Multi-Agent Temporary Logic $TS4^{U}_{K_n}$ Based at Non-linear Time and Imitating Uncertainty via Agents' Interaction

David McLean[1] and Vladimir Rybakov[1,2]

[1] School of Computing, Mathematics and DT,
Manchester Metropolitan University,
John Dalton Building, Chester Street, Manchester M1 5GD, U.K.
[2] Institute of Mathematics, Siberian Federal University,
79 Svobodny Prospect, Krasnoyarsk, 660041, Russia
{D.McLean,V.Rybakov}@mmu.ac.uk

Abstract. This paper considers AI problems concerning reasoning in multi-agent environment. We introduce and study multi-agents' non-linear temporal logic $\mathbf{TS4^{U}_{K_n}}$ based on arbitrary (in particular, non-linear, finite or infinite) frames with reflexive and transitive accessibility relations, and individual symmetric accessibility relations R_i for agents. Main accent of our paper is modeling of logical uncertainty for statements via interaction of agents (passing knowledge). Conception of interacting agents is implemented via arbitrary finite paths of transitions by agents accessibility relations. We address problems decidability and satisfiability for $\mathbf{TS4^{U}_{K_n}}$. It is proved that $\mathbf{TS4^{U}_{K_n}}$ is decidable (and, in particular, the satisfiability problem for it is also decidable). We suggest an algorithm for checking satisfiability based on computation possibility of refutation special inference rues in finite models of effectively bounded size.

Keywords: multi-agents' logic, interacting agents, temporal logic, non-linear temporal logic.

1 Introduction

Basically a multi-agent system (MAS) is a system composed of multiple interacting intelligent agents within certain environment. Study of MASs (with autonomous or interacting, say competitive) agents is an active area in modern AI. Technique and research outputs are various, diverse and work well in many contemporary areas (though, it seems, most popular area is applications in IT, – cf. Nguyen et al [17–19], Arisha et al [1], Avouris [2], Hendler [11]). Area of modeling reasoning (initially, – an individual human reasoning) is an old branch of AI, which now includes technique for modeling multi-agents reasoning. These techniques use a logical language for reasoning about agents' knowledge and properties (e.g. various technique of mathematical (symbolic) logic is widely used (cf. [10, 12, 13])); in particular, multi-agent modal logics were implemented.

L. Rutkowski et al. (Eds.): ICAISC 2013, Part II, LNAI 7895, pp. 375–384, 2013.

Historically, multi-agent epistemic logics have found various applications in fields ranging from AI domains such as robotics, planning, and motivation analysis in natural language, to negotiation and game theory in economics, to distributed systems analysis and protocol authentication in computer security. Ability of intelligent agents to reason about knowledge is an essential feature of those applications. For instance, technique of non-classical logic (as modal, temporal or epistemic) gives inference capabilities to deduce implicit knowledge from the explicitly represented facts. This approach allows to describe the properties (specifications) with explicit, mathematically preciseness, which simplifies identification. Historically, usage of logical language in knowledge representation is known since reasonably long ago (cf. Brachman and Schmolze (1985, [7]), Moses and Shoham (1993, [14]), Nebel (1990, [15]). It also had some applications in industry Rychtycki (1996, [30]).

To represent knowledge and to specify it, the question what is a shared knowledge and what is a common knowledge for all agents has been risen. It seems, first ideas concerning this specification appeared in Barwise (1988, [8]), Niegerand and Tuttle (1993, [16]), Dvorek and Moses (1990, [9]). Modern approach to common knowledge logics was suggested in the book Fagin R., Halpern J., Moses Y., Vardi M. (1995, [10]). This book contains a series of theorems on completeness for various common knowledge logics w.r.t. possible worlds models.

In study of multi-agents' reasoning, an essential point is how to represent interaction of agents, exchange of information (cf. e.g., Sakama et al [20]). Study of multi-modal agents logics and temporal agents-logics, representing these features, were undertaken in a series of works by V.Rybakov. A kernel part in these works was a representation the case when the logics describe interacting agents. In Rybakov, 2009, [26] some technique to handle interactions was found, and, as a consequence, it was proved that the multi-agent Linear Temporal Logic (with UNTIL and NEXT and with interacting agents, or dually, common knowledge) is decidable, that the satisfiability problem for this logic is also decidable, and some algorithms solving the problem were found (cf. also Rybakov [25]). Besides, research of just multi-agent logics (as modal and temporal) with aim to find solution of satisfiability problem (and decidability corresponding logics) was earlier undertaken in Rybakov [27, 28], Babenyshev and Rybakov [3–6].

