Chapter 24
Covariance Intersection Fusion Robust Steady-State Kalman Filter for Two-Sensor Systems with Time-Delayed Measurements

Wenjuan Qi, Peng Zhang, Wenqing Feng and Zili Deng

Abstract For two-sensor systems with time-delayed measurements and uncertain noise variances, this paper presents a measurements transformation approach which transforms the systems with time-delayed measurements into the equivalent systems without measurement delays. Further the local robust steady-state Kalman filter with conservative upper bounds of unknown noise variances is presented, and then the covariance intersection (CI) fusion robust steady-state Kalman filter is also presented. The robustness of these filters is proved based on the Lyapunov equation. It is proved that the robust accuracy of the CI fuser is higher than that of each local robust Kalman filter. A Monte-Carlo simulation example shows its correctness and effectiveness.

Keywords Multi-sensor information fusion · Covariance intersection fusion · Robust Kalman filter · Time-delayed measurements · Uncertain noise variances

24.1 Introduction

The multi-sensor information fusion has received great attentions and has been widely applied in many high-technology fields, such as tracking, signal proceeding, GPS position, robotics and so on. Usually, the standard systems without time-delayed observations are considered, but in many fields, such as the communication and control engineering, the systems with observations delays exist [1, 2].

The optimal Kalman filtering needs to exactly know system model and noise variances, the robust Kalman filters are designed to solve the filtering problems for

W. Qi · P. Zhang · W. Feng · Z. Deng (✉)
Department of Automation, Heilongjiang University, Electronic and Engineering College 130, XueFu Road 74, Harbin 150080, China
e-mail: dzl@hlju.edu.cn

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uncertain systems. In recent years, several results have been derived for any admissible uncertainty of model parameters [3, 4] based on the Riccati equations.

Recently, the covariance intersection fusion (CI) method has been presented by Julier and Uhlman [5], which can handle the systems with unknown variances and cross-covariances.

In this paper, the two-sensor systems with uncertain noise variances and time-delayed measurements are considered. The local steady-state robust Kalman filter is presented and the covariance intersection (CI) fusion robust Kalman filter is proposed by the convex combination of the local robust Kalman filters. The robustness of the filters is proved based on the Lyapunov equation.

24.2 Measurement Transformation

Consider the two-sensor uncertain system with time-delayed measurements

$$x(t+1) = \Phi x(t) + \Gamma w(t) \tag{24.1}$$

$$z_i(t) = H_i x(t-k_i) + e_i(t), i = 1, 2, \tag{24.2}$$

where t is the discrete time, $x(t) \in R^n$ is the state, $z_i(t) \in R^{m_i}$ is the measurement of the ith subsystem, $k_i \geq 0$ is the time-delay, $w(t) \in R^r, e_i(t) \in R^{m_i}$ are uncorrelated white noises with zeros mean and unknown actual variances $\bar{Q}$ and $\bar{R}_i$, respectively. Φ, Γ and H_i are known constant matrices. Assume that Q and R_i are conservative upper bounds of $\bar{Q}$ and $\bar{R}_i$, respectively, i.e.

$$\bar{Q} \leq Q, \bar{R}_i \leq R_i, i = 1, 2 \tag{24.3}$$

where $A \leq B$ means that $B - A \geq 0$ is a semi-positive definite matrix. Assume that each subsystem is completely observable and completely controllable.

Introducing the new measurements $y_i(t)$ and the measurement noises $v_i(t)$

$$y_i(t) = z_i(t+k_i), v_i(t) = e_i(t+k_i), i = 1, 2 \tag{24.4}$$

From (24.2), we have the observation equations without time-delayed

$$y_i(t) = H_i x(t) + v_i(t), i = 1, 2 \tag{24.5}$$

where $v_i(t)$ also has the variances $\bar{R}_i$. From (24.4), we have

$$\hat{x}_i^z(t|t) = \hat{x}_i(t|t-k_i), i = 1, 2 \tag{24.6}$$

where $\hat{x}_i^z(t|t)$ are the estimates of $x(t)$ based on $(z_i(t), z_i(t-1), \cdots)$ $\hat{x}_i(t|t-k_i)$ are the estimates of $x(t)$ based on $(y_i(t-k_i), y_i(t-k_i-1), \cdots)$.

