Chapter 1
Formation Control of Autonomous Underwater Vehicles Based on Finite-Time Consensus Algorithms

Jian Yuan, Zhong-Hai Zhou, Hua Mu, Yu-Ting Sun and Lei Li

Abstract Formation control of autonomous underwater vehicles is investigated. A finite-time consensus algorithm for second-order system is proposed, the consensus on velocities of AUV (linear velocity and angular velocity) and positions (displacement and attitude) are carried out with the finite-time consensus. Because of the limited communication range, the communication ranges of AUVs are predefined, so the AUVs system is modeled as a networked system with variable communication topologies. We demonstrate the formation control of multiple AUVs with different communication ranges, which show that the finite-time consensus on positions and velocities is obtained. Finally, we demonstrate the formation control of multiple AUVs with constraints on maximum velocity. Simulation results verify the effectiveness of the proposed control algorithms.

Keywords Autonomous underwater vehicle · Formation control · Finite-time consensus

1.1 Introduction

Cooperative control of multiple autonomous underwater vehicles (AUV) plays an important role on marine scientific investigation and marine development. The formation of multi-AUV can significantly enhance applications on the marine sampling, imaging, surveillance and communications. Compared to the formation

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control of multi-robot, the formation control of multi-AUV is particularly difficult, especially on controlling attitude and direction of AUV; what is more, the communication method among AUVs is acoustic. When communication distance increases, the communication qualities deteriorate quickly; this mainly makes time-delay, signal attenuation and distortion. Although formation control of multiple AUVs obtains a wide range of attention in recent years, the fruits on formation control problem are less than ones on land multi-robot problems. For example, Fiorelli conducted a collaborative and adaptive sampling research of multi-AUV at the Monterey Bay [1]; Yu and Ura carried out the cable-based modular fast-moving and obstacle-voidance experiments, and presented an interconnected multi-AUV system with three-dimension sensors. On the aspect of formation control framework [2–4] proposed a four-layer cooperative control strategy based on hierarchical structure; Ref. [5] proposed a hierarchical control framework based on hybrid model. In addition, Yang converted a nonholonomic system to a chain one and designed a controller to implement a leader-follower formation for multiple AUVs in [6]. Formation control of land multi-robot has appeared more sophisticated control methods. Kalantar and Zimmer [7] studied the distributed formation control. References [8, 9] adopted the centralized formation control based on virtual structure; Refs. [10–13] used a distributed formation framework based on virtual structure to realize the formation control. Do studied the control of robot formation with limited communications [14], Dong and Farrell [15] studied collaboration control of robots with nonholonomic constraints, Kumar et al. [16] studied the cooperative control of robots with nonholonomic constraints based on omni-direction vision and designed distributed controllers and estimators. The authors proposed synchronization methods for two categories of complex networks: linearly and nonlinearly time-delay coupled networks with multiple agents in [17]. Convergence analysis of decentralized slot time synchronization algorithms for wireless Ad Hoc networks is shown with multiple-agent technologies in [18]. The formation control for multiple autonomous underwater vehicles is rather different than the control methods for other vehicles, because the formation control for AUVs is of its characteristics, such as the large-scale distribution in space. The finite-time consensus controller designing based on finite-time control and consensus problem has important theoretical and practical significance. Wang and Xiao [19] proposed a finite-time formation control framework for the large-scale multiple agents. The formation information is divided into the global information and local information, the former decides the formation shape of the whole formation, and only the leader can obtain such whole information but followers can only obtain the local information. This framework can greatly reduce the amount of communication between the agents. And then they designed a nonlinear consensus algorithm to time-varying, time invariant formation and trajectory tracking control. Wang and Hong [20] proposed a few kinds of algorithms for agents' network with a dynamic coupled topology. And then they designed a finite-time consensus protocol with Lyapunov function and graph theory and proposed a time invariant nonsmooth controller.

The decentralized controller methods for the autonomous underwater vehicle are applied more and more, but they ignore the coupling relationship between them. Another method is that an AUV is modeling as an agent, but this method ignores attitude characteristics of AUV's (pitch, roll and yaw). In this paper, we consider the cooperative control problem in three dimensional spaces. The proposed algorithm is based on finite-time consensus algorithms to realize the formation control for underwater vehicles. Finally, the simulation results show the effectiveness of the control strategy.

