Chapter 18
Backstepping Based Type-2 Adaptive Fuzzy Control for a Generic Hypersonic Flight Vehicle

Fang Yang, Ruyi Yuan, Jianqiang Yi, Guoliang Fan and Xiangmin Tan

Abstract A backstepping controller is designed for the altitude subsystem of a generic hypersonic flight vehicle. The derivatives of the virtual signals in backstepping control design are obtained by command filters with magnitude, bandwidth and rate limit constraints. Dynamic inversion control is used in velocity subsystem design. General uncertainties are estimated online using interval type-2 adaptive fuzzy logic system. Simulation results demonstrate the effectiveness and robustness of the proposed controller and also validate type-2 fuzzy logic is more capable of handling uncertainties than type-1 fuzzy logic.

Keywords Backstepping · Hypersonic flight control · Type-2 fuzzy logic · Adaptive control · Command filter · Dynamic inversion control

18.1 Introduction

Hypersonic flight vehicle (HFV) flies at a speed of more than 5 Mach and through a large altitude range. Although HFV has many application advantages, its flight control law design is highly challenging. There exist complex interactions between the propulsion system, aerodynamics and structural dynamics [1]. Furthermore, flight tests data are lacking and aerodynamic characteristics are difficult to measure and estimate. In summary, uncertainty is a major problem which needs to be dealt with in control design.

Backstepping (BS) control method provides a recursive method for stabilizing the origin of a system in strict-feedback form. [2] combines backstepping method with fuzzy adaptive approximator to track the velocity and altitude command. But

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the derivative process of the virtual control signal is very complicated and it does not consider the signal constraint. [3] introduces command filter to obtain the derivative of the virtual control signal and also consider its magnitude and rate constraints. But the adaptive law is tightly dependent on the coefficients model and when the uncertainty becomes bigger, the robustness becomes weaker.

Type-2 fuzzy set (T2-FS) is characterized by membership functions (MFs) that are themselves fuzzy while type-1 fuzzy set's (T1-FS) MF is crisp. Figure 18.1 shows the membership function example of T2-FS and the structure of type-2 fuzzy logic system (T2-FLS). Here we use interval T2-FLS (IT2-FLS) whose type reducer just involves computing the lower and upper MFs of the antecedent and consequent sets [4].[5, 6] use an adaptive IT2-FLS to approximate unknown nonlinear system online and it does not need any prior knowledge about the nonlinear terms.

This paper proposes a backstepping based type-2 adaptive fuzzy controller for a generic hypersonic flight vehicle (GHFV). Uncertainties are estimated online by IT2-FLS. The derivatives of signals are obtained by command filters. Simulation results show its effectiveness and robustness under big uncertainties and noises.

18.2 Problem Statement and Preliminaries

The mathematical model of the longitudinal dynamics of the GHFV is given by

$$\begin{aligned}&\dot{V} = \frac{T\cos\alpha - D}{m} - g\sin\gamma,\ \dot{\gamma} = \frac{L + T\sin\alpha}{mV} - \frac{g\cos\gamma}{V}\\&\dot{h} = V\sin\gamma,\ \dot{\alpha} = q - \dot{\gamma},\ \dot{q} = M_y/I_y\end{aligned} \tag{18.1}$$

where the forces, the moment and the coefficients are

$$\begin{aligned}&L = \frac{1}{2}\rho V^2 sC_L,\ D = \frac{1}{2}\rho V^2 sC_D,\ T = \frac{1}{2}\rho V^2 C_T,\ My = \frac{1}{2}\rho V^2 s\bar{c}C_M\\&C_L = C_L^{\alpha}\alpha,\ C_T = C_T^{\phi}\phi + C_T^0,\ C_M = C_M^{\delta_e}\delta_e + C_M^0\end{aligned} \tag{18.2}$$