This paper studies the multi-agents' non-linear temporal logic $\mathbf{TS4}^{\mathbf{U}}_{\mathbf{K_n}}$ based on arbitrary (in particular, non-linear, finite or infinite) frames with reflexive and transitive accessibility relations, and individual symmetric accessibility relations R_i for agents. The impellent aim of our paper is how to represent logical uncertainty in multi-agent reasoning. We suggest to model logical uncertainty for statements via interaction of agents (passing knowledge).

Conception of interacting agents is implemented via arbitrary finite paths of transitions by agents accessibility relations. This approach uses technique developed by V.Rybakov in the mentioned above papers, it essentially uses [26], and the current paper extends results from [29] in order to handle logical uncertainty via interaction of agents. Main computational problems we dealing with are problems of decidability and satisfiability for $\mathbf{TS4}^{\mathbf{U}}_{\mathbf{K_n}}$. We show that $\mathbf{TS4}^{\mathbf{U}}_{\mathbf{K_n}}$

is decidable (in particular, the satisfiability problem for it is also decidable). An algorithm for checking satisfiability based on computation truth values of special inference rues in finite models of effectively bounded size is constructed. Paper contains some preliminary information for understanding obtained results.

2 Basic Notation, Definitions, Known Facts

We start from recalling notation and known facts concerning modal, multi-modal and temporal logics (so, some familiarity with these areas is assumed, though we give below the definitions to follow the paper). First, we the need special Kripke/Hintikka-like frames $\mathbf{F}$ defined as follows: $\mathbf{F} := \langle W, R, R_1, \ldots, R_n \rangle$, where W is a set of states (symbols of states, modeling web sites), R is a binary relation on W (modeling, for example, web connections, or runs of computations): aRb means that there is a connection from state a to state b (e.g. by clicking link buttons, some amount of steps in a computational procedure, etc.).

Relation R is assumed to be reflexive and transitive (which corresponds well to understanding time in a run of a computation, and models transitions in runs of computations, passing via web connections, etc). Thus, the following holds: $\forall a \in W,\ aRa;\quad \forall a, b, c \in W,\ aRb\ \&\ bRc\ \Rightarrow\ aRc$. States (worlds) in $\mathbf{F}$ – symbols from W – form with respect to R clusters. A cluster $C(a)$ generated by $a \in W$ is the set $\{b \mid b \in W, aRb\ \&\ bRa\ \}$.

Any relation R_i (agent i accessibility relation) is reflexive, transitive and symmetric relation (i.e. $aR_ib \Rightarrow bR_ia$) on $C(a)$ for any $a \in W$. An interpretation of such approach to model agents' relation via internet connections is as follows: being logged at web-site a, i-agent may access by R_i some web sites from $C(a)$ (in accordance with possession of access rules/passwords) - and switch between sites in its disposal freely, back and forth. Yet i cannot jump to another sites outside $C(a)$ without permitting (convoy) from administrator.

For computational runs the interpretation is similar: there are several computational threads imitated as relations R_i – any thread is a computational agent, relation R holds a cluster of local computations around an time tick. We would like to built a framework for description multi agent reasoning based at such sematic approach.

Our approach is based at a hybrid of a non-linear temporal logic and some knowledge multi-agent logic. Language of our logic consists of standard language of Boolean logic extended with temporal and agent knowledge operations. So, it contains potentially infinite set of propositional letters P; the logical operations include usual Boolean logical operations and usual unary agent knowledge operations K_i, $1 \leq i \leq m$; also it contains the operation for knowledge via agents' interaction **KnI** (this operation may be expressed as a dual counterpart of the common knowledge operation introduced, e.g. in Fagin et al [10]), and the unary logical operation U with meaning 'uncertain'.