Define the local steady-state cross-covariance as

$$P_{ij}^z = E\left[\tilde{x}_i^z(t|t)\tilde{x}_j^{zT}(t|t)\right], P_{ij}\left(k_i, k_j\right) = E\left[\tilde{x}_i(t|t-k_i)\tilde{x}_j^T\left(t|t-k_j\right)\right] \tag{24.7}$$

where $\tilde{x}_i^z(t|t) = x(t) - \hat{x}_i^z(t|t)$, $\tilde{x}_i(t|t-k_i) = x(t) - \hat{x}_i(t|t-k_i)$, from (24.6), we can get $P_{ij}^z = P_{ij}(k_i, k_j)$. When $i = j$, defining $P_i^z = P_{ii}^z$, $P_i(k_i) = P_{ii}(k_i, k_i)$, we have $P_i^z = P_i(k_i)$.

The problem is to find the local robust steady-state Kalman filter $x_i^z(t|t)$ and the CI fused robust steady-state Kalman filter $x_{CI}^z(t|t)$.

24.3 Local Robust Steady-State Kalman Filter

For two-sensor system (24.1) and (24.5), the local conservative steady-state Kalman one-step predictor with conservative variances Q and R_i are given by Sun and Deng [6], Kailath et al. [7]

$$\hat{x}_i(t+1|t) = \psi_{pi}\hat{x}_i(t|t-1) + K_{pi}y_i(t) \tag{24.8}$$

$$\psi_{pi} = \Phi - K_{pi}H_i, K_{pi} = \Phi\Sigma_i H_i^T\left(H_i\Sigma_i H_i^T + R_i\right)^{-1} \tag{24.9}$$

where Ψ_{pi} is a stable matrix and conservative one-step predictor error variance Σ_i satisfies the steady-state Riccati equation

$$\Sigma_i = \Phi\left[\Sigma_i - \Sigma_i H_i^T\left(H_i\Sigma_i H_i^T + R_i\right)^{-1}H_i\Sigma_i\right]\Phi^T + \Gamma Q\Gamma^T \tag{24.10}$$

From (24.8), it can be rewritten as the Layapunov equation

$$\Sigma_i = \psi_{pi}\Sigma_i\psi_{pi}^T + \Gamma Q\Gamma^T + K_{pi}R_iK_{pi}^T \tag{24.11}$$

Defining the actual steady-state one-step predictor error variance as

$$\bar{\Sigma}_i = \mathrm{E}\left[\tilde{x}_i(t+1|t)\tilde{x}_i^{\mathrm{T}}(t+1|t)\right], \tilde{x}_i(t+1|t) = x(t+1) - \hat{x}_i(t+1|t) \tag{24.12}$$

Theorem 1 *The Kalman one-step predictor* (24.8)–(24.11) *is robust for all admissible actual variances* $\bar{Q}$ *and* $\bar{R}_i$ *satisfying* $\bar{Q} \le Q, \bar{R}_i \le R_i$, *in the sense that*

$$\bar{\Sigma}_i \le \Sigma_i \tag{24.13}$$

Proof From (24.1), we have $\hat{x}_i(t+1|t) = \Phi\hat{x}_i(t|t)$, applying (24.12) yields $\tilde{x}_i(t+1|t) = \Phi\tilde{x}_i(t|t) + \Gamma w(t)$, where $\tilde{x}_i(t|t) = \left[I_n - K_{fi}H\right]\tilde{x}_i(t|t-1) - K_{fi}v_i(t)$ and $K_{fi} = \Sigma_i H_i^T\left(H_i\Sigma_i H_i^T + R_i\right)^{-1}$, we have the actual prediction error formula

$$\tilde{x}_i(t+1|t) = \psi_{pi}\tilde{x}_i(t|t-1) + \Gamma w(t) - K_{pi}v_i(t) \tag{24.14}$$

According to (24.12), applying (24.14) yields the actual steady-state one-step predictor error variance as

$$\bar{\Sigma}_i = \psi_{pi}\bar{\Sigma}_i\psi_{pi}^T + \Gamma\bar{Q}\Gamma^T + K_{pi}\bar{R}_iK_{pi}^T \tag{24.15}$$

