On the Identification and Establishment of Topological Spatial Relations

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Abstract. Human beings use spatial relations to describe many daily tasks in their language. However, to date in robotics the navigation problem has been thoroughly investigated as the task of guiding a robot from one spatial coordinate to another. Therefore, there is a difference of abstraction between the language of human beings and the algorithms used in robot navigation. This article introduces the research performed on the use of topological relations for the formalization of spatial relations and navigation. The main result is a new heuristic, called Heuristic of Topological Qualitative Semantic (HTQS), which allows the identification and establishment of spatial relations.

Keywords: qualitative navigation, spatial relations, Heuristic of Topological Qualitative Semantic (HTQS).

1 Introduction

Robots have proven to be useful tools for police officers, surgeons and cleaning staff. However, in each of these cases robots are designed for specific tasks. Currently, the construction of multifunctional robots is studied: robots able to navigate in environments where the behavior must change such as offices or homes, and to directly work with humans in various activities. Until now, many successes in robotics are related to the reactive paradigm [2]. But it seems difficult for the reactive paradigm to enable the development of multifunctional robots, as the manner to interact in the environment will constantly change depending on the task at hand. It should be kept in mind that the natural way of communicating spatial tasks by humans is by making use of spatial relations. A multifunctional robot would continually receive tasks such as:

"Take the package that is on the table and leave it in the closet."

This way, it seems that it is necessary for a multifunctional robot to function in human's life to possess a high degree of spatial reasoning, and concretely about spatial relations.

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I believe that within the symbolic paradigm it is possible to develop decision-making algorithms founded on the topological relations and apply these to navigation. So, I have begun a research program to study the usefulness of topological notions for navigation and get multifunctional robots. The starting hypothesis of this work is that topological relations are a useful and effective tool in achieving navigation based on identifying and establishing spatial relations. In this article, I present the first theoretical results about identifying and establishing spatial relations through topological notions of my program research.

This paper is structured as follows. In section 2 spatial representation methods used in AI and their application are examined. Then, section 3 introduces the Heuristic of Topological Qualitative Semantics. Afterwards, section 4 describes a method for identifying topological relations. Finally, in section 5 we will discuss the results and future work.

2 Representation of Space for Navigation

The methods of robot decision making can be divided into quantitative and qualitative, depending on the type of spatial representation used. In qualitative methods, space is not represented by a metric space. But specifically the best way to represent the space is not a resolved issue yet in qualitative methods. Even when the first years of research on general methods of qualitative spatial reasoning were unsuccessful, "the poverty conjecture" was set out [13]. This conjecture states that: "There is no purely qualitative, general-purpose representation of spatial properties". But research conducted in recent times seems to refute or at least weaken "the poverty conjecture" [3].

In AI the problems of satisfiability, decidability, complexity of different variants, composition of spatial relations, and its mixture with fuzzy techniques for handling uncertainty given a qualitative description of a static space have been largely treated [20,18,22]. In the last times the same kind of problems also for dynamic spaces and reasoning about the movement [23,4]. However, the issue is the creation of an algorithm to make decisions.

Depending on the complexity of the environment and the objective assigned to a robot, the architecture for decision making may come to consist of three levels: implementation, navigation (or local navigation) and planning (or global navigation). Among the objects of a space there are three types of spatial information: topological, orientational and positional. Proposals for building qualitative spatial representations are based on any of the above types of spatial information in a qualitative basis. There is the opinion in the robotics field that the qualitative topological information is appropriate for planning, but too abstract to allow the realization of navigation [21]. So, until now the problem of qualitative planning has been also largely treated [12][19][16][17]but there are few works about navigation with qualitative methods and they are focus on soccer[11][5]. So, there is not general methods to translate the information captured by the robot sensors to a high-level representation with a full group of spatial relations. Therefore, in the way for getting multifunctional robots it is necessary to investigate the use of spatial relations for navigation. Based on before we define a new type of navigation problem, which will be called "navigation problem based on spatial relations", which can be expressed as follows:

Given a starting point A establish the spatial relation(s) $G(G_1, G_2, ...)$ *among objects* $[O, O']$ $([O_1, O_1'], [O_2, O_2'], ...)$ *by using [the robot's] knowledge and sensory information received.*