Temporal unary operations are $\Diamond^+$ (with meaning 'possible in future' by a sequence of computational steps) and $\Diamond^-$ (with meaning *possible, so to say* *in past, – by a sequence of backtracks*). The formation rules for formulas are standard: any propositional letter is a formula,

(i) if α and β are formulas, then $\alpha \wedge \beta$ is a formula;

(ii) if α and β are formulas, then $\alpha \vee \beta$ is a formula;

(iii) if α and β are formulas, then $\alpha \to \beta$ is a formula;

(iv) if α is a formula, then $\Diamond^{+}\alpha$ is a formula;

(v) if α is a formula, then $\Diamond^{-}\alpha$ is a formula;

(vi) if α is a formula, then for any i $K_i\alpha$ is a formula;

(vii) if α is a formula, then $\mathbf{KnI}\alpha$ is a formula;

(vii) if α is a formula, then $\mathrm{U}\alpha$ is a formula.

Intuitive meaning of this operations is as follows.

$K_i\varphi$ can be read: agent i knows φ in the current state;

$\Diamond^{+}\varphi$ says that there is a state (web site) b accessible from the current state a by a sequence of links, were the statement (formula) φ is true at b. So to say, there is a state, accessible in future, where φ is true.

$\Diamond^{-}\varphi$ means that there is a state b accessible from the current state a by a sequence of backtracks, were the statement (formula) φ is true at b.

$\mathbf{KnI}\varphi$ means: in the current state, the statement φ *may be known by interaction between agents.*

$\mathrm{U}\varphi$ has meaning the statement φ is uncertain (has uncertain truth value).

Next step of our construction are rules to compute truth values of formulas at states of arbitrary frames (where some truth valuation for formulas' letters is given; such frames with given valuations we will call models). So, given a frame $\mathbf{F} := \langle W, R, R_1, \ldots, R_n \rangle$, and a set of propositional letters P, a valuation V of P in $\mathbf{F}$ is a mapping of P into the set of all subsets of the set W, in symbols, $\forall p \in P, V(p) \subseteq W$. If, for an element $a \in W$, $a \in V(p)$ we say *the fact p is true in the state a.* In the notation below $(\mathbf{F}, a) \Vdash_V \varphi$ is meant to say the formula φ in true at the state a in the model $\mathbf{F}$ w.r.t. the valuation V. The rules for computation of truth values of formulas are as follows:

$$\forall p \in P,\ \forall a \in W \quad (\mathbf{F}, a) \Vdash_V p \iff a \in V(p);$$

$$(\mathbf{F}, a) \Vdash_V \varphi \wedge \psi \iff [(\mathbf{F}, a) \Vdash_V \varphi \text{ and } (\mathbf{F}, a) \Vdash_V \psi];$$

$$(\mathbf{F}, a) \Vdash_V \varphi \vee \psi \iff [(\mathbf{F}, a) \Vdash_V \varphi \text{ or } (\mathbf{F}, a) \Vdash_V \psi];$$

$$(\mathbf{F}, a) \Vdash_V \varphi \to \psi \iff [not[(\mathbf{F}, a) \Vdash_V \varphi] \text{ or } (\mathbf{F}, a) \Vdash_V \psi];$$

$$(\mathbf{F}, a) \Vdash_V \neg\varphi \iff \text{ not } [(\mathbf{F}_C, a) \Vdash_V \varphi];$$

$$(\mathbf{F}, a) \Vdash_V K_i \varphi \iff \forall b \in W[(aR_i b) \implies (\mathbf{F}, b) \Vdash_V \varphi];$$

$$(\mathbf{F}, a) \Vdash_V \Diamond^+ \varphi \iff \exists b \in W\,[(aRb) \text{ and } (\mathbf{F}, b) \Vdash_V \varphi];$$

$$(\mathbf{F}, a) \Vdash_V \Diamond^- \varphi \iff \exists b \in W\,[(aR^- b) \text{ and } (\mathbf{F}, b) \Vdash_V \varphi];$$

$$(\mathbf{F}, a) \Vdash_V \mathbf{KnI}\varphi \Leftrightarrow \exists a_{i1}, a_{i2}, \ldots, a_{ik} \in W$$

$$[aR_{i1} a_{i1} R_{i2} a_{i2} \ldots R_{ik} a_{ik}] \& (\mathbf{F}, a_{ik}) \Vdash_V \varphi;$$

$$(\mathbf{F}, a) \Vdash_V \mathrm{U}\varphi \iff [(\mathbf{F}, a) \Vdash_V \mathbf{KnI}\varphi \quad \text{and} \quad (\mathbf{F}, a) \Vdash_V \mathbf{KnI}\neg\varphi];$$