Defining $\Delta\Sigma_i = \Sigma_i - \bar{\Sigma}_i$, subtracting (24.15) from (24.11) yields the Lyapunov equation

$$\Delta\Sigma_i = \psi_{pi}\Delta\Sigma_i\psi_{pi}^T + \Gamma(Q - \bar{Q})\Gamma^T + K_{pi}(R_i - \bar{R}_i)K_{pi}^T \tag{24.16}$$

Applying (24.3), noting that ψ_{pi} is a stable matrix, and applying the property of the Lyapunov equation [1] yield that $\Delta\Sigma_i \geq 0$, i.e. $\bar{\Sigma}_i \leq \Sigma_i$. □

For (24.1) and (24.5), the steady-state multi-step Kalman predictors are given by [6, 7]

$$\hat{x}_i(t + k_i|t) = \Phi^{k_i-1}\hat{x}_i(t+1|t), k_i \geq 2 \tag{24.17}$$

The local steady-state multi-step predictor error variances are given as

$$P_i(k_i) = \Phi^{k_i-1}\Sigma_i\left(\Phi^{k_i-1}\right)^T + \sum_{j=0}^{k_i-2}\Phi^j\Gamma Q\Gamma^T\left(\Phi^j\right)^T, k_i \geq 2 \tag{24.18}$$

Defining the actual steady-state multi-step predictor error variance as

$$\bar{P}_i(k_i) = \mathrm{E}\left[\tilde{x}_i(t+k_i|t)\tilde{x}_i^{\mathrm{T}}(t+k_i|t)\right], \tilde{x}_i(t+k_i|t) = x(t+k_i) - \hat{x}_i(t+k_i|t) \tag{24.19}$$

Theorem 2 The conservative Kalman multi-step predictor (24.17)–(24.18) is robust for all admissible actual variances $\bar{Q}$ and $\bar{R}_i$ satisfying $\bar{Q} \leq Q, \bar{R}_i \leq R_i$. i.e.

$$\bar{P}_i(k_i) \leq P_i(k_i) \tag{24.20}$$

Proof Iterating $N - 1$ steps for (24.1), we obtain the non-recursive formula as

$$x(t + k_i) = \Phi^{k_i-1}x(t+1) + \sum_{j=0}^{k_i-2}\Phi^j\Gamma w(t-1) \tag{24.21}$$

Substituting (24.17) and (24.21) into $\tilde{x}_i(t+k_i|t) = x(t+k_i) - \hat{x}_i(t+k_i|t)$, we have

$$\tilde{x}_i(t+k_i|t) = \Phi^{k_i-1}\tilde{x}(t+1|t) + \sum_{j=0}^{k_i-2}\Phi^j\Gamma w(t-1) \tag{24.22}$$

Substituting (24.22) into (24.19) yields the actual steady-state filtering error variance as

$$\bar{P}_i(k_i) = \Phi^{k_i-1}\bar{\Sigma}_i\left(\Phi^{k_i-1}\right)^T + \sum_{j=0}^{k_i-2}\Phi^j\Gamma\bar{Q}\Gamma^T\left(\Phi^j\right)^T, k_i \geq 2 \tag{24.23}$$

Defining $\Delta P_i(k_i) = P_i(k_i) - \bar{P}_i(k_i)$, subtracting (24.23) from (24.18) yields

$$\Delta P_i(k_i) = \Phi^{k_i-1}(\Sigma_i - \bar{\Sigma}_i)\left(\Phi^{k_i-1}\right)^T + \sum_{j=0}^{k_i-2}\Phi^j\Gamma(Q - \bar{Q})\Gamma^T\left(\Phi^j\right)^T \tag{24.24}$$

Applying (24.3) and (24.13) yields $\Delta P_i(k_i) \geq 0$, (24.20) holds. □

24.4 CI Fusion Robust Steady-State Kalman Filter

For two-sensor system (24.1) and (24.2), applying the CI fused algorithm [5], the CI fusion robust steady-state Kalman filters are given as

$$\hat{x}^z_{CI}(t|t) = P^z_{CI}\Big(\omega\big(P^z_1\big)^{-1}\hat{x}^z_1(t|t) + (1-\omega)\big(P^z_2\big)^{-1}\hat{x}^z_2(t|t)\Big) \tag{24.25}$$

$$P^z_{CI} = \Big[\omega\big(P^z_1\big)^{-1}+(1-\omega)\big(P^z_1\big)^{-1}\Big]^{-1} \tag{24.26}$$

Applying (24.25), (24.26), (24.6) and (24.7) yields

$$\hat{x}^z_{CI}(t|t) = P^z_{CI}\big(\omega P_1^{-1}(k_1)\hat{x}_1(t|t-k_1) + (1-\omega)P_2^{-1}(k_2)\hat{x}_2(t|t-k_2)\big) \tag{24.27}$$

$$P^z_{CI} = \big[\omega P_1^{-1}(k_1) + (1-\omega)P_2^{-1}(k_2)\big]^{-1} \tag{24.28}$$

with the constraint $\omega \geq 0$, when $k_i = 1$, we have $P_1(1) = \Sigma_1, P_2(1) = \Sigma_2$.

The weighting coefficient ω is obtained by minimizing the performance index

$$\min_{\omega} trP^z_{CI} = \min_{\omega\in[0,1]} tr\Big\{\big[\omega P_1^{-1}(k_1) + (1-\omega)P_2^{-1}(k_2)\big]^{-1}\Big\} \tag{24.29}$$

where the symbol tr denotes the trace of matrix. The optimal weights ω can be quickly obtained by the 0.618 method or the Fibinacci method.

Theorem 3 *The covariance intersection fused filter* (24.27) *and* (24.28) *has the actual error variance* $\bar{P}_{CI}$ *as*

$$\begin{aligned}\bar{P}^z_{CI} &= E\big[\tilde{x}^z_{CI}(t|t)\tilde{x}^{zT}_{CI}(t|t)\big]\\ &= P^z_{CI}\big[\omega^2 P_1^{-1}(k_1)\bar{P}_1(k_1)P_1^{-1}(k_1) + \omega(1-\omega)P_1^{-1}(k_1)\bar{P}_{12}(k_1,k_2)P_2^{-1}(k_2)\\ &\quad +\omega(1-\omega)P_2^{-1}(k_2)\bar{P}_{21}(k_2,k_1)P_1^{-1}(k_1) + (1-\omega)^2P_2^{-1}(k_2)\bar{P}_2(k_2)P_2^{-1}(k_2)\Big]P^z_{CI}\end{aligned} \tag{24.30}$$

where $\bar{P}_{12}(k_1,k_2) = E\big[\tilde{x}_1(t|t-k_1)\tilde{x}_2^T(t|t-k_2)\big]$ and

$$\begin{aligned}\bar{P}_{12}(k_1,k_2) =& \Phi^{k_1-1}\psi_{p1}^{k_2-k_1}\bar{\Sigma}_{12}\big(\Phi^{k_2-1}\big)^T\\ &+\sum_{r=k_1-1}^{k_2-2}\Phi^{k_1-1}\psi_{pi}^{k_1-r-1}\Gamma\bar{Q}\Gamma^T(\Phi^r)^T + \sum_{r=0}^{k_1-2}\Phi^r\Gamma\bar{Q}\Gamma(\Phi^r)^{T}, k_2\geq k_1\geq 2\end{aligned} \tag{24.31}$$

$$\begin{aligned}\bar{P}_{12}(k_1,k_2) =& \Phi^{k_1-1}\bar{\Sigma}_{12}\psi_{p2}^{k_1-k_2}\big(\Phi^{k_2-1}\big)^T\\ &+\sum_{r=k_2-1}^{k_1-2}\Phi^r\Gamma\bar{Q}\Gamma^T\psi_{p2}^{(k_2-r-1)T}\Psi^{(k_2-1)T} + \sum_{r=0}^{k_2-2}\Phi^r\Gamma\bar{Q}\Gamma(\Phi^r)^{T}, k_1\geq k_2\geq 2\end{aligned} \tag{24.32}$$

Especially

$$\bar{P}_{12}(1,1) = \bar{\Sigma}_{12}, \bar{\Sigma}_{12} = \psi_{p1}\bar{\Sigma}_{12}\psi_{p2}^T + \Gamma\bar{Q}\Gamma^T \tag{24.33}$$

$$\bar{P}_{12}(k_1,1) = \Phi^{k_1-1}\bar{\Sigma}_{12}\psi_{p2}^{k_1-1} + \sum_{r=0}^{k_1-2}\Phi^r\Gamma\bar{Q}\Gamma^T\Psi_{p2}^{rT} \tag{24.34}$$

$$\bar{P}_{12}(1,k_2) = \psi_{p1}^{k_2-1}\bar{\Sigma}_{12}\left(\Phi^{k_2-1}\right)^T + \sum_{r=0}^{k_2-2}\psi_{pi}^r\Gamma\bar{Q}\Gamma^T(\Phi^r)^T \tag{24.35}$$

Proof From (24.28), we have $x(t) = P_{CI}^z\left[\omega P_1^{-1}(k_1) + (1-\omega)P_2^{-1}(k_2)\right]x(t)$. Using (24.27), we easily obtain the CI actual fused filtering error

$$\tilde{x}_{CI}^z(t|t) = P_{CI}^z\left[\omega P_1^{-1}(k_1)\tilde{x}_1(t|t-k_1) + (1-\omega)P_2^{-1}(k_2)\tilde{x}_2(t|t-k_2)\right] \tag{24.36}$$

which yields (24.30). Equations (24.31)–(24.35) have been proved in Ref. [6]. □

Remark 1 Applying (24.20), Ref. [5] proved that the two-sensor CI fuser is robust for all admissible $\bar{Q}$ and $\bar{R}_i$ satisfying (24.3), i.e.

$$\bar{P}_{CI}^z \leq P_{CI}^z \tag{24.37}$$

24.5 Accuracy Analysis

Theorem 4 For the two-sensor system (24.1)–(24.2) with time-delayed measurements, the local steady-state robust Kalman filter and CI fuser have the accuracy relations

$$\bar{P}_i^z = \bar{P}_i(k_i), P_i^z = P_i(k_i) \tag{24.38}$$

$$tr\bar{P}_i^z \leq trP_i^z, i = 1,2 \tag{24.39}$$

$$tr\bar{P}_{CI}^z \leq trP_{CI}^z \leq trP_i^z, i = 1,2 \tag{24.40}$$

Proof From the robustness (24.20) the accuracy relation (24.39) holds. From (24.37), the first inequality of (24.40) holds. Applying (24.29), taking $\omega = 1$ yields $trP_{CI}^z = trP_1^z$ and $\omega = 0$ yields $trP_{CI}^z = trP_2^z$, Hence when $\omega \in [0,1]$, we have the accuracy relation $trP_{CI}^z \leq trP_i^z, i = 1,2$ □.

Remark 2 Inequalities (24.39) and (24.40) show that the robust accuracy of the CI fuser is higher than that of each local robust filter.

24.6 Simulation Example

Consider the two-sensor tracking system (24.1)–(24.2)with time-delayed measurements, where $\Phi = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.5\mathrm{T}_0^2 \\ T_0 \end{bmatrix}$, $H_1 = [\,1 \quad 0\,]$, $H_2 = I_2$, $T_0 = 0.25$ is the sampled period, $x(t) = [x_1(t), x_2(t)]^T$ is the state, $x_1(t)$ and $x_2(t)$ are the position and velocity of target at time tT_0. $w(t)$ and $v_i(t)$ are independent Gaussion white noises with zero mean and unknown variances Q and R_i respectively. In the simulation, we take $Q = 0.5,\ R_1 = 0.58,\ R_2 = diag(4, 0.25),\ \bar{Q} = 0.45,\ \bar{R}_1 = 0.5,\ \bar{R}_2 = diag(3,\ 0.16),\ k_1 = 1,\ k_2 = 2$

In order to give a geometric interpretation of the accuracy relations, the covariance ellipse is defined as the locus of points $\{x : x^T P^{-1} x = c\}$, where P is the variance matrix and c is a constant. Generally, we select $c = 1$. It has been proved in [8] that $P_1 \le P_2$ is equivalent to that the covariance ellipse of P_1 is enclosed in that of P_2. The accuracy comparison of the covariance ellipses is shown in Fig. 24.1. From Fig. 24.1, we see that the ellipse of the actual variances $\bar{\Sigma}_1 or \bar{P}_2(2)$ is enclosed in that of $\Sigma_1 or P_2(2)$, respectively, which verify the consistent (24.13) and (24.20). The ellipse of actual CI fused variance $\bar{P}_{CI}$ is enclosed in that of P_{CI}, which verifies the robustness of (24.37).