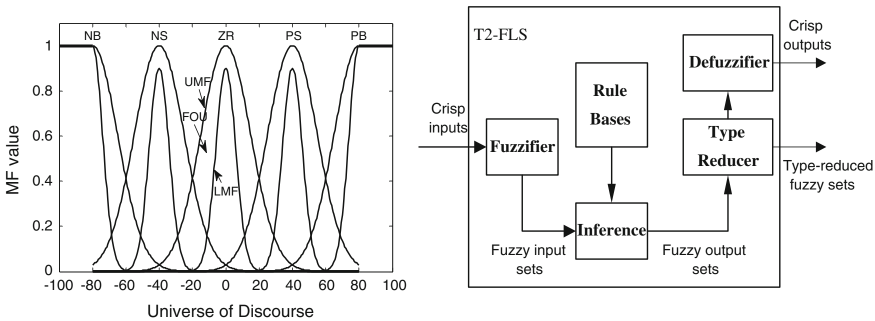

Fig. 18.1 Mebership function example of T2-FS (*left*) and structure of T2-FLS (*right*)

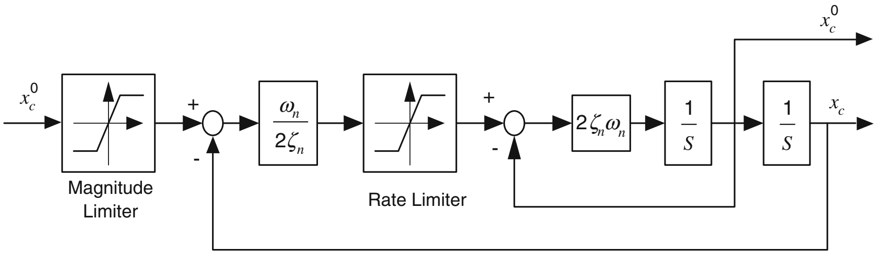

Fig. 18.2 Command filter with magnitude, bandwidth and rate limit constraints

Detailed coefficients can be seen in [7].

In usual backstepping control design, the analytic computation of virtual control signal derivatives is tedious. Here instead, we use command filters to obtain the derivatives of virtual control signals as well as reference command signals. Command filters are also used to filter out high-frequency noise and get the transient process of the reference command signals. As Fig. 18.2 shows, command filter puts magnitude, bandwidth and rate limitations on the signal [8]. ζ_n, ω_n are damping ratio and bandwidth. When command limiting is not in effect, the transfer function from the input to the output is

$$\frac{x_c(s)}{x_c^0(s)} = \frac{\omega_n^2}{s^2 + 2\zeta_n\omega_n s + \omega_n^2} \tag{18.3}$$

18.3 Control Design

18.3.1 Velocity Subsystem Dynamic Inversion Controller Design

The overall control scheme is shown in Fig. 18.3. The model error, parameter variation, external disturbances are all taken as the general uncertainty Δ^* and are estimated online by IT2-FLS. The error dynamics of the velocity subsystem is

$$\dot{\tilde{V}} = \dot{V} - \dot{V}_c = \bar{q}S\cos\alpha C_T^{\phi}\Big/m * \phi + \bar{q}S(\cos\alpha C_T^0 - C_D)\big/m - g\sin\gamma - \dot{V}_c \tag{18.4}$$

Denote $f_1 = \bar{q}S\cos\alpha C_T^{\phi}\Big/m,\ \ g_1 = \bar{q}S(\cos\alpha C_T^0 - C_D)\big/m$. When using nominal parameters, f_1, g_1 are nominal values. In practice there exist measurement uncertainties, model uncertainties and external disturbances. We take all of them as ideal general uncertainty Δ_1^*. An IT2-FLS is used to estimate Δ_1^* online and get $\widehat{\Delta_1}$ instead. So (18.4) can be written as

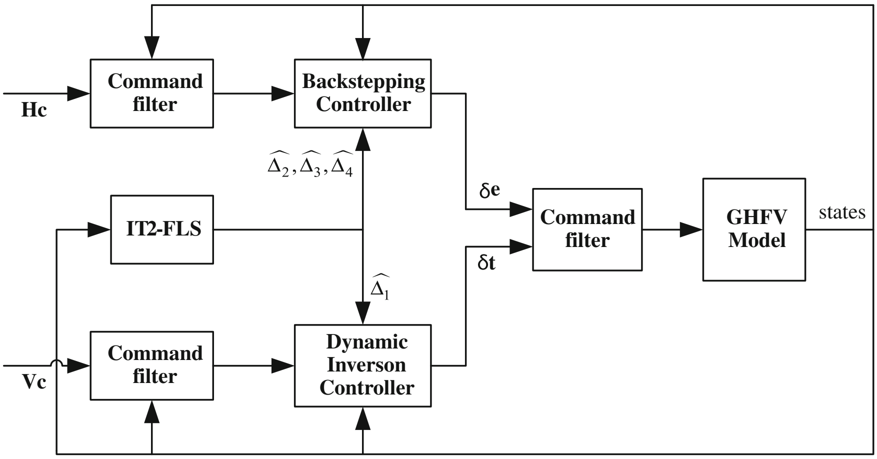

Fig. 18.3 The block diagram of the overall control scheme

$$\dot{\tilde{V}} = f_1\phi + g_1 - g\sin\gamma - \dot{V}_c - \Delta_1^* \tag{18.5}$$

In order to stabilize the tracking error, by dynamic inversion control, the throttle setting signal is chosen as

$$\phi_c = \left[-g_1 + g\sin\gamma + \dot{V}_c - k_1\tilde{V} + \widehat{\Delta_1}\right] \Big/ f_1 \tag{18.6}$$

Where k_1 is a positive constant and (18.5) can be written as

$$\dot{\tilde{V}} = -k_1\tilde{V} + (\widehat{\Delta_1} - \Delta_1^*) \tag{18.7}$$

18.3.2 Altitude Subsystem Backstepping Controller Design

The error dynamics of the altitude is

$$\dot{\tilde{h}} = \dot{h} - \dot{h}_c = V\sin\gamma - \dot{h}_c \tag{18.8}$$

In order to stabilize (18.8), the flight path angle command can be chosen as

$$\gamma_c = \sin^{-1}\frac{\dot{h}_c - k_2\tilde{h}}{V} \tag{18.9}$$

where k_2 is a positive constant and (18.8) can be written as

$$\dot{\tilde{h}} = -k_2\tilde{h} \tag{18.10}$$

The error dynamics of the flight path angle is

$$\dot{\tilde{\gamma}} = \dot{\gamma} - \dot{\gamma}_c = \overline{q}SC_L^{\alpha}/mV^{*}\alpha + \overline{q}S\sin\alpha C_T/mV - g\cos\gamma/V - \dot{\gamma}_c \tag{18.11}$$

Denote $f_2 = \overline{q}SC_L^{\alpha}/mV$, $g_2 = \overline{q}S\sin\alpha C_T/mV$. As $T\sin\alpha \ll L$, γ is mainly driven by α, so we choose α as the virtual control signal. Similar to the design in velocity subsystem, we consider the general uncertainty Δ_2^* and its estimate $\widehat{\Delta_2}$. Then the virtual control signal is chosen as

$$\alpha_c = \left[-g_2 + \frac{g\cos\gamma}{V} + \dot{\gamma}_c - k_3\tilde{\gamma} + \widehat{\Delta_2}\right]\Big/f_2 \tag{18.12}$$

where k_3 is a positive constant and (18.11) can be written as

$$\dot{\tilde{\gamma}} = -k_3\tilde{\gamma} + (\widehat{\Delta_2} - \Delta_2^*) \tag{18.13}$$

The error dynamics of the angle of attack is

$$\dot{\tilde{\alpha}} = \dot{\alpha} - \dot{\alpha}_c = q - (L + T\sin\alpha)/mV + g\cos\gamma/V - \dot{\alpha}_c \tag{18.14}$$

Denote $g_3 = (L + T\sin\alpha)/mV$ and choose the virtual control signal as

$$q_c = g_3 - g\cos\gamma/V + \dot{\alpha}_c - k_4\tilde{\alpha} + \widehat{\Delta_3} \tag{18.15}$$

where k_4 is a positive constant and (18.14) can be written as

$$\dot{\tilde{\alpha}} = -k_4\tilde{\alpha} + (\widehat{\Delta_3} - \Delta_3^*) \tag{18.16}$$

The error dynamics of the pitch rate is

$$\dot{\tilde{q}} = \dot{q} - \dot{q}_c = \overline{q}ScC_M^{\delta_e}\Big/I_y * \delta_e + \overline{q}ScC_M^0/I_y - \dot{q}_c \tag{18.17}$$