The latter one is an essential step in our approach. So, we assume that a statement φ has uncertain truth value in the current world (state) if agents may, passing to each other information, conclude that φ might be true in some state of the current environment, but that φ can also be false in some state, and this state is also achievable for agents trough a finite transition by agents' accessibility relations.

Now we recall some definitions necessary for description technique applied in the sequel. *Given a model* $\mathcal{M} := \langle \mathbf{F}, V \rangle$ *based at a frame* $\mathbf{F}$ *with a base set* W *and a valuation* V*, and a formula* φ*,*

(i) φ *is* satisfiable *in* $\mathcal{M}$ *(denotation –* $\mathcal{M} \Vdash_{Sat} \varphi$*) if there is a state* b *of* $\mathcal{M}$ *(*$b \in W$*) where* φ *is true:* $(\mathbf{F}, b) \Vdash_V \varphi$.

(ii) φ *is* valid *in* $\mathcal{M}$ *(denotation –* $\mathcal{M} \Vdash \varphi$*) if, for any* b *of* W*, the formula* φ *is true at* b *(*$(\mathbf{F}, b) \Vdash_V \varphi$*) w.r.t.* V*.*

For a frame $\mathbf{F}$ and a formula φ, φ is satisfiable in $\mathbf{F}$ (denotation $\mathbf{F} \Vdash_{Sat} \varphi$) if there is a valuation V in the frame $\mathbf{F}$ such that $\langle \mathbf{F}, V \rangle \Vdash_{Sat} \varphi$. φ is valid in $\mathbf{F}$ (notation $\mathbf{F} \Vdash \varphi$) if $not(\mathbf{F} \Vdash_{Sat} \neg\varphi)$.

Definition 1. *The logic* $\mathbf{TS4^U_{K_n}}$ *is the set of all formulas which are valid in all frames* $\mathbf{F}$ *(i.e. valid at all frames w.r.t. all valuations). A formula* φ *is said to be a theorem of* $\mathbf{TS4^U_{K_n}}$ *if* $\varphi \in \mathbf{TS4_{K_n}}$*.*

We say a formula φ is *satisfiable* iff there is a valuation V in a Kripke frame $\mathbf{F}$ which makes φ satisfiable: $\langle \mathbf{F}, V \rangle \Vdash_{Sat} \varphi$. Clearly, a formula φ is satisfiable iff

$\neg\varphi$ is not a theorem of $\mathbf{TS4^U_{K_n}}$: $\neg\varphi \notin \mathbf{TS4^U_{K_n}}$, and vice versa, φ is a theorem of $\mathbf{TS4^U_{K_n}}$ ($\varphi \in \mathbf{TS4^U_{K_n}}$) if $\neg\varphi$ is not satisfiable.

3 Decidability of $\mathbf{TS4^U_{K_n}}$

In this section we study computational problems for $\mathbf{TS4^U_{K_n}}$ and describe main technical result of this paper: solution of the decidability and the satisfiability problems for $\mathbf{TS4^U_{K_n}}$. Actually we will use the technique and the scheme of solution used already earlier in our work [29], and the current paper just extend the latter one to handle uncertainty via interaction of agents. Here, as earlier, for technical reason (which makes all constructions much shorter and efficient) we will use transformation or formulas to simple inference rules. Most gain from this transformation is that we will then consider only very simple and uniform formulas - formulas without nested operations (this simplifies the proofs and allows to avoid the necessity to consider nested operations, and hence proofs by induction over formula complexity). First we recall some technical definitions. A (sequential) (inference) rule is an expression (statement)