In order to verify the above theoretical accuracy relations, taking $\rho = 200$ runs, the curves of the mean square errors (MSE) of local and fused Kalman filters are shown in Fig. 24.2, which verifies the accuracy relations (24.39), (24.40) and the accuracy relations in Table 24.1.

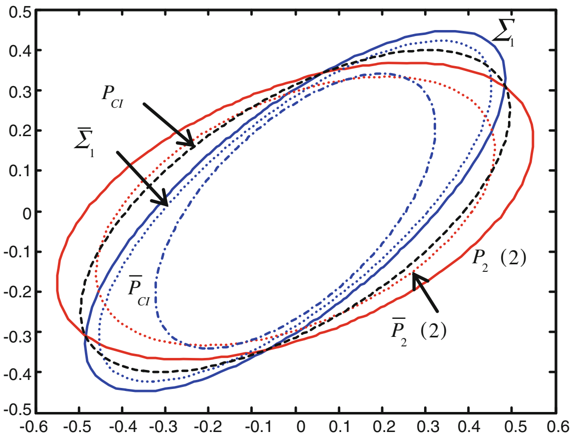

Fig. 24.1 The covariance *ellipses* of robust Kalman filters

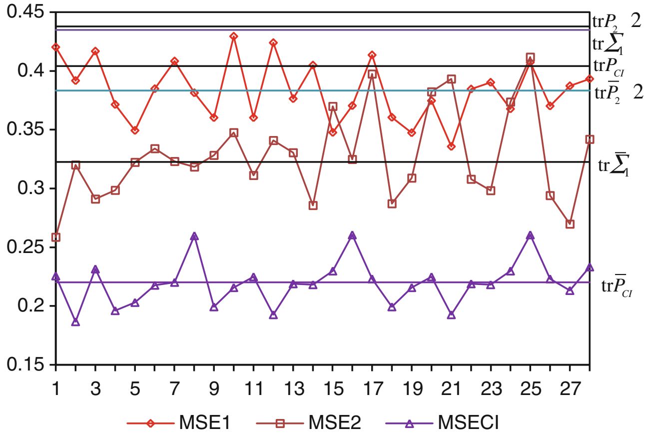

Fig. 24.2 The MSE *curves* of local and fused filters

Table 24.1 The accuracy comparison of local and fused filters

$tr\Sigma_1$	$\mathrm{tr}P_2(2)$	$tr\bar{\Sigma}_1$	$tr\bar{P}_2(2)$	trP_{CI}	$tr\bar{P}_{CI}$
0.4348	0.4377	0.3833	0.3225	0.4042	0.2202

24.7 Conclusion

For two-sensor systems with uncertain noise variances and time-delayed measurements, the local and CI robust fused robust steady-state Kalman filters have been presented, and their robustness was proved based on the Lyapunov equation. The robust accuracy of CI fuser is higher than that the robust accuracy of each local filter

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References

1. Kailath T, Sayed AH, Hassibi B (2000) Linear estimation. Prentice Hall, New York
2. Lu X, Zhang HS, Wang W (2005) Kalman filtering for multiple time-delay systems. Automatica 87(4):1455–1461
3. Zhu X, Soh YC, Xie L (2002) Design and analysis of discrete-time robust Kalman filters. Automatica 38:1069–1077

4. Theodor Y, Sharked U (1996) Roust discrete-time minimum-variance filtering. IEEE Trans Sig Process 44(2):181–189
5. Julier SJ, Uhlman JK (2009) General decentralized data fusion with covariance intersection. In: Liggins ME, Hall DL, Llinas J (eds) Handbook of multisensor data fusion, theory and practice. CRC Press, Boca Raton, pp 319–342
6. Sun X-J, Deng Z-L (2009) Information fusion wiener filter for the multisensory multichannel ARMA signals with time-delayed measurements. IET-Sig Process 3:403–415
7. Kailath T, Sayed AH, Hassibi B (2000) Linear estimation. Prentice Hall, New York
8. Deng Z, Zhang P, Qi W, Liu J, Gao Y (2012) Sequential covariance intersection fusion Kalman filter. Inf Sci 189:293–309