

# Chapter 96

## The Relations Tracking Method of Establishing Reachable Matrix in ISM

Xue-feng He and Yue-wu Jing

**Abstract** Interpretative Structural Modeling (ISM) is a common Structure modeling technology. For this technology, the establishment of reachable matrix is a quite important step. By analyzing the relations among many factors in a system, the relations tracking method for reachable matrix is presented in this paper. Through a comparative analysis of several methods, the relations tracking method is proved to be rapid and effective for establishment of reachable matrix meanwhile avoiding complex matrix operations, and can be used to enhance the applicability of ISM.

**Keywords** Adjacency matrix · Directed graph · Interpretative structural modeling · Reachable matrix

### 96.1 Introduction

The Interpretative Structural Modeling (ISM) is a common structure modeling technology and widely used in many fields (Li 2011; Thakkar et al. 2004, 2007; Singh 2007; Kanungo and Bhatnagar 2002; Bolaños et al. 2005). In terms of the workflow of ISM, when directed graph is obtained, the establishment of reachable matrix is a quite important step (Wang 1998). Usually, the main methods for establishing reachable matrix are formula method (Wang 2000), experience

---

X. He (✉)

School of Economics and Management, Southwest University  
of Science and Technology, Mianyang, China  
e-mail: hxfjyw@163.com

Y. Jing

Institute of Technology, China Academy of Engineering Physics,  
Mianyang, China  
e-mail: jywxfh163.com

dialogue method (Zhenkui 1998) and Warshall algorithm (Lipschutz and Lipson 2002; Wang and Ge 1996) and so on. In this paper, the relations tracking method for reachable matrix is presented, compared with the other methods, this method can avoid complex matrix operations.

## 96.2 Relations Tracking Method

### 96.2.1 Definition

Relations tracking method, just as its name implies, the relationships among many factors in a system should be tracked firstly and then the reachable matrix may be established. In this paper we use Fig. 96.1 as an example to illustrate.

### 96.2.2 Steps

The first step is to find out the direct reachable set of each node.

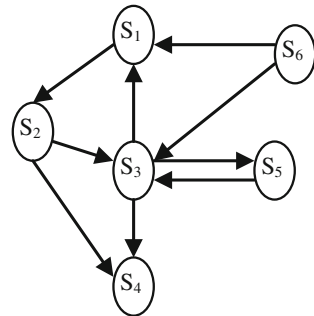
Direct reachable set is the element set that a node can reach directly not including the node itself, expressed with  $D\langle i \rangle$ . For example, for Fig. 96.1, the direct reachable set of node  $S_2$  is  $D\langle 2 \rangle = \{3, 4\}$ . Similarly,  $D\langle 4 \rangle = \emptyset$ . All the direct reachable sets are shown in Table 96.1.

The second step is to find out the tracking reachable set of each node.

Tracking reachable set is the element set that each node can reach, whether directly or indirectly, which includes the node itself, expressed with  $R\langle i \rangle$ . For reachable matrix, this step is quite important.

The core idea of relations tracking method is as follows. Each node can be viewed as a source node and the direct reachable set of the node will be obtained, then each node in the direct reachable set can be viewed as a branch node which can be used as the next level branch node, thus the tree branch of each node can be obtained and all the nodes of tree branch constitute the tracking reachable set of

Fig. 96.1 Directed connection diagram



**Table 96.1** The direct reachable sets

$S_i$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$
$D < i >$	2	3,4	1, 4, 5	$\emptyset$	3	1,3

each node. If a direct reachable set is an empty set, then the tracking reachable set is the node itself. For example, in Table 96.1, the direct reachable set of  $S_4$  is empty, then  $R < 4 > = \{4\}$ .

The important principles: in the process of branching, if a node is repeated, then the node should be omitted, that is the node is no longer continue to branch.

For example, for the tracking reachable set of  $S_1$ , the tree branch obtained is shown in Fig. 96.2.

Thus,  $R < 1 > = \{1, 2, 3, 4, 5\}$ , similarly,  $R < 2 > = \{1, 2, 3, 4, 5\}$ ,  $R < 3 > = \{1, 2, 3, 4, 5\}$ ,  $R < 5 > = \{1, 2, 3, 4, 5\}$ ,  $R < 6 > = \{1, 2, 3, 4, 5, 6\}$ , the tree branches of  $S_2, S_3, S_5$  and  $S_6$  are shown in Fig. 96.3.

The third step is to write out the reachable matrix.

The reachable matrix  $M$  is shown below.

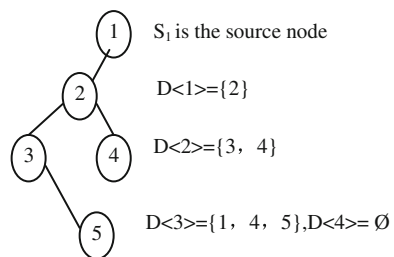
$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

This method can avoid fundamentally complex matrix operations, and it just needs to track the relations among nodes.