1.2 A Modeling of AUV in Three-Dimension Space

Firstly, we establish two coordinate frames: the geocentric inertial coordinate frame *O-xyz* and the body-fixed coordinate frame *E-xhz*. The origin *O* in inertial coordinate frame is defined in some point at sea level, the *O-x* and *O-y* are in horizontal level, and *O-x* is parallel with the longitude and points northwardly, *O-y* is parallel with the latitude and points eastwardly, and *O-x* is perpendicular to horizontal level and points to the earth's core. The three axes form the right-hand screw relation. The origin *O* in body-fixed coordinate frame is defined in some point at the AUV's cancroids. *E-x* is defined in longitudinal section plane and points to the orientation of linear velocity, *E-h* is perpendicular to the portrait section plane and points to the right-hand orientation, and *E-z* is in the longitudinal section plane and points to the down orientation. Also the three axes form the right-hand screw relation. *E-x*, *E-h* and *E-z* are inertia principal axis of an AUV. The coordinates of AUV *i* in inertial coordinate frame are shown in Fig. 1.1.

The transformation matrix from the inertial coordinate frame to the body-fixed coordinate frame is

$$\boldsymbol{T}_i = \begin{bmatrix} \boldsymbol{T}_{vi} & 0 \\ 0 & \boldsymbol{T}_{wi} \end{bmatrix}$$

where

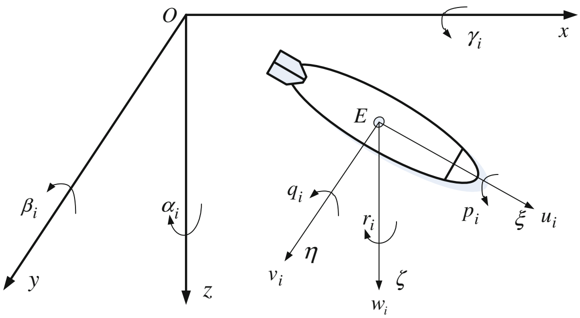

Fig. 1.1 The coordinates of AUV *i* in coordinate frame

$$T_{vi} = \begin{bmatrix} \cos\beta_i \cos\alpha_i & -\sin\beta_i \cos\gamma_i + \cos\beta_i \sin\gamma_i \sin\alpha_i & \sin\beta_i \sin\gamma_{ii} + \cos\beta_i \cos\gamma_i \sin\alpha_i \\ \sin\beta_i \cos\alpha_i & \cos\beta_i \cos\gamma_i + \sin\beta_i \sin\gamma_i \sin\alpha_i & -\cos\beta_i \sin\gamma_i + \sin\beta_i \cos\gamma_i \sin\alpha_i \\ -\sin\alpha_i & \sin\gamma_i \cos\alpha_i & \cos\gamma_i \cos\alpha_i \end{bmatrix}$$

$$T_{wi} = \begin{bmatrix} 1 & \tan\beta_i \sin\gamma_i & \tan\beta_i \cos\gamma_i \\ 0 & \cos\gamma_i & -\sin\gamma_i \\ 0 & \sin\gamma_i / \cos\beta_i & \cos\gamma_i / \cos\beta_i \end{bmatrix}$$

where a_i is the roll angle of AUV i, b_i is the pitch angle of AUV i and g_i is the yaw angle of AUV i. We define $\boldsymbol{X}_i = [x_{si} \quad y_{si} \quad z_{si} \quad f_i \quad q_i \quad j_i]^{\mathrm{T}}$, which denotes the positions and attitudes, and $\boldsymbol{v}_i = [u_i \quad v_i \quad w_i \; p_i \quad q_i \quad r_i]^{\mathrm{T}}$ which denotes the linear velocity and the angular velocity in body-fixed coordinate frame. So $\dot{X}_i = T_i^{-1} \cdot v_i = V_i$. Then we define $\boldsymbol{U}_i = \dot{\boldsymbol{V}}$ as acceleration in the inertial coordinate frame. So the dynamics equation is modeled as the following second-order system

$$\begin{aligned} \dot{X}_i &= V_i \\ \dot{V}_i &= U_i \end{aligned} \tag{1.1}$$

1.3 Finite-Time Second-Order Consensus Control Algorithm Based on Positions of Virtual Leader

The proposed formation control scheme is only based on the position information of the virtual leader, which makes a consensus on the distance $\boldsymbol{H}_i$ with its position vector minus the range from its position to the virtual leader. That means the position vector of the virtual leader belonging to every AUV $\boldsymbol{X}_{vl}^i$ ($\boldsymbol{X}_{vl}^i = \boldsymbol{X}_i(1:3) - \boldsymbol{H}_i$) will reach a consensus: $\boldsymbol{X}_{vl}^i \to \boldsymbol{X}_{vl}^j$, $i \neq j$. So AUVs form a fixed shape shown in Fig. 1.3. The proposed control algorithm still require linear velocity vector, angular velocity vector and angular position vector in inertial coordinate frame to reach a consensus in finite time and obtain a consensus on position vectors of the virtual leader for every AUVs. So there is a time t_s when $t \geq t_s$ the following equations hold