$$\mathbf{r} := \frac{\varphi_1(x_1,\ldots,x_n),\ldots,\varphi_l(x_1,\ldots,x_n)}{\psi(x_1,\ldots,x_n)},$$

where $\varphi_1(x_1,\ldots,x_n),\ldots,\varphi_l(x_1,\ldots,x_n)$ and $\psi(x_1,\ldots,x_n)$ are formulas constructed out of letters $x_1,\ldots,x_n$. The letters $x_1,\ldots,x_n$ are the variables of $\mathbf{r}$, we use the notation $x_i \in Var(\mathbf{r})$. A meaning of a rule $\mathbf{r}$ is that the statement (formula) $\psi(x_1,\ldots,x_n)$ (which is called conclusion) follows from statements (formulas) $\varphi_1(x_1,\ldots,x_n)$, $\ldots$, $\varphi_l(x_1,\ldots,x_n)$ which are called premisses.

Definition 2. *A rule* $\mathbf{r}$ *is said to be* valid *in a Kripke model* $\langle\mathbf{F},V\rangle$ *(notation* $\mathbf{F}\Vdash_V r$*) if* $[\forall a\ ((\mathbf{F},a)\Vdash_V \bigwedge_{1\leq i\leq l}\varphi_i)] \Rightarrow \forall a\ ((\mathbf{F},a)\Vdash_V\psi)$. *Otherwise we say* $\mathbf{r}$ *is refuted in* $\mathbf{F}$, *or refuted in* $\mathbf{F}$ *by* V, *and write* $\mathbf{F}\nVdash_V\mathbf{r}$. *A rule* $\mathbf{r}$ *is valid in a frame* $\mathbf{F}$ *(notation* $\mathbf{F}\Vdash\mathbf{r}$*) if, for any valuation* V, $\mathbf{F}\Vdash_V\mathbf{r}$

Given a formula φ we can convert it into the rule $x \to x/\varphi$ and employ a technique of reduced normal forms for inference rules as follows. Evidently,

Lemma 1. *A formula* φ *is a theorem of* $\mathbf{TS4^U_{K_n}}$ *iff the rule* $(x\to x/\varphi)$ *is valid in any frame* $\mathbf{F}$.

A rule $\mathbf{r}$ is said to be in *reduced normal form* if $\mathbf{r} = \varepsilon/x_1$ where

$$\varepsilon := \bigvee_{1\leq j\leq l}\Big(\bigwedge_{1\leq i,k\leq n,i\neq k}[x_i^{t(j,i,0)}\wedge(\Diamond^+x_i)^{t(j,i,1)}\wedge(\Diamond^-x_i)^{t(j,i,2)}\wedge$$
$$\bigwedge_{1\leq q\leq n}(\neg\mathbf{K}_q\neg x_i)^{t(j,i,q,1)}\wedge\mathbf{KnI}x_i^{t(j,i,3)}\wedge(\mathrm{U}x_i)^{t(j,i,4)}]\Big),$$

all x_s are certain letters (variables), $t(j,i,z), t(j,i,k,z) \in \{0,1\}$ and, for any formula α above, $\alpha^0 := \alpha$, $\alpha^1 := \neg\alpha$.

Definition 3. *Given a rule* $\mathbf{r_{nf}}$ *in reduced normal form,* $\mathbf{r_{nf}}$ *is said to be a normal reduced form for a rule* $\mathbf{r}$ *iff, for any frame* $\mathbf{F}$*,* $\mathbf{F} \Vdash \mathbf{r} \Leftrightarrow \mathbf{F} \Vdash \mathbf{r_{nf}}$ *.*

Reasoning by the same scheme of proof as in Lemma 3.1.3 and Theorem 3.1.11 from [22] we obtain

Theorem 1. *There exists an algorithm running in (single) exponential time, which, for any given rule* $\mathbf{r}$*, constructs its normal reduced form* $\mathbf{r_{nf}}$*.*

For readers interested in details of this technique we put below a draft of proof for Theorem 1. Actually we shall specify the general algorithm described in Lemma 3.1.3 and Theorem 3.1.11 [22] to the language of our logic.