3 Heuristic of Topological Qualitative Semantic

Initially, Freksa proposed the idea of representing the relation between relations by a graph, it is called conceptual neighborhood[15], and uses it to reason with incomplete or coarse knowledge about temporal relations and spatial relations [14]. Independently, Egenhofer propose a similar idea to reasoning about changes in topological relations[6]. Our research has come from the work of Egenhofer and Al-Taha[6]. They introduced a way to relate the topological relations by defining a distance between them and their representation in a graph. In the graph, each one of the topological relations is represented by a node and each node has an arc with that node with a minimal distance to their respective topological relations. This type of graph is called the Closest Topological Relationship Graph (CTRG). The same authors also created a revised version that includes arcs between nodes when there is a transformation that enables passing two objects from a topological relation to another. In their work, Egenhofer and Al-Taha studied the possible use of CTRG for inference and prediction. The conclusions were that the CTRG could be used to infer and predict with certainty only in a few cases. The problem discovered by Egenhofer and Al-Taha is the CTRG threw more than one possible deformation diagram in some cases.

The first target has been develop an algorithm of the kind of analysis of means and ends for topological relations. Thanks to Ernst's investigations [10], we know that an analysis of means and ends will find a solution if you can define a complete order relation on the differences. A graph is a binary relation on a set. Therefore, as the CTRG is a graph with cycles, it is impossible to create an analysis of means and ends from it. So, it is needed to obtain a linear binary relation on the topological relations. The reason for the CTRG to be non-linear is the wide range of changes that the objects may suffer. Clearly, Geography Information System takes into account changes in the size of an object, as rivers, lakes, sea or forest stands may increase or decrease surface drastically. However, in a common navigation environment, such as a building, this does not happen to objects (that is, to robots). Therefore, one can take into account the following two conditions: temporal isosize and temporal isoshape to generate different lineal graphs. But the linealization is not the final solution. There are cases for which the 8 relations of the "9-intersection model" [8] are not enough to make decisions because it keeps multiple trajectories from one an initial node and another final node. Hence, more relation are necessary to distinguish that trajectories. I propose the following 13 relations: Disjoint-0, Meet-0, Overlap-0, CoveredBy-0, Covers-0, Inside, Equal, Contains, Disjoint-1, Meet-1, Overlapping-1, CoveredBy-1, Covers-1. The relative topological relations are jointly exhaustive and pairwise disjoint. The formal definition of topological relations will be seen later on. In order to distinguish between the set of relations to be defined and R_8, the new set of relations is called *relative topological relations* and denoted by S_{13}. This relations will be used to create a kind of lineal graphs. Each of those graphs is called *topological reasoning linear graph*(TRLG).

The decision-making method must indicate what action must be done to establish a spatial relation. The method found here is a heuristic. The heuristic finds a solution to establishing the topological relations problem. Let us call it Heuristic of Topological Qualitative Semantic (HTQS). To explain in what consists the HTQS a generic example to illustrate its operation will be given. Imagine you have two objects: an object that has the capacity to act, o_A, and a reference object, o_R, in respect of which one wants that o_A be able to fix a concrete relative topological relation. Thus, suppose that o_A can work with three actions:

$$\Lambda(o_A) = \{f_1|_x(x) = x + 1, \quad f_2|_x(x) = x, \quad f_3|_x(x) = x - 1\}$$

and o_R can not move and remains static.

$$\Lambda(o_R) = \{f_4|_x(x) = x\}$$

The first data structure used is a table that is constructed by applying a means-ends analysis on the actions. Thus, the functions are labeled with the way in which the quantitative positions in space are changed, assigning an order relation to be satisfied when the action applies. If isosize and isoshape conditions are met, there are only three possible cases. Thus, the functions of o_A are labeled creating the results contained in the next table.

function	label
f_1	$<$
f_2	$=$
f_3	$>$

The second data structure used are the TRLG. The nodes of each TRLG are labeled with an incremental enumeration. In this example we take that o_A and o_R are bigger than one unit and equal in size. Since we are in $\mathbb{Z}$ the form of an object is necessarily the same if the size is the same. Therefore, the TRLG used is shown in Fig. 1.

Once you have the above data structures, the algorithm to select the function has the next steps:

1. The relative topological relation between the objects o_A and o_R is calculated from their current positions, s_c, and the number associated to the node of s_c , n_{s_c}, is stored.
2. The number associated to the node of the relative topological relation which is the target between the objects o_A and o_R, n_{s_t}, is gotten.

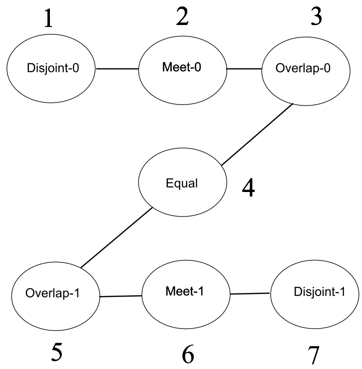

Fig. 1. Incremental enumeration on an TRLG

3. It is checked what order relation holds between n_{s_c} and n_{s_t} from the following:
   - $n_{s_c} < n_{s_t}$
   - $n_{s_c} = n_{s_t}$
   - $n_{s_c} > n_{s_t}$
4. It is looked at the means-ends table using the order relation that holds between n_{s_c} and n_{s_t} to pick the action that is labeled with the same order relation. This is the action selected by heuristics.

The heuristic is for a one dimensional space but it can be easily generalized to a topological n-dimensional space. To do this, several results from topology can be used to build the topological space $\mathbb{Z}^n$. It is known that, given a topology on X and another on Y, there is a canonical way to create a topology on the cartesian product $X \times Y$, the so-called product topology. Thus, from the base $\mathcal{B} = \{(x_1, x_2) : x_1 < x_2 \quad x_1, x_2 \in \mathbb{Z}\}$, which generates the order topology on $\mathbb{Z}$, we can construct the product topology on $\mathbb{Z} \times \mathbb{Z}$. The definition of the product topology on $X \times Y$ can be applied equally to $X_1 \times X_2 \times \cdots \times X_n$. Therefore, there is no problem in building a generalization to dimension n of the topological space.

The next subject is the topological relations in a space $X_1 \times X_2 \times \cdots \times X_n$. Because space is constructed by the Cartesian product, the n-dimensional topological relation between two objects is defined by a tuple of n components, being the component k the topological relation that occurs in dimension k between the two objects. Thus:

$$\forall A, B \in X_1 \times \cdots \times X_n \quad A r_x B \Leftrightarrow A\langle r_{x'}, r_{x''}, ..., r_{x^n}\rangle B$$

where r_{x^k} is the relation between A and B in dimension k.

Therefore, the generalization to n dimensions of the HTQS consists in applying it successively on each dimension. Nevertheless, this generalization has several restrictions on the real world because for the generalization it is necessary that the sides of each object have right angles to each other. This way, 2-dimensional objects are rectangles, 3-dimensional objects are cubes, and so on. This ensures that one can change the corresponding topological relation in one dimension without change the topological relation in another one. That is, it meets the principle of optimality, and the combination of a solution for every dimension is solution to the complete problem.

3.1 HTQS Over Non-dense Sets

The criteria to assign a TRLG to two objects in dense topological spaces is the size relationship between two objects . However we focus on non-dense spaces, because the intention is implement the algorithm in robots which as sensor systems as data structures are going to be discrete. In no-dense spaces the size ratio between objects is not sufficient to assign a TRLG, since in each case subcases arise by the difference between the dense and non-dense sets. Table 1 presents all subcases for non-dense topological spaces, with their corresponding TRLG, assuming isosize and isoshape in the objects.

Table 1. This table shows the conditions between two objects to assign a TRLG in a topological space generated with a non-dense set. The notation can be consulted int the 3.

Case	Subcase	Lineal topological reasoning graph
$\|A\| > \|B\|$	$\|A\| = (\|B\| + 1)$	$\langle s_1, s_2, s_3, s_5, s_{10}, s_{11}, s_{12}, s_{13} \rangle$
	$\|A\| > (\|B\| + 1)$	$\langle s_1, s_2, s_3, s_5, s_8, s_{10}, s_{11}, s_{12}, s_{13} \rangle$
$\|A\| < \|B\|$	$(\|A\| + 1) = \|B\|$	$\langle s_1, s_2, s_3, s_4, s_9, s_{11}, s_{12}, s_{13} \rangle$
	$\|A\| > (\|B\| + 1)$	$\langle s_1, s_2, s_3, s_4, s_6, s_9, s_{11}, s_{12}, s_{13} \rangle$
$\|A\| = \|B\|$	$\|A\| = \|B\| = 1$	$\langle s_1, s_7, s_{13} \rangle$
	$\|A\| = \|B\| > 1$	$\langle s_1, s_2, s_3, s_7, s_{11}, s_{12}, s_{13} \rangle$

4 Calculation of Relative Topological Relations

The previous section described the heuristic to solve a establishing topological relations problem. The first step of the heuristic says that the relative topological relation is calculated using the positions of the two objects. But so far how to perform the calculation has not been explained. This step is important because it connects the mechanisms of perception of the environment of a mobile robot with the navigation algorithm. Thus, this section introduces a method to calculate the relative topological relation of two regions of space.

At first, one might consider the use of the "9-intersection model" for the calculation[7]. But there are two impediments that prevent implementation. The first obstacle is that the "9-intersection model" was proposed for dense topological spaces, such as $\mathbb{R}^n$, and fails for non-dense spaces. To solve this problem it has been proposed to use interior and border definitions different from those contained in the topology [9]. But these new definitions from the field of digital topology lose the simplicity that makes the "9-intersection model" so interesting. The second and most important impediment is that the number of relative topological relations is 13, compared to the 8 relations that the "9-intersection model" can define for regions in $\mathbb{R}^2$. Therefore, the "9-intersection model" reconstructed with the definitions of digital topology has no ability to define the 13 relative topological relations that are necessarily to be defined. Due to the two obstacles just mentioned, we have created a variant of an old formalism. The old formalism is the method for finding relations between time intervals used in the Allen algebra [1]. Freksa noted that the 13 relationships defined by Allen's formalism could be interpreted as spatial in a spatial context [14]. Certainly, Allen applies his method to convex intervals in $\mathbb{R}$, and these do not correspond with opens sets of the topological connected space over $\mathbb{R}$. Nevertheless, the convex intervals over $\mathbb{Z}$ coincide with the open sets of the order topology on $\mathbb{Z}$. Thus, instead of using the Allen method to calculate the relations between two time intervals, it is used to calculate a topological relation. Indeed, the interpretation given to $\mathbb{Z}$ is that of a spatial dimension. Allen's method also hasn't the second impediment cited for the "9-intersection model" because it can characterize the relative topological relations. Although it is needed to do some variations due that Allen was using $\mathbb{R}$, a dense set, as model of time, but $\mathbb{Z}$ is a not dense set. The Allen's formalism is the next set of conditions

$$\begin{pmatrix} \min(X) < \min(Y) & \min(X) < \max(Y) \\ \max(X) < \min(Y) & \max(X) < \max(Y) \\ \min(X) = \min(Y) & \min(X) = \max(Y) \\ \max(X) = \min(Y) & \max(X) = \max(Y) \end{pmatrix}$$

Now, we want to use the formalism on a no-dense set as model of space. For this reason we introduce two changes to modify the definition of the relation "Meets". This changes are in my opinion a better definition of the relation Meets for space because two objects do not share points of space, but they have adjacent points. The new formalism, called *order propositions matrix* and denoted by $P^{\leqq}(X,Y)$ consists of the following matrix:

$$P^{\leqq}(X,Y) = \begin{pmatrix} \min(X) < \min(Y) & \min(X) < \max(Y) \\ \max(X) < \min(Y) & \max(X) < \max(Y) \\ \min(X) = \min(Y) & \min(X) - 1 = \max(Y) \\ \max(X) + 1 = \min(Y) & \max(X) = \max(Y) \end{pmatrix}$$

Each of the elements of the matrix is a proposition that takes a value of $\mathbb{B} = \{0,1\}$, depending on whether the proposition is false(0) or true(1). Table 2 shows the characterization of each of the relative topological relations through the order propositions matrix.

Thus, the values taken by matrix $P^{\leqq}(X,Y)$ directly show the relative topological relations between two sets, and therefore, the node of the TRLG they deserve. The reader should realize that Allen's method enables characterizing 13 binary relations (see Table 3). These thirteen relations are composed of 6 binary

Table 2. Table containing the relative topological relations defined by means of the order propositions matrix

Disjoint-0	Meet-0	Overlap-0	CoveredBy-0	Covers-0
$P^{<} = \begin{pmatrix} 1\,1 \\ 1\,1 \\ 0\,0 \\ 0\,0 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 1\,1 \\ 1\,1 \\ 0\,0 \\ 1\,0 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 1\,1 \\ 0\,1 \\ 0\,0 \\ 0\,0 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 0\,1 \\ 0\,1 \\ 1\,0 \\ 0\,0 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 1\,1 \\ 0\,0 \\ 0\,0 \\ 0\,1 \end{pmatrix}$
Inside	Equal	Contains		
$P^{<} = \begin{pmatrix} 0\,1 \\ 0\,1 \\ 0\,0 \\ 0\,0 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 0\,1 \\ 0\,0 \\ 1\,0 \\ 0\,1 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 1\,1 \\ 0\,0 \\ 0\,0 \\ 0\,0 \end{pmatrix}$		
Covers-1	CoveredBy-1	Overlap-1	Meet-1	Disjoint-1
$P^{<} = \begin{pmatrix} 0\,1 \\ 1\,0 \\ 0\,0 \\ 0\,1 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 0\,1 \\ 0\,0 \\ 0\,0 \\ 0\,1 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 0\,1 \\ 0\,0 \\ 0\,0 \\ 0\,0 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 0\,0 \\ 0\,0 \\ 0\,1 \\ 0\,0 \end{pmatrix}$	$P^{<} = \begin{pmatrix} 0\,0 \\ 0\,0 \\ 0\,0 \\ 0\,0 \end{pmatrix}$

Table 3. The 13 binary relations enabled by Allen's method

Allen's Relations	Symbol	Relative Topological Relations (S_{13})	Symbol
Before	r_1	Disjoint-0	s_1
Meets	r_2	Meet-0	s_2
Overlaps	r_3	Overlap-0	s_3
Starts	r_4	CoveredBy-0	s_4
Finished-by	r_6^-	Covers-0	s_5
During	r_5	Inside	s_6
Equal	r_7	Equal	s_7
Includes	r_5^-	Contains	s_8
Finishes	r_6	CoveredBy-1	s_9
Started-by	r_4^-	Covers-1	s_{10}
Overlapped-by	r_3^-	Overlapping-1	s_{11}
Meet-by	r_2^-	Meet-1	s_{12}
After	r_1^-	Disjoint-1	s_{13}

relations, their corresponding 6 inverse binary relations, and the binary relation "equal". The inverse relation of "equal" is itself. These relations defined for the temporal dimension can also be used in spatial dimension.

If we recall the definition of inverse

$$R^{-} = \{(y, x) : (x, y) \in R\}$$

one realizes that any spatial location between two objects meets a relation and its inverse. When describing the temporal relationship between two events, it is indifferent to use a relation or its inverse, since the observer is describing from the outside, and both relations contain the same information. But when a particular agent must make a decision to move, the observer is the agent; so it matters if you use a relation or its inverse, since actions are labeled for a specific order within pairs of objects in the relations. Thus, for HTQS only one of two relations is useful, the other leads to a misunderstanding of the algorithm.

So, what is the relation that must be used in the HTQS? The answer is that it is chosen according to what object is applying HTQS to make decisions. In $P^{\leqq}(X, Y)$, X must always be the agent making the decision, and Y is the object respect of which the agent must make a decision.

5 Discussion and Future Work

Language and human thought make use of spatial relations for the description of many tasks performed daily. But so far, the approaches most commonly used for representing space in navigation algorithms have been through numeric spatial coordinates. The creation of algorithms to perform navigation tasks, identifying and establishing spatial relations, is a logical step in the objective of getting multifunctional robots that can be integrated into human society. The application of topological relations for the representation of spatial relations has been used in cartography and geography for the construction of commercial GIS. However, topological relations to date have not been investigated in robot navigation, despite the importance of spatial relations for human navigation and communication. Therefore, I have decided to start a research to study the usefulness of topological notions to achieve multifunctional robots. This article has presented the first results of my research in decision making to establish spatial relations.

The next step in my research programa will be the creation of an architecture will use HTQS to make decisions. The architecture must implement a knowledge base to store information to choose what will be its targets and it let's to avoid the physical restrictions of the environment. Also another issue will be the mathematical method to identify the topological relation between two objects. The actual representation imposes important restrictions in the generalization to a n-dimensional space, an deep analysis can let improve the situation finding another method to representation. One important issue will be the technique of computer vision which will be linked with the spatial representation.

To sum up, my research has got a method to making decisions based on identifying and establishing spatial relations. I have linked concepts of different fields. Namely the work of Egenhofer and colleagues that belongs to the realm of GIS, the research of Ernst on heuristic analysis of means and ends, and Allen's algebras for temporal reasoning. Three results, which apparently were not related, are the foundation that has allowed the development of the HTQS heuristics. The new theoretical results presented here are the basis on which I am developing new navigation algorithms. Thus, the work presented here offers the first results in the agenda of my research program to develop a navigation architecture based on topological notions for the creation of robots that use spatial relations in their interactions with humans and methods to make decisions.

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