$$\begin{cases} \left\| X_{vl}^i - X_{vl}^j \right\| = 0 \\ \left\| X_i(4:6) - X_j(4:6) \right\| = 0 \\ \left\| V_j - V_i \right\| = 0, \; i \neq j, \; j \in N_j \end{cases}$$

where N_j denotes the set of all neighbor AUVs, $\|\cdot\|$ denotes the vector norm and $\boldsymbol{H}_i$ denotes the distance vector from AUV i to the virtual leader. The expression of $\boldsymbol{H}_i$ with seven AUVs is as follows

$$H_i = \begin{cases} \sin(3\pi/4)[r(i-4) \quad 0.5r \quad r(4-i)]^{\mathrm{T}}, & i = 1, 2, 3, 4 \\ \cos(3\pi/4)[r(4-i) \quad 0.5r \quad r(4-i)]^{\mathrm{T}}, & i = 5, 6, 7 \end{cases}$$

where r denotes the predefined unit distance.

For the second-order system in Eq. (1.1), we propose a following consensus protocol with the second-order dynamic equation

$$\boldsymbol{U}_i = \sum_{j \in N_j} (a\mathbf{sgn}(\boldsymbol{X}_{vl}^j - \boldsymbol{X}_{vl}^i)\left|(\boldsymbol{X}_{vl}^j - \boldsymbol{X}_{vl}^i)\right|^{\chi} + b\mathrm{sgn}V_j - \boldsymbol{V}_i)\left|\boldsymbol{V}_j - \boldsymbol{V}_i\right|^{\chi}) \tag{1.2}$$

where $0 < a,\ b \leq 1$ and $0 < \chi < 1$ are the convergence coefficients, $|\cdot|$ denotes the absolute value of every element, and $\mathbf{sgn}(\cdot)$ denotes the vector sign function with the expression $\mathbf{sgn}(\boldsymbol{\sigma}) = [sign(\sigma_1) \cdots sign(\sigma_i) \cdots sign(\sigma_n)]^{\mathrm{T}}$ where $\boldsymbol{\sigma} = [\sigma_1 \cdots \sigma_i \cdots \sigma_n \in R^n]$ denotes a vector with

$$sign(\sigma_i) = \begin{cases} 1, \ \sigma_i > 0 \\ 0, \ \sigma_i = 0 \\ -1, \ \sigma_i < 0 \end{cases}$$

We define $\mathbf{sig}(\boldsymbol{\sigma})^{\rho} = \mathbf{sgn}(\boldsymbol{\sigma})|\boldsymbol{\sigma}|^{\rho} = [sign(\sigma_1)|\sigma_1|^{\rho} \cdots sign(\sigma_i)|\sigma_i|^{\rho} \cdots sign(\sigma_n)|\sigma_n|^{\rho}]^{\mathrm{T}}$.

So Eq. (1.2) can be written as

$$\boldsymbol{U}_i = \sum_{j \in N_j} (a\mathbf{sig}(\boldsymbol{X}_{vl}^j - \boldsymbol{X}_{vl}^i)^{\chi} + b\mathbf{sig}(\boldsymbol{V}_j - \boldsymbol{V}_i)^{\chi}).$$

In the following section the boundedness of the closed loop system is verified with consensus protocol in (1.2).

Proof (The sign $\cdot$ denotes the product of corresponding elements of a vector)

We define a vector Lyapunov function

$$\mathrm{V} = \sum_{i=1}^{n} \sum_{j=1}^{n} \int_0^{X_{vl}^j - X_{vl}^i} a\mathbf{sig}(\mathbf{s})^{\chi} \cdot d\mathbf{s} + \sum_{i=1}^{n} V_i \cdot \dot{V}_i \Big/ 2.$$

Solve the differential coefficients along the closed loop system trajectory, and we notice that $\mathbf{sig}(\boldsymbol{X}_{vl}^j - \boldsymbol{X}_{vl}^i)^c$ is odd function, we obtain

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n} \boldsymbol{V}_i \cdot \dot{\boldsymbol{V}}_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a\,\mathbf{sig}(\boldsymbol{X}_{vl}^{i} - \boldsymbol{X}_{vl}^{j})^{\chi} \cdot d\mathbf{s} \\
&= \sum_{i=1}^{n} V_i \cdot \sum_{i=1}^{n} (a\,\mathbf{sig}(\boldsymbol{X}_{vl}^{j} - \boldsymbol{X}_{vl}^{i})^{\chi} + b\,\mathbf{sig}(\boldsymbol{V}_j - \boldsymbol{V}_i)^{\chi}) - \sum_{i=1}^{n}\sum_{j=1}^{n} a\,\mathbf{sig}(\boldsymbol{X}_{vl}^{j} - \boldsymbol{X}_{vl}^{i})^{\chi} \cdot \boldsymbol{V}_i \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} b\boldsymbol{V}_i \cdot \mathbf{sig}(\boldsymbol{V}_j - \boldsymbol{V}_i)^{\chi} \\
&= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} (b+b)\boldsymbol{V}_i \cdot \mathbf{sig}(\boldsymbol{V}_j - \boldsymbol{V}_i)^{\chi}
\end{aligned}$$

Notice that $(\boldsymbol{V}_i - \boldsymbol{V}_j) \cdot \mathbf{sig}(\boldsymbol{V}_j - \boldsymbol{V}_i)^{\chi} = \left|\boldsymbol{V}_i - \boldsymbol{V}_j\right|^{1+\chi}$, so

$$\dot{\mathrm{V}} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b(\boldsymbol{V}_i - \boldsymbol{V}_j) \cdot \mathbf{sig}(\boldsymbol{V}_j - \boldsymbol{V}_i)^{\chi} \le \mathbf{0}.$$

Because of the limited communication range of every AUV, which means the AUV communicate only with the AUVs in communication range (noted as *SetDis*). In the formation shaping procedure the communication topology is dynamic which means the matrix $A(t)$ is time variant.

1.4 Numerical Simulations

We simulate some squares in space as AUVs with velocity and attitude. The effectiveness of the control scheme is shown with two simulation results. Choose $a = 1$, $b = 1$, $\chi = 0.3$ and $SetDis = 30(\mathrm{m})$. Simulation results is shown in