Assume we are given with a rule

$$\mathbf{r} = \frac{\varphi_1(x_1,...,x_n),...,\varphi_m(x_1,...,x_n)}{\psi(x_1,...,x_n)}$$

It is evident that $\mathbf{r}$ is equivalent to the rule

$$\mathbf{r_0} = \frac{\varphi_1(x_1,...,x_n) \wedge ... \wedge \varphi_m(x_1,...,x_n) \wedge x_c \equiv \psi(x_1,...,x_n)}{x_c}$$

where x_c is a new variable. Therefore we can restrict the case to the rules in the form $\mathbf{c} = \varphi(x_1,...,x_n)/x_c$.

If $\varphi = \alpha \circ \beta$, where $\circ$ is a binary logical operation and both formulas α and β are not simply variables or unary logical operations applied to variables (which both we call final formulas), take two new variables x_α and x_β and the rule

$$\mathbf{r_1} := (x_\alpha \circ x_\beta) \wedge (x_\alpha \equiv \alpha) \wedge (x_\beta \equiv \beta)/x_c.$$

If one from formulas α or β is final and another one not, we apply this transformation to the non-final formula. It is clear that $\mathbf{r}$ and $\mathbf{r_1}$ are equivalent w.r.t. validity in frames.

If $\varphi = *\alpha$, where $*$ is a unary logical operation and α is not a variable, take a new variable x_α and the rule

$$\mathbf{r_1} := *x_\alpha \wedge (x_\alpha \equiv \alpha)/x_c.$$

Again $\mathbf{r}$ and $\mathbf{r_1}$ are equivalent. We continue this transformation over the resulting rules

$$\frac{\bigwedge_{j \in J_1} \gamma_j \wedge \bigwedge_{i \in I_1} x_{\alpha_i} \equiv \alpha_i}{x_c}$$

until all formulas α_i and γ_j in the premise of the resulting rules will be either atomic formulas, i.e. logical operations applied to variables, or variables. Evidently this transformation is *polynomial.* Further, we transform the premise of the resulting rule in the disjunctive normal form and make disjunctive normal form to be perfect (having the disjunctive members to be uniform length and containing all the components required in the definition of reduced normal forms) and obtain as the result an equivalent rule $\mathbf{r_2}$. This transformation, as well as all known ones for reduction of Boolean formulas to disjunctive normal forms, is *exponential* in time. As the result the final rule $\mathbf{r_f}$ has the required form. This concludes the proof. □

Recall now that the decidability of our logic (in particular decidability of the satisfiability problem) will follow (by this theorem) if we find an algorithm recognizing rules in reduced normal form which are valid in all frames $\mathbf{F}$. As in earlier works, most important starting point is to develop a technique to handle interactions of agents. This technical step is carried out in the following lemma (which, as earlier, might be proved using a tick similar to the one used in proof of Lemma 8 in Rybakov [26]).

Lemma 2. *A rule $\mathbf{r_{nf}}$ in reduced normal form is refuted in a frame $\mathbf{F}$ if and only if $\mathbf{r_{nf}}$ can be refuted in a frame with time clusters of size square exponential from $\mathbf{r_{nf}}$.*

Based at this lemma, and applying then a technique developed from standard filtration technique, we may prove

Lemma 3. *A rule $\mathbf{r_{nf}}$ in reduced normal form is refuted in a frame $\mathbf{F}$ iff $\mathbf{r_{nf}}$ can be refuted in a finite frame $\mathbf{F}_1$ by a special valuation V, where the size of the frame $\mathbf{F}_1$ has effective upper bound computable from the size of $\mathbf{r_{nf}}$.*

Using Theorem 1, Lemma 1 and Lemma 3 we derive our main result:

Theorem 2. *The logic $\mathbf{TS4^U_{K_n}}$ is decidable; the satisfiability problem for $\mathbf{TS4^U_{K_n}}$ is decidable.*

Possible Applications: The technique of this paper and obtained algorithm may be applied in research analyzing statements about multi-agent reasoning. E.g. verification '*if a statement (specification) is consistent (i.e. satisfiable)*' may be performed via suggested algorithm. Though computational complexity of this algorithm is high, the verification yet might be (in many cases) done by direct inspection via construction of a frame disproving a rule (satisfying the formula), where the frame could have permissible size. Besides, suggested framework may clarify essence of reasoning via analyzing interdependence of statements with logically oriented substance.