Denote $f_3 = \overline{q}ScC_M^{\delta_e}\Big/I_y$, $g_4 = \overline{q}ScC_M^0/I_y$ and choose the elevator deflection signal as

$$\delta_{ec} = \left[-g_4 + \dot{q}_c - k_5\tilde{q} + \widehat{\Delta_4}\right]\Big/f_3 \tag{18.18}$$

where k_5 is a positive constant and (18.17) can be written as

$$\dot{\tilde{q}} = -k_5\tilde{q} + (\widehat{\Delta_4} - \Delta_4^*) \tag{18.19}$$

18.3.3 Interval Type-2 Adaptive Fuzzy Controller Design

In order to obtain the general uncertainty $\widehat{\Delta_k}(k = 1,2,3,4)$, we use an interval type-2 adaptive fuzzy controller with pre-determined IT2 antecedent sets and adaptive T1 consequent sets. The fuzzy rule bases consist of a collection of IF–THEN rules in the following form

$$R^i: \text{ If } \begin{matrix} \widetilde{V} \\ \widetilde{\gamma} \\ \widetilde{\alpha} \\ \widetilde{q} \end{matrix} \text{ is } \begin{matrix} \widetilde{A_1^i} \\ \widetilde{A_2^i} \\ \widetilde{A_3^i} \\ \widetilde{A_4^i} \end{matrix} \text{ and } \begin{matrix} \dot{\widetilde{V}} \\ \dot{\widetilde{\gamma}} \\ \dot{\widetilde{\alpha}} \\ \dot{\widetilde{q}} \end{matrix} \text{ is } \begin{matrix} \widetilde{B_1^i} \\ \widetilde{B_2^i} \\ \widetilde{B_3^i} \\ \widetilde{B_4^i} \end{matrix}, \text{ then } \begin{matrix} \widehat{\Delta_1} \\ \widehat{\Delta_2} \\ \widehat{\Delta_3} \\ \widehat{\Delta_4} \end{matrix} \text{ is } \begin{matrix} C_1^i \\ C_2^i \\ C_3^i \\ C_4^i \end{matrix} \quad i = 1, 2, \ldots, M$$

where M is the number of rules, $\widetilde{A_k^i}$ and $\widetilde{B_k^i}$ are IT2 antecedent sets, C_k^i is T1 consequent sets. $\widetilde{A_k^i}$ and $\widetilde{B_k^i}$ each has 5 IT2-FS like Fig. 18.1, so $M = 5^2 = 25$. By using the singleton fuzzification, product inference, centre-average defuzzification and the center-of-sets type reducer, the type-reduced T1-FS is given by

$$\Delta_{k\cos} = \int_{y_k^1} \cdots \int_{y_k^M} \int_{f_k^i} \cdots \int_{f_k^M} \prod\nolimits_{i=1}^{M} \mu_{C_k^i}(y_k^i) \prod\nolimits_{i=1}^{M} \mu_{F_k^i}(f_k^i) \Big/ \frac{\sum_{i=1}^{M} f_k^i y_k^i}{\sum_{i=1}^{M} f_k^i} \tag{18.20}$$

where y_k^i is the centroid of C_k^i and f_k^i is the firing value associated with the ith rule. Here the centroid y_k^i is defined as the point which has the maximum membership value 1. As $\mu_{F_k^i}(f_k^i) = 1$, $\mu_{C_k^i}(y_k^i) = 1$, so

$$\Delta_{k\cos} = \int_{y_k^1} \cdots \int_{y_k^M} \int_{f_k^i} \cdots \int_{f_k^M} 1 \Big/ \frac{\sum_{i=1}^{M} f_k^i y_k^i}{\sum_{i=1}^{M} f_k^i} = [\Delta_{kl}, \Delta_{kr}] \tag{18.21}$$

In the adaptive controller, y_k^i is the adaptive parameter and is denoted by new symbol θ_k^i. The fuzzy basis function is

$$\underline{\xi_k}^i = f_{kl}^i \Big/ \sum\nolimits_{i=1}^{M} f_{kl}^i, \overline{\xi^k}^i = f_{kr}^i \Big/ \sum\nolimits_{i=1}^{M} f_{kr}^i \tag{18.22}$$

f_{kl}^i, f_{kr}^i denote the firing values used to compute the bounds Δ_{kl}, Δ_{kr} which can be obtained using the Karnik–Mendel algorithm [9]. Denote $\underline{\xi_k} = (\underline{\xi_k}^1, \underline{\xi_k}^2, \ldots, \underline{\xi_k}^M)^T$, $\overline{\xi^k} = (\overline{\xi^k}^1, \overline{\xi^k}^2, \ldots, \overline{\xi^k}^M)^T$, $\xi_k = (\underline{\xi_k} + \overline{\xi^k}) \big/ 2$, $\theta_k = (\theta_k^1, \theta_k^2, \ldots, \theta_k^M)^T$. The general uncertainty Δ_k can be achieved by

$$\widehat{\Delta_k} = (\Delta_{kl} + \Delta_{kr})/2 = \theta_k^T \xi_k \tag{18.23}$$

18.3.4 Adaptive Law

Denote $e = [\widetilde{V}\, \widetilde{h}\, \widetilde{\gamma}\, \widetilde{\alpha}\, \widetilde{q}]^T$, $K = diag(k_1, k_2, k_3, k_4, k_5)$, $\widetilde{\theta_k} = \widehat{\theta_k} - \theta_k^*$, $k = 1, 2, 3, 4$, $P = diag(p_1, p_2, p_3, p_4, p_5)$, $\widetilde{\Delta} = [\widetilde{\Delta_1}, 0, \widetilde{\Delta_2}, \widetilde{\Delta_3}, \widetilde{\Delta_4}]^T = [\widetilde{\theta_1}^T \xi_1, 0, \widetilde{\theta_2}^T \xi_2, \widetilde{\theta_3}^T \xi_3,$

$\widetilde{\theta}_4{}^T\xi_4]^T$, $\psi = [\psi_1, \psi_2, \psi_3, \psi_4]^T = [p_1\widetilde{V}, p_3\widetilde{\gamma}, p_4\widetilde{\alpha}, p_5\widetilde{q}]^T$. Then the state error dynamics can be written as

$$\dot{e} = -Ke + \widetilde{\Delta} \tag{18.24}$$

Consider the following Lyapunov function candidate

$$V_L = \frac{1}{2}e^T Pe + \sum_{i=1}^{4} \frac{1}{2\eta_i} \widetilde{\theta}_i{}^T \widetilde{\theta}_i \tag{18.25}$$

Where P is positive diagonal weight matrix and η_i is positive learning rate. The time derivative of V_L is

$$\dot{V}_L = \dot{e}^T Pe + \sum_{i=1}^{4} \dot{\widetilde{\theta}}_i^T \widetilde{\theta}_i \Big/ \eta_i = -e^T KPe + \sum_{i=1}^{4} \widetilde{\theta}_i{}^T (\dot{\hat{\theta}}_i \Big/ \eta_i + \psi_i \xi_i) \tag{18.26}$$

So by Lyapunov synthesis approach, the parameter adaptive law can be

$$\dot{\hat{\theta}}_i = Proj(-\eta_i \psi_i \xi_i), \quad i = 1, 2, 3, 4 \tag{18.27}$$

Where $Proj(\bullet)$ is the projection operator which guarantees all parameters are in their allowed compact sets.

18.4 Simulations

At the trimmed condition $V_0 = 4590.3\,\text{m/s}$, $h_0 = 33528\,\text{m}$, $\gamma_0 = 0\,\text{rad}$, $q_0 = 0\,\text{rad/s}$, $\alpha_0 = 0.04799$ rad, we choose the reference periodic signal as $600\sin(0.04\pi t)$ in altitude change. Frequencies of the command filters are $\omega_v = 1$, $\omega_h = 5$, $\omega_\gamma = 20$, $\omega_\alpha = 10$, $\omega_q = 10$, $\omega_e = 40$, $\omega_t = 40$ and all damping ratios equal 1. Weights p_i and learning rates η_i are 1. $k_1 = 38.4096$, $k_2 = 0.4730$, $k_3 = 0.4217$, $k_4 = 0.6768$, $k_5 = 1.1971$. $m = m_0(1 + U + 0.8 * GWN)$, $I_y = I_{y0}(1 + U + 0.8 * GWN)$, $\rho = \rho_0(1 + U + 0.8 * GWN)$, $s = s_0(1 + U + 0.8 * GWN)$, $\overline{c} = \overline{c_0}(1 + U + 0.8 * GWN)$, where $U = -30\,\%$ is the parameter uncertainty and GWN represents the Gaussian white noise whose power is 0.002.

Simulations are conducted in 3 cases: (a) BS controller; (b) BS + IT2-AFC; (c) BS + T1-AFC. Figure 18.4 shows the results when there is no uncertainty. We see that V and h can track the reference commands very well with or without adaptive fuzzy controllers and that all three controllers have nearly the same control effect.

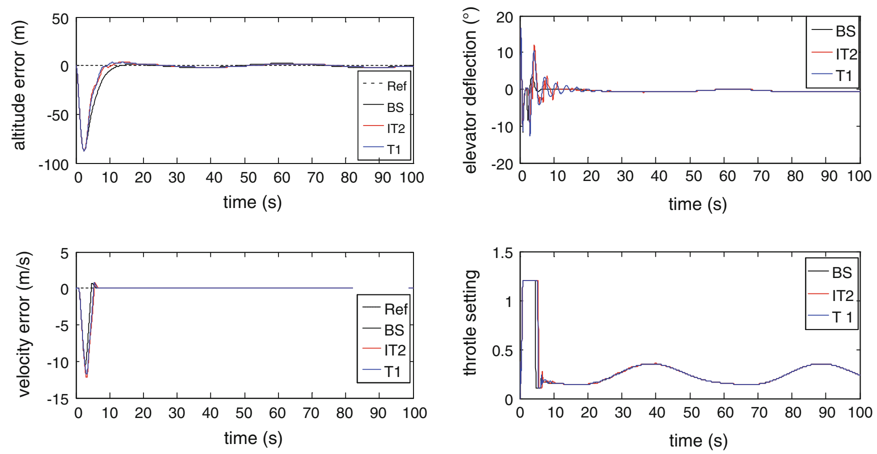

Fig. 18.4 Velocity and altitude tracking error (*left*) and control signals (*right*) without uncertainty

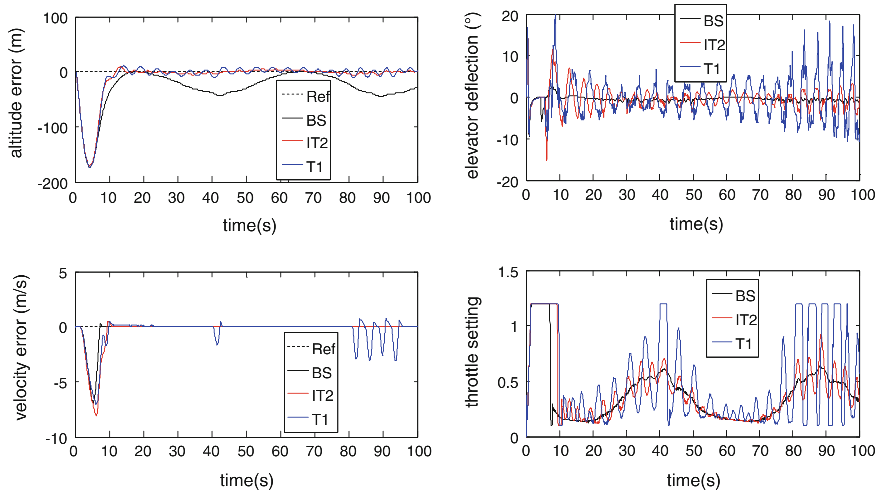

Fig. 18.5 Velocity and altitude tracking error (*left*) and control signals (*right*) with uncertainty

Figure 18.5 shows the results when there exists big uncertainty. The pure backstepping controller has large steady-state altitude tracking error. When we use T1-AFC to compensate the general uncertainty, it has large oscillating tracking errors and large control chattering while the tracking errors and control chattering of IT2-AFC are much smaller.

18.5 Conclusion

In this paper, we designed a backstepping based interval type-2 adaptive fuzzy logic controller for a generic hypersonic flight vehicle. Command filters were used to obtain the derivatives of the wanted signals. Simulation results validated its effectiveness and robustness and also showed that T2-FLS is more capable of handling uncertain problem than T1-FLS.

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