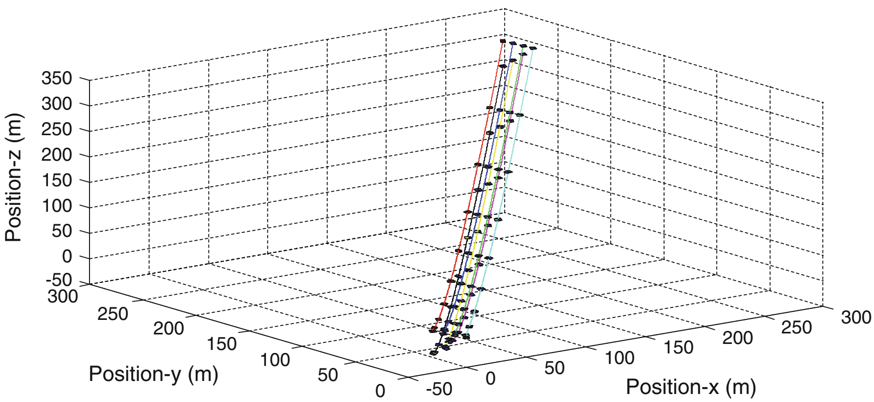

Fig. 1.2 Formation control with 7 AUVs

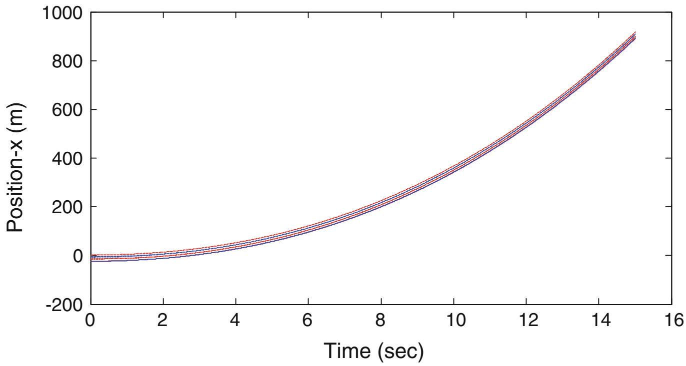

Fig. 1.3 Consensus on position x

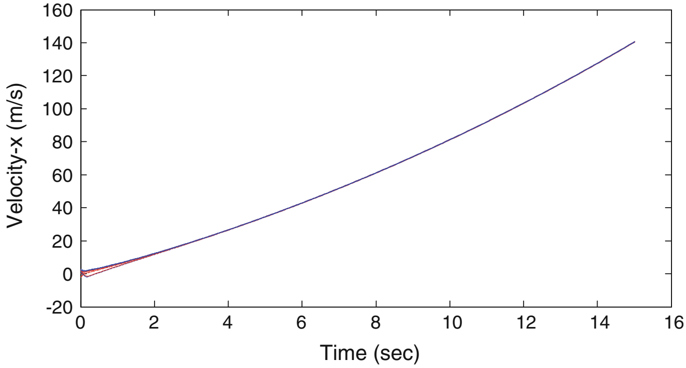

Fig. 1.4 Consensus on linear velocity x

Figs. 1.2, 1.3 and 1.4. From the simulation results, we notice that the AUVs form the desired shape in the finite time and the attitudes of all AUVs reach a consensus in finite time.

1.5 Conclusions

The formation control problem based the proposed consensus algorithm in two cases is investigated. The effectiveness of the proposed finite-time consensus is verified. Typically the velocity of AUV can not instantaneously increase to infinity; it will definitely lengthen the synchronization time or lead the formation can not achieve the desired formation shape in the presence of maximum speed constraint. So the formation fault diagnosis and formation reconfiguration problem is the next focus of our research.

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References

1. Fiorelli E (2006) Multi-AUV control and adaptive sampling in Monterey bay. IEEE J Oceanic Eng 31(4):935–948
2. Yu SC, Ura T (2004) A system of multi-AUV interlinked with a smart cable for autonomous inspection of underwater structures. Int J Offshore Polar Eng 14(4):264–272
3. Yu SC, Ura T (2004) Experiments on a system of multi-AUV interlinked with a smart cable for autonomous inspection of underwater structures. Int J Offshore Polar Eng 14(4):273–283
4. Xiang XB (2007) Coordinated control for multi-AUV systems based on hybrid automata. In: Proceedings of IEEE international conference on robotics and biomimetics, pp 2121–2126
5. Tangirala S, Kumar R, Bhattacharyya S et al (2005) Hybrid-model based hierarchical mission control architecture for autonomous underwater vehicles, American control conference, vol 1. Portland, pp 668–673
6. Yang EF, Gu DB (2007) Nonlinear formation-keeping and mooring control of multiple autonomous underwater vehicles. IEEE/ASME Trans Mechatron 2(2):164–178
7. Kalantar S, Zimmer UR (2007) Distributed shape control of homogeneous swarms of autonomous underwater vehicles. Auton Robots 22(1):37–53
8. Lewis MA, Tan KH (1997) High precision formation control of mobile robots using virtual structure approach. Auton Robots 4:387–403
9. Lawton AR, Young BJ, Beard RW (2000) A decentralized approach to elementary formation maneuvers, IEEE international conference on robotics and automation, vol 3. San Francisco, pp 2728–2733
10. Leonard NE, Fiorelli E (2001) Virtual leader, artificial potentials and coordinated control of groups, IEEE conference on decision and control, vol 3. Orlando, pp 2968–2973
11. Lawton JR, Beard RW, Young BJ (2003) A decentralized approach to formation maneuvers. IEEE Trans Robot Autom 19(6):933–941
12. Ren W, Sorensen N (2008) Distributed coordination architecture for multi-robot formation control. Robot Auton Syst 56(4):324–333
13. Ren W, Beard RW (2004) Decentralized scheme for spacecraft formation flying via the virtual structure approach. J Guidance Control Dyn 27(1):73–82
14. Do KD (2008) Formation tracking control of unicycle-type mobile robots with limited sensing ranges. IEEE Trans Control Syst Technol 16(3):527–538
15. Dong WJ, Farrell JA (2009) Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty. Automatica 45:706–710
16. Das K, Fierro R, Kuma V et al (2002) A vision-based formation control framework. IEEE Trans Robot Autom 18(5):813–825
17. Wu XF, Xu C (2011) Mean synchronization of pinning complex networks with linearly and nonlinearly time-delay coupling. Int J Digit Content Technol Appl. 5(3):33–46
18. Yang Q, Shi J, Tang B (2010) Convergence analysis of decentralized slot synchronization algorithm for wireless ad hoc networks. J Convergence Inf Technol 5(10):223–232
19. Xiao F, Wang L, Chen J, Gao Y (2009) Finite-time formation control for multi-agent systems. Automatica 45(11):2605–2611
20. Wang X, Hong Y (2010) Distributed finite-time χ-consensus algorithms for multi-agent systems with variable coupling topology. J Syst Sci Complexity 23(2):209–218