4 Conclusion, Future Work

This paper constructs a framework to study logical properties of reasoning in multi-agents' environment and to construct tools for computation satisfiable and valid statements. Main aim here is to model logical uncertainty via interaction of agents. We suggest the logic $\mathbf{TS4^U_{K_n}}$ based at non-linear frames describing this approach and show that $\mathbf{TS4^U_{K_n}}$ is decidable (and hence the satisfiability problem for this logic is also decidable). Suggested algorithms are based at computation refutability of rules in reduced from at special finite frames of effectively bounded size. Developed approach is rather flexible and allows to work with a variety of logics from AI and CS.

Future research in suggested line may include investigation of AI-logics describing reasoning modeled by frames based at non-transitive time, which may reflect computations (transitions) with bounded introspection (to future and past). Aiomatizability for $\mathbf{TS4^U_{K_n}}$ is an open problem, besides complexity issues and possible ways of refining the complexity bounds in the suggested algorithm are also interesting. Problems of decidability w.r.t. admissible inference rules for $\mathbf{TS4^U_{K_n}}$ and similar logics are not investigated yet. Interesting direction is to model logical uncertainty via different truth values of the statements in 'neighborhoods' of a current state (but not merely in current time cluster, as in this paper). For example, this might be actual in the case of intransitive logics, where only 'tomorrow' and 'yesterday' clusters are taken to consideration.

References

1. Arisha, K., Ozcan, F., Ross, R., Subrahmanian, V.S., Eiter, T., Kraus, S.I.: A platform for collaborating agents. IEEE Intelligent Systems 14(2), 64–72 (1999)
2. Avouris, N.M.: Co-operation knowledge-based systems for environmental decision-support. Knowledge-Based Systems 8(1), 39–53 (1995)
3. Babenyshev, S., Rybakov, V.: Logic of Plausibility for Discovery in Multi-agent Environment Deciding Algorithms. In: Falsafi, B., VijayKumar, T.N. (eds.) PACS 2000. LNCS, vol. 2008, pp. 210–217. Springer, Heidelberg (2001)
4. Babenyshev, S., Rybakov, V.: Decidability of Hybrid Logic with Local Common Knowledge Based on Linear Temporal Logic LTL. In: Beckmann, A., Dimitracopoulos, C., Löwe, B. (eds.) CiE 2008. LNCS, vol. 5028, pp. 32–41. Springer, Heidelberg (2008)
5. Babenyshev, S., Rybakov, V.: Logic of Discovery and Knowledge: Decision Algorithm. In: Falsafi, B., VijayKumar, T.N. (eds.) PACS 2000. LNCS, vol. 2008, pp. 711–718. Springer, Heidelberg (2001)
6. Babenyshev, S., Rybakov, V.: Describing Evolutions of Multi-Agent Systems. In: Velásquez, J.D., Ríos, S.A., Howlett, R.J., Jain, L.C. (eds.) KES 2009, Part I. LNCS, vol. 5711, pp. 38–45. Springer, Heidelberg (2009)
7. Brachman, R.J., Schmolze, J.G.: An overview on the KL-ONE knowledge representation system. Cognitive Science 9(2), 179–226 (1985)
8. Barwise, J.: Three Views of Common Knowledge. In: Vardi (ed.) Proc. Second Confeuence on Theoretical Aspects of Reasoning about Knowledge, pp. 365–379. Morgan Kaufmann, San Francisco (1988)
9. Dwork, C., Moses, Y.: Knowledge and Common Knowledge in a Byzantine Environment: Crash Failures. Information and Computation 68(2), 156–183 (1990)
10. Fagin, R., Halpern, J., Moses, Y., Vardi, M.: Reasoning About Knowledge, p. 410. The MNT Press, Cambridge (1995)
11. Hendler, J.: Agents and the semantic web. IEEE Intelligent Systems 16(2), 30–37 (2001)
12. Kifer, M., Lozinski, L.: A Logic for Reasoning with Inconsistency. J. Automated Deduction 9, 171–215 (1992)