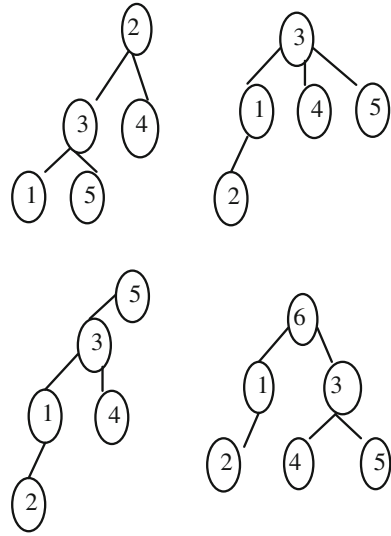
### 96.2.3 The Explanation of Relations Tracking Method

The relations tracking method reflects the essence of reachable matrix. In terms of this method, the tracking reachable set can be obtained directly based on the directed graph, so the reachable matrix obtained from relations tracking method is just requested. From reachable matrix  $M$  it can be known that the elements in row 1, 2, 3 and 5 are same, which suggest that  $S_1, S_2, S_3, S_5$  may form loops.

**Fig. 96.2** The tree branch of  $S_1$



**Fig. 96.3** The tree branches of  $S_2, S_3, S_5, S_6$



The relations tracking method can avoid the repeated searching because a repeated node will be no longer continue to branch. Breadth First Search is a method looking for the shortest path between two nodes in a directed graph (Wang et al. 1994; Lu and Feng 2006; Yuan and Wang 2011), although the repeated searching can be avoided, it requires that a clear hierarchical relation be established firstly. However, in the ISM process, the hierarchical relationship is just obtained after reachable matrix.

### 96.3 Comparative Analysis of Several Methods

#### 96.3.1 Formula Method

With formula method, the reachable matrix is obtained in terms of  $(A + I)r = (A + I)r - 1 = M$ .  $A$  is adjacency matrix,  $I$  is unit matrix,  $M$  is reachable matrix.  $A$  can be obtained based on directed graph.

Usually, in terms of the formula, after the sequential operation,  $A_i$  can be obtained, that is  $A_1 \neq A_2 \neq \dots \neq A_{r-1} = A_r = A_{r+1}$ ,  $A_r = (A + I)^r$ ,  $r \leq n - 1$ ,  $n$  is order number. Because  $M = (A + I)^r$ , then  $M = A_r = A_r - 1$ . Also taken Fig. 96.1 as an example, the operation process is shown below.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A_1 = A + I$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$A_2 = (A + I)^2 = A_1^2$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A_3 = (A + I)^3 = A_2 \cdot A_1$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A_4 = (A + I)^4 = A_3 \cdot A_1 = A_3 = M$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Formula method is a traditional method and used widely, but the biggest drawback of this method is the complicated matrixes calculation which is only acceptable when elements are quite few. Actually a system is often large with many elements, and the relationships among elements are quite complicated. Therefore complicated matrix calculation decreases the practicality of this method (Tian and Wang 2003).

### 96.3.2 Warshall Algorithm

The steps of this algorithm are shown below. ① $P \leftarrow A$ . ② $k \leftarrow 1$ . ③ $i \leftarrow 1$ . ④if  $p_{ik} = 1$ , then  $p_{ij} \leftarrow p_{ij} \vee p_{kj}$ ,  $j = 1, 2, \dots, n$ . ⑤ $i \leftarrow i+1$ . If  $i \leq n$ , then turn to step ④. ⑥ $k \leftarrow k+1$ . If  $k \leq n$ , then turn to step ③, otherwise stop.  $A$  is adjacency matrix,  $P$  is reachable matrix.

Among these steps, the step ④ is crucial. For Fig. 96.1, the operation process is shown as follows.

$$\textcircled{1} P \leftarrow A \quad P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\textcircled{2} \underline{k \leftarrow 1} \quad i \leftarrow 1 \quad p_{11} = 0, \quad i \leftarrow 2 \quad p_{21} = 0$$

$$i \leftarrow 3 \quad p_{31} = 1 \quad P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & \underline{1} & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$i \leftarrow 4 \quad p_{41} = 0, \quad i \leftarrow 5 \quad p_{51} = 0$$

$$i \leftarrow 6 \quad p_{61} = 1 \quad P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & \underline{1} & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\underline{k \leftarrow 2}$$

$$i \leftarrow 1 \quad p_{12} = 1 \quad P = \begin{bmatrix} 0 & 1 & \underline{1} & \underline{1} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$i \leftarrow 2 \quad p_{22} = 0,$$

$$i \leftarrow 3 \quad p_{32} = 1 \quad P = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 1 & 1 & \underline{1} & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$i \leftarrow 4 \quad p_{42} = 0, \quad i \leftarrow 5 \quad p_{52} = 0$$

$$i \leftarrow 6 \quad p_{62} = 1 \quad P = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & \underline{1} & 1 & 0 \end{bmatrix}$$

$$\underline{k \leftarrow 3}$$

$$i \leftarrow 1 \quad p_{13} = 1 \quad P = \begin{bmatrix} \underline{1} & 1 & 1 & 1 & \underline{1} & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$i \leftarrow 2 p_{23} = 1 \quad P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ \underline{1} & \underline{1} & 1 & 1 & \underline{1} & 0 \\ \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$i \leftarrow 3 p_{33} = 1 \quad P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$i \leftarrow 4 p_{43} = 0$$

$$i \leftarrow 5 p_{53} = 1 \quad P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \underline{1} & \underline{1} & 1 & \underline{1} & \underline{1} & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$i \leftarrow 6 p_{63} = 1 \quad P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & 0 \end{bmatrix}$$

Because the calculation process is quite tedious, so the partial steps ( $k = 4$  and  $5$ ) are omitted.

When  $k = 6$ , the comparing result is as follows.

$$\begin{aligned} k \leftarrow 6 \quad i \leftarrow 1 \quad p_{16} &= 0 \\ i \leftarrow 2 \quad p_{26} &= 0 \\ i \leftarrow 3 \quad p_{36} &= 0 \\ i \leftarrow 4 \quad p_{46} &= 0 \\ i \leftarrow 5 \quad p_{56} &= 0 \\ i \leftarrow 6 \quad p_{66} &= 0 \end{aligned}$$



The essence of Warshall algorithm is matrix comparing and matrix updating based on the comparisons, at last the reachable matrix can be obtained. The comparing scope of formula method is between matrix and matrix, while the comparing scope of Warshall algorithm is limited in a matrix. However, Warshall algorithm also involves a large number of repeated comparing. From the operations above it can be known that when  $k = 3$ ,  $i = 5$ , matrix P do not change, but according to this algorithm, the comparing of remainder must go on.

Another problem of Warshall algorithm is, reachable matrix obtained from Warshall algorithm cannot reflect the circumstance that a node gets to itself. However, the reachable matrix obtained from formula method can reflect the circumstance that each node gets to itself. The reason is that the operation of formula method is made after the adjacency matrix and unit matrix added together. This paper argues that when reachable matrix obtained from Warshall algorithm plus unit matrix, the problem can be solved effectively.

Compared with formula method and Warshall algorithm, relations tracking method take the form of tree branch to track the reachable set, thus the complicated operations can be avoided. Of course, with computer programming the establishment of reachable matrix may be more efficient.

## 96.4 Conclusion

When establishing ISM, the calculation of reachable matrix is always quite crucial and tedious, in order to solve this problem, the relations tracking method for reachable matrix is presented, compared with the other methods, this method can avoid complex matrix operations and consequently enhance the practical operability of ISM.

**Acknowledgments** This research is supported by the Sichuan Province Cyclic Economy Research Center (XHJJ-1222).

## References

- Bolaños R, Fontela E, Nenclares A, Pastor P (2005) Using interpretive structural modeling in strategic. *Manage Decis* 43(6):77–895
- Kanungo S, Bhatnagar VV (2002) Beyond generic models for information system quality: the use of interpretive structural modeling (ISM). *Syst Res Behav Sci* 19(6):531–549
- Li C (2011) System optimize of urban rail transit based on interpretation structure model. *J Beijing Jiaotong Univ* 35(5):58–63
- Lipschutz S, Lipson M (2002) *Discrete mathematics (in America)*. Science Press, Beijing
- Lu Q, Feng X (2006) Search path building algorithm based on breadth-first search. *Mod Comput* 12:87–89

- Singh MD (2007) Knowledge management barriers: an interpretive structural modeling approach. In: presented at the *2007 IEEE International Conference on Industrial Engineering and Engineering Management*, Singapore, 2007
- Thakkar J, Kanda A, Deshmukh SG (2004) Evaluation of buyer-supplier relationships using an integrated mathematical approach of interpretive structural modeling (ISM) and graph theoretic matrix: the case study of Indian automotive SMEs. *J Manuf Technol Manage* 19(1):92–124
- Thakkar J, Deshmukh SG, Gupta AD, Shankar R (2007) Development of a balanced scorecard: an integrated approach of interpretive structural modeling (ISM) and analytic network process (ANP). *Int J Prod Perform Manage* 56(1):25–29
- Tian Z, Wang H (2003) The application of interpretative structural modeling in the design of blