Chapter 22
The Microeconomic Basis for Solow Model on Economic Growth

Zhen He

Abstract Solow's economic growth model, also called the neoclassical economic growth model, is the cornerstone of the Modern Growth Theory. However, except that it's an exogenous growth modal, the aggregate production function and the social capital output per capita curve are "assumed" in it, so that it lacks of the foundation of microeconomics. The goal of this essay is to offer the strong microeconomics theory support for the aggregate production function and social capital output per capita curve which are the structural elements of Solow model.

Keywords Aggregate production function · Microeconomics theory · Social capital output per capita curve · Solow's economic growth model

22.1 Introduction

Robert M. Solow, who made a great contribution to the theory of economic growth, was awarded Nobel Prize in Economics in 1987 because he first created the Macroeconomic growth model. The economic growth model made by Solow is also called Solow economic growth model, Solow model or the exogenous economic growth model. At the same time, his model was called the neoclassical economic growth model as well, because it is the economic growth model in the framework of the neoclassical economics, known as the foundation of the modern economic growth theory.

Nevertheless, Solow model's obvious defect is that it's an exogenous growth model. The reason is that the savings rate, the rate of the population growth and the rate of technology advance are all assumed as exogenous variables in it, which

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means they are actually decided by the behavior of the people inside the economy. So scholars have been devoting themselves to those variables' endogenization, and have presented many endogenous economic growth models (Skousen 2001).

This dissertation thinks that Solow model still lacks the foundation of microeconomics, for some endogenous variables, such as the interest rate, etc., in the endogenous model are still macro variables, even if realizing Solow model's endogenization. But there again, to make exogenous model endogenization is to find the personal behavior hidden in the micro level of the model.

The micro foundation of macroeconomics theories is important, and the issue that the economists have been paying attention to. Fisher, Mises, Friedman successively made up the Missing Monetary Link between the macroeconomics and the microeconomics, establishing the foundation of the personal behavior hidden in macro currency and the microeconomics theory. Hicks made a great contribution to microcosmicizing Keynes's macroeconomics theory, being called the earliest pioneer for microcosmicizing the macroeconomics.

Solow Model lacks the foundation of the microeconomics lie in that: under the premise that there is no description in the connection with the microeconomics, it abruptly assumes a aggregate production function in an economic system shown below:

$$Y = A \cdot F(K, L) \tag{22.1}$$

In function (22.1), Y equals the aggregate output of the economic system, K equals the aggregate capital stock, L equals the aggregate labor, A equals the technology progress variable. The problem is whether the aggregate production function 'assumed' by Solow model can be derived from the production function of every manufacturer representing the personal behavior of the manufacturer.

Besides, for the visualization, only based on the aggregate production function but no description of the connection with the microeconomics theory, Solow model describes the Fig. 22.1 which shows the curve of the capital output social per capita (or fatigue), under the premise that it assumes Capital and labor

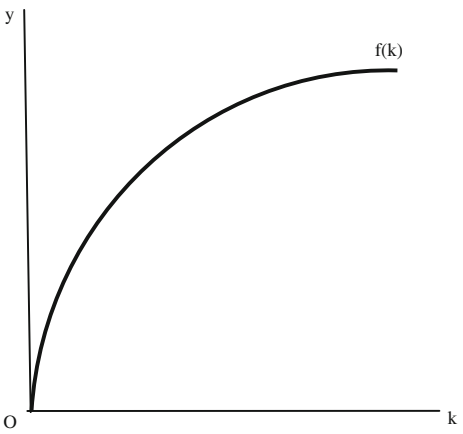

Fig. 22.1 The social production per capita function curve in Solow model

marginal production decline, constant returns to scale and "Inada Conditions". Some scholars straightly use empirical data to describe the curve of the aggregate production function (Cai 2002). Here is the problem: can the curve of the capital output social per capita (or fatigue) be theoretically derived?

22.2 The Microeconomics Foundation of the Aggregate Production Function

Generally, the form of Cobb-Douglas function is

$$f(x) = A\prod\nolimits_{i=1} x_i^{a_i} \tag{22.2}$$

In function (22.2), A is a constant; a_i is the elasticity of f(x) to x_i. If x_i is the consumption goods, then f(x) is a utility function; if x_i is the factors of the production, then f(x) a production function (Eatwell et al. 1987).

Assume the labor input of the manufacturer i is L_i, the capital is K_i, and the output is Y_i, the elasticity of Y_i to L_i and K_i are α and β, so we have the Cobb-Douglas production function as the following:

$$Y_i = A_i L_i^{\alpha} K_i^{\beta} \tag{22.3}$$

The marginal output of labor $MP_L = \frac{\Delta Y_L}{\Delta L}$, the marginal output of capital $MP_K = \frac{\Delta Y_K}{\Delta K}$, separately represent the increment of the output caused by the increment in the capital K (or the labor L) when the labor L (or the capital K) is constant. By Eq. (22.3), the marginal output of capital and labor input but the manufacturer i is

$$\frac{\Delta Y_{Li}}{\Delta L_i} = \alpha \cdot \frac{Y_i}{L_i} \tag{22.4}$$

$$\frac{\Delta Y_{K_i}}{\Delta K_i} = \beta \cdot \frac{Y_i}{K_i} \tag{22.5}$$

The Microeconomics theory indicates: the condition of the manufacturer's production equilibrium is that the ratio equals each other of each production factor's marginal output over its price. Assume the price of the labor is w, the price of the capital is r, the condition of the manufacturer i's production equilibrium is:

$$\frac{MP_{L_i}}{w} = \frac{MP_{K_i}}{r}$$

or

$$\frac{\Delta Y_{L_i}}{w \cdot \Delta L_i} = \frac{\Delta Y_{K_i}}{r \cdot \Delta K_i} \tag{22.6}$$

The function (22.6) is a monetary expression. When not using money, the function (22.6) can be:

$$\frac{\Delta Y_{L_i}}{\Delta L_i} = \frac{\Delta Y_{K_i}}{\Delta K_i} \tag{22.7}$$

The function (22.7) indicates: the condition of the manufacturer i's production equilibrium is that the ratio between each factor's input–output equals each other or has the same proportion (Homans 1961; Adams 1965; He 2011a).

For function (22.4) and (22.5) to be substitute into function (22.7), we have:

$$\alpha_i \cdot \frac{Y_i}{L_i} = \beta_i \cdot \frac{Y_i}{K_i} \tag{22.8}$$

After the same variables being cancelled, the function (22.8) will be:

$$\frac{K_i}{L_i} = \frac{\beta}{\alpha} \tag{22.9}$$

In which, α, β are the exogenous variables. Hendrik Samuel Houthakker found: when the manufacturer input production factors in Lyon Cardiff fixed ratio, and the fixed ratio of each manufacturer fits the Pareto distribution, the aggregate production function has the form of Cobb-Douglas production function (Sato et al. 1975). Based on function (22.9), the aggregate production function about labor and capital will be:

$$Y = AL^{\alpha}K^{\beta} \tag{22.10}$$

In the function (22.10), the aggregate labor L (or the aggregate capital K) is the sum of the each manufacturer's labor L_i (or capital K_i) in a certain time period, or the aggregate capital (or the aggregate labor) input in the economic system in the corresponding time period.

The function (22.10) is the general form of the social production function or the aggregate production function shown in the function (22.1). But, the function (22.10) is theoretically derived based on the single manufacturer production function, so it is strongly supported by microeconomics theory.

22.3 The Microeconomics Foundation of the Social Capital Output Per Capita Curve

To let the people learn and understand the economics theory straightly, economists usually use charts to represent the relevant economic models. Therefore, Solow model generally needs to describe the curve of the social capital output per capita (or fatigue).

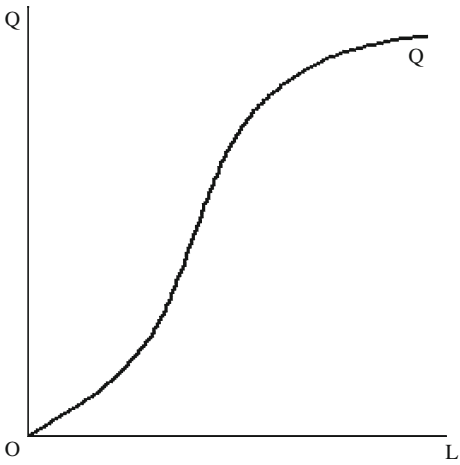

Fig. 22.2 The total output curve with one factor input

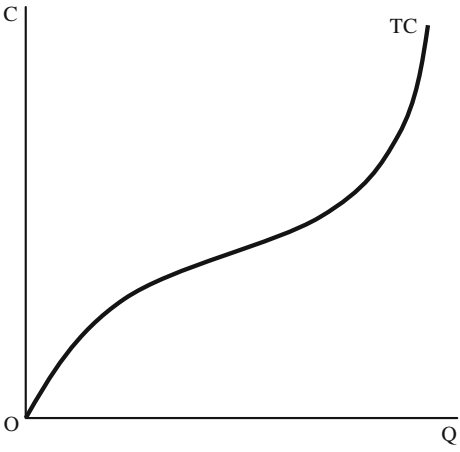

Fig. 22.3 The curve of the manufacturer's short-term total cost

Having the aggregate production function derived from microeconomics theory, basing on the assumptions including Capital and labor marginal production decline, constant returns to scale and "Inada Conditions" (He 2010), etc., there should be a capital production per capita curve that can be described. Nevertheless, we will derive it from the production function curve in the micro level.

What the Fig. 22.2 represents is situation that shows the output Q when the manufacturer inputs one factor. The Producers choose theory in Microeconomics tells us: in short term, the input labor is a variable production factor. Assume the price of the labor is constant, or, in the coordinate system L-Qas shown in Fig. 22.2, the price can be considered as an exogenous production factor, so the changing in the producer's costs C equals the changing in the labor input number. Therefore, we can switch the horizontal ordinate L to C, and switch the position of both x-axis and y-axis, there becomes Fig. 22.3, which is the reverse of the

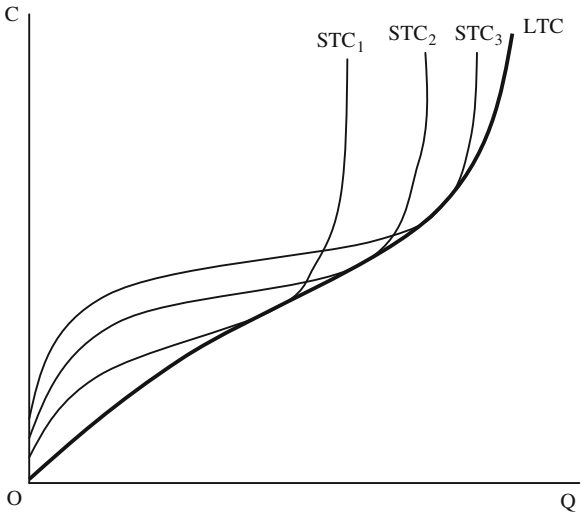

Fig. 22.4 The long-term total cost curve is the envelope of the short-term

manufacturer's total cost curve and the manufacturer's total output curve (Huang 2005; He 2011b), the graphic that is rotated counter-clockwise 90° using y-axis as symmetry axis in Fig. 22.2. Because there is no price variable in either Figs. 22.2 or 22.3, the price can be considered as an exogenous factor or a constant.

Solow model is a long-term economic model. In fact, the Producers choose theory in Microeconomics indicates: in the long term, both the labor input and the capital input are the variable production variables, the long-term total cost curve LTC is the envelope curve of the short-term total cost curve, as shown in Fig. 22.4.

Similar to the relationship of the short-term total cost curve and the short-term total output curve, in the $k_i - y_i$ coordinate system as shown in Fig. 22.5, the price can be considered as an exogenous variable. This indicates: in the long term, the changing in the manufacturer's long-term total cost equals the changing in the quantity of the factor input; the long-term total cost curve and the long-term total output curve are reverse. To present with a circular function curve, the long-term labor input and the capital input can be presented as the capital per capita (or fatigue), and the output can be presented as the output per capita. Assume Fig. 22.4 represents the long-term total cost curve of the manufacturer i, we can switch k_i to C on x-axis, and y_i to Q on y-axis in Fig. 22.4, so there will be Fig. 22.5 that shows the capital output per capita (or fatigue) curve of the manufacturer i. This is the graphic that Fig. 22.4 is rotated counter-clockwise 90 degree, and then using y-axis as symmetry axis in Fig. 22.4.

Assume that there are n manufacturers in a society, the social aggregate production function has the homogeneity in the changing proportion of the output and the input; meanwhile, assume that the variable of the social capital input per capita is k, the social output per capita is y. Same with the principle that the industry Supply (or Production) curve is derived from the Supply (Production) curve of every manufacturer within the industry, we will have: the social capital

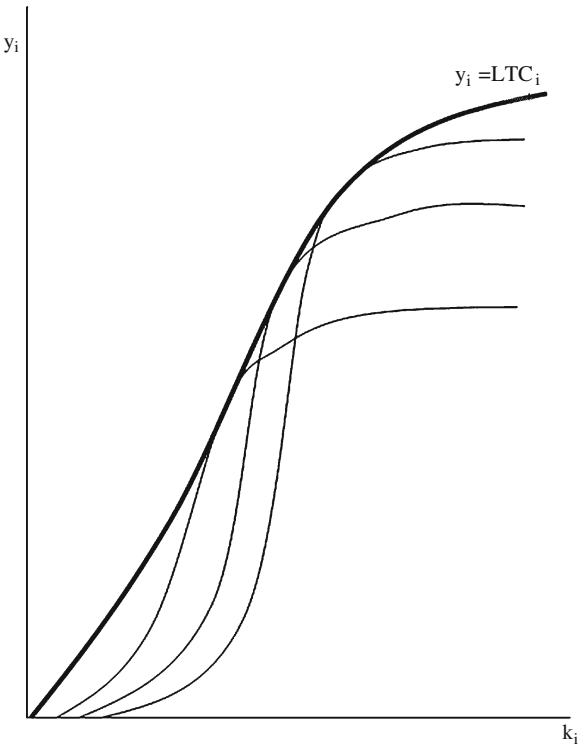

Fig. 22.5 T capital output per capita curve of manufacturer i

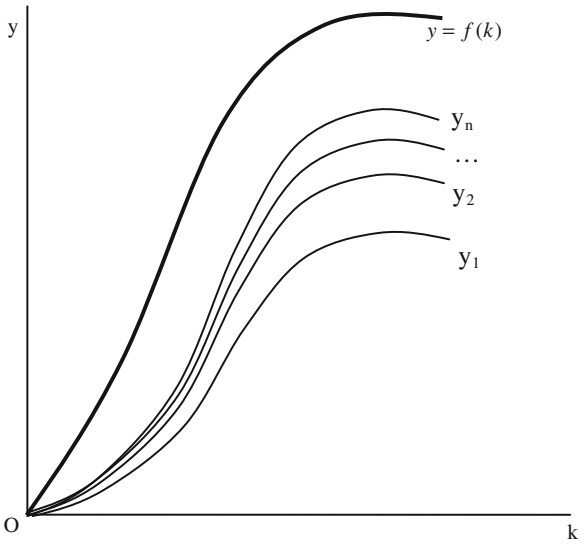

Fig. 22.6 The social capital output per capita curve

output per capita curve is like the situation shown in Fig. 22.5, meaning it is the curve that is gained after summing the x-axis and y-axis in the Supply (Production) curve of every manufacturer within the industry. Figure 22.6 can be derived from Fig. 22.5. Attention please, Fig. 22.6 is the theoretical form of the social capital output per capita curve to Fig. 22.1, and the different thing is that Fig. 22.6 has a strong support of the Microeconomics theory.

22.4 Conclusion

Due to the analysis above, the conclusion is as followings:

1. As the structured elements of Solow model, the aggregate production function and the social capital output per capita curve can now have the theoretical foundation of Microeconomics.
2. The Micro optimality can ensure the Macro optimality. From function (22.9) to function (22.10), we can see: if each manufacturer achieves the inputs with fixed proportion of optimal condition in Micro economy, meaning if there are an optimal α and β, it will make sure that the Macro economy will achieve the optimal condition. In fact, we can prove that there does exist the optimal α and β, meaning there will be an optimal proportion with L and K, an optimal proportion with A and K as well, and an optimal criteria curve for the capital output per capita curve (He 2010). Via the optimal proportion, it will ensure the strong connection between the physical capital, human capital, and technology advance (or R&D) to contribute to the economic growth together.

Of course, this article has its lack, which is no matter the derivation is from the production function in the micro level to the production function in the macro level, or no matter the derivation is from the production function curve in micro level to the production curve in the macro level, they are using the simple aggregation from the micro to the macro. The more appropriate and scientific way is to use the Superposition principle in Physics, and of course before that there should be the Field Theory and the Superstring Theory. All these need another article to discuss.

References

Adams JS (1965) Inequity in social exchange. In: Berkowitz L (ed) Advances in experimental social psychology. Academic Press, New York, pp 267–299

Cai J-M (2002) Macroeconomics. People's Publisher, Beijing, pp 55–59 (Chinese)

Eatwell J, Milgate M, Newman P (1987) The new Palgrave: a dictionary of economics, vol 1. The Macmillan Press Limited, London, pp 460–461

He J-S (2010) Economics: return to adam smith's happiness and harmony framework, vol 2. Truth and Wisdom Press and Shanghai People's Publisher, Shanghai, pp 45–47 (Chinese)

He Z (2010) Chapter 25 quarter 3 the pareto optimal criterion for harmony social interest. He J-S, Economics: return to adam smith's happiness and harmony framework, vol 2. Truth and Wisdom Press and Shanghai People's Publisher, Shanghai, pp 267–292 (Chinese)

He J-S (2011a) A concise introduction to economics: return to adam smith's happiness and harmony framework. Truth and Wisdom Press and Shanghai People's Publisher, Shanghai, p 348 (Chinese)

He J-S (2011b) A concise introduction to economics: return to adam smith's happiness and harmony framework (footnote on the page 38, 87). Truth and Wisdom Press and Shanghai People's Publisher, Shanghai (Chinese)

Homans GC (1961) Social behavior: its elementary forms. Harcourt, Brace and World, Inc., New York, p 75

Huang Y-J (2005) Microeconomics, 2nd, High Education Press, Beijing, pp 87–88. (Chinese)

Sato K (1975) Production functions and aggregation, vol XXI. North-Holland, Amsterdam, pp 10–32

Skousen M (2001) The making of modern economics: the lives and ideas of the great thinkers. M. E. Sharpe Inc., New York