13. Kraus, S., Lehmann, D.L.: Knowledge, Belief, and Time. Theoretical Computer Science 98, 143–174 (1988)
14. Moses, Y., Shoham, Y.: Belief and Defeasible Knowledge. Artificial Intelligence 64(2), 609 - 322 (1993)
15. Nebel, B.: Reasoning and Revision in Hybrid Representation Systems. LNCS, vol. 322 (1940)
16. Neiger, G., Tuttle, M.R.: Common knowledge and consistent simultaneous coordination. Distributed Computing 5(3), 334–352 (1993)
17. Nguyen, N.T., Jo, G.-S., Howlett, R.J., Jain, L.C. (eds.): KES-AMSTA 2008. LNCS (LNAI), vol. 4953. Springer, Heidelberg (2008)
18. Nguyen, N.T., Huang, D.S.: Knowledge Management for Autonomous Systems and Computational Intelligence. Journal of Universal Computer Science 15(4) (2009)
19. Nguyen, N.T., Katarzyniak, R.: Actions and Social Interactions in Multi-agent Systems. Special issue for International Journal of Knowledge and Information Systems 18(2) (2009)
20. Sakama, C., Son, T.C.: Interactind Answer Set. In: Dix, J., Fisher, M., Novák, P. (eds.) CLIMA X. LNCS, vol. 6214, pp. 122–140. Springer, Heidelberg (2010)
21. Rybakov, V.V.: A Criterion for Admissibility of Rules in the Modal System S4 and the Intuitionistic Logic. Algebra and Logic 23(5), 369–384 (1984) (Engl. Translation)
22. Rybakov, V.V.: Admissible Logical Inference Rules. Studies in Logic and the Foundations of Mathematics, vol. 136. Elsevier Sci. Publ., North-Holland (1997)
23. Rybakov, V.V.: Logical Consecutions in Discrete Linear Temporal Logic. Journal of Symbolic Logic (ASL, USA) 70(4), 1137–1149 (2005)
24. Rybakov, V.: Until-Since Temporal Logic Based on Parallel Time with Common Past. Deciding Algorithms. In: Artemov, S., Nerode, A. (eds.) LFCS 2007. LNCS, vol. 4514, pp. 486–497. Springer, Heidelberg (2007)
25. Rybakov, V.: Logic of knowledge and discovery via interacting agents - Decision algorithm for true and satisfiable statements. Inf. Sci. 179(11), 1608–1614 (2009)
26. Rybakov, V.: Linear Temporal Logic LTK_K extended by Multi-Agent Logic K_n with Interacting Agents. J. Log. Comput. 19(6), 989–1017 (2009)
27. Rybakov, V.V.: Representation of Knowledge and Uncertainty in Temporal Logic LTL with Since on Frames Z of Integer Numbers. In: König, A., Dengel, A., Hinkelmann, K., Kise, K., Howlett, R.J., Jain, L.C. (eds.) KES 2011, Part I. LNCS, vol. 6881, pp. 306–315. Springer, Heidelberg (2011)
28. Rybakov, V.V.: Agents' Logics with Common Knowledge and Uncertainty: Unification Problem, Algorithm for Construction Solutions. In: König, A., Dengel, A., Hinkelmann, K., Kise, K., Howlett, R.J., Jain, L.C. (eds.) KES 2011, Part I. LNCS, vol. 6881, pp. 171–179. Springer, Heidelberg (2011)
29. Rybakov, V.: Multi-Agent Logic based on Temporary Logic $TS4_{K_n}$ serving Web Search. In: Grana, M., et al. (eds.) KES 2012, Advances in Knowledge-Based and Intelligent Information and Engineering Systems. Frontiers in Artificial Intelligence and Applications, vol. 243, pp. 108–117 (2012)
30. Rychtyckyi, N.: DLMS: An evaluation of KL-ONE in the automobile industry. In: Aiello, L.C., Doyle, J., Shapiro, S. (eds.) Proc. of he 5-th Int. Conf. on Principles of Knowledge Representation and Reasoning (KR 1996), pp. 588–596. Morgan Kaufmann, San Francisco (1996)