosomes $\mathbf{X}$ were initialized according with method described in [10].

Detailed description of the evolutionary strategy $(\mu + \lambda)$, used to train neuro-fuzzy systems, can be found in [4, 10].

5 Experimental Results

In our paper we considered the well-known harmonic oscillator as an example to demonstrate the usefulness of the proposed modelling method:

$$\frac{d^2x}{dt^2} + 2\zeta\omega\frac{dx}{dt} + \omega^2 x = 0, \tag{10}$$

where ζ, ω are oscillator parameters, and $x(t)$ is a reference value of the modelled process as function of time. The main parameter of the oscillator ω (angular

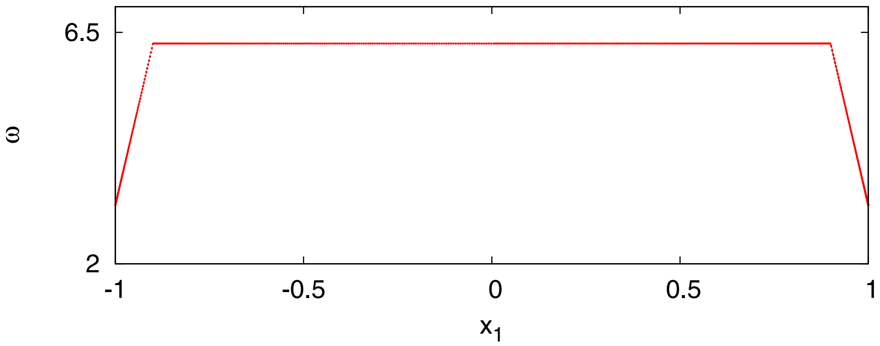

Fig. 2. The value of ω parameter as the function of state variable x_1

frequency) is intentionally modified in some operating points in simulation to make the object nonlinear, as depicted in Fig. 2. In our simulations ζ is the constant value $\zeta = 0$.

We used the following state variables: $x_1(t) = dx(t)/dt$ and $x_2(t) = x(t)$. In such case the system matrix $\mathbf{A}$ is described as follows:

$$\mathbf{A} = \begin{bmatrix} 0 & -\omega^2 \\ 1 & 0 \end{bmatrix}, \tag{11}$$

and matrix of the corrections $\mathbf{P_A}$ is in the form:

$$\mathbf{P_A} = \begin{bmatrix} 0 & p_{12}(\mathbf{x}) \\ 0 & p_{22}(\mathbf{x}) \end{bmatrix}. \tag{12}$$

In our method we assume that the system matrix A is known, so the goal of the modelling was to recreate the unknown parameters of correction matrix $p_{12}(\mathbf{x})$ and $p_{22}(\mathbf{x})$ in such a way that the model reproduces the reference data as accurately as possible. Because in general case the analytical dependences used to generate the reference data are not known in order to recreate unknown parameters we used multi-input, single-output fuzzy systems. As the input of each system the measurable output signals of modelled process $\hat{x}_1(k)$ and $\hat{x}_2(k)$ were used. The outputs of fuzzy systems were used as the values of correction matrix parameters $p_{12}(\mathbf{x})$ and $p_{22}(\mathbf{x})$. The accuracy of the model was determined by comparing values of its output signals $\hat{x}_1(k)$ and $\hat{x}_2(k)$ with reference values $x_1(k)$ and $x_2(k)$. The error was computed according to formula (9).

In our simulations the neuro-fuzzy systems (7) with Gaussian membership functions and algebraic t-norm were used. In order to determine membership functions parameters (mean value and standard deviation) we used evolutionary strategy (μ, λ) which is characterized by the following parameters: $\mu = 50, \lambda = 300, p_m = 0.077$ and $p_c = 0.7$ and the number of generations $= 2000$.

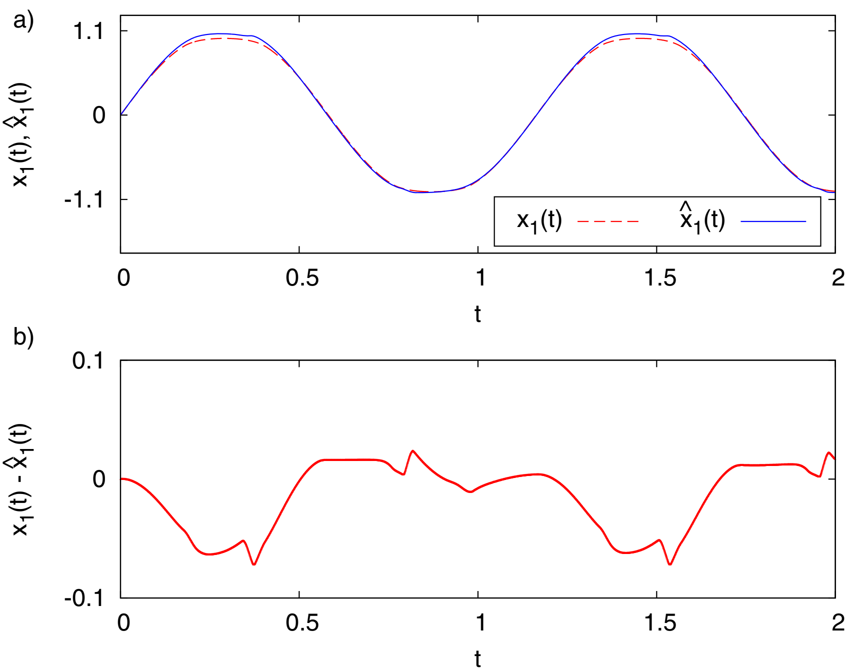

Fig. 3. Comparison between the reference and estimated data

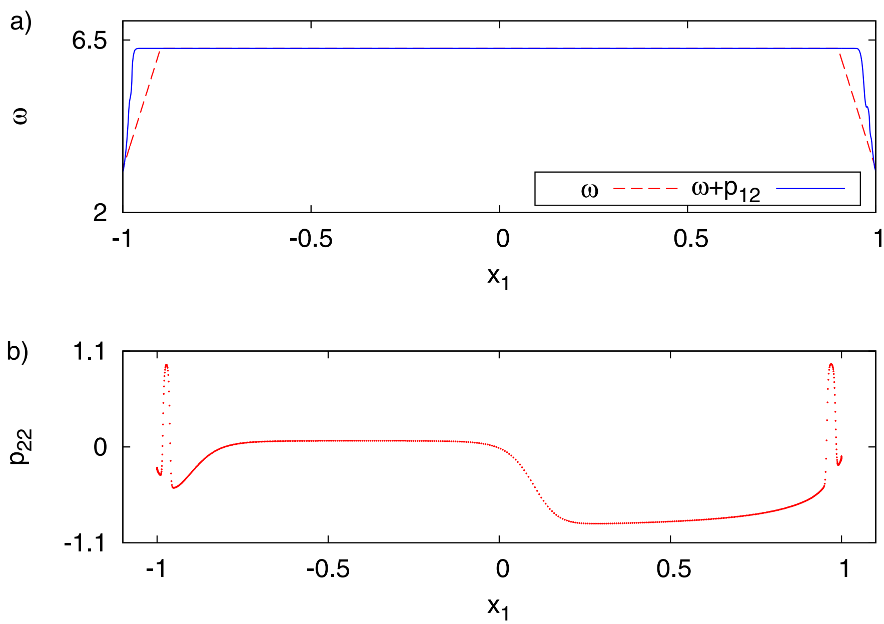

Fig. 4. Comparison between the actual and modelled by fuzzy system values of matrix coefficients

The neuro-fuzzy systems (7) obtained as results of evolutionary process are characterized by 5 rules, 2 inputs ($\hat{x}_1(k)$ and $\hat{x}_2(k)$) and single output. The experimental results are depicted in Fig. 4-6. The accuracy of nonlinear modelling obtained in out simulations (the average RMSE) was lower than 0.002.

It should be noted that difference between the reference and estimated data are negligible (Fig. 3). In addition, differences between actual and modelled by the fuzzy system (7) values of matrix $\mathbf{P}_\mathbf{A}$ coefficients are also acceptable.

6 Conclusion

In this paper a new method for modelling of nonlinear dynamic systems was proposed. This method is based on the local linear approximation performed continuously for successive operating points. It allows to use the very convenient and well-known method from linear control theory to analyse the obtained model.

Moreover, the proposed hybrid modelling method, based on fuzzy logic system and state variables technique, gives the potential possibility to the interpretation of accumulated knowledge.

The simulation shows the fully usefulness of the proposed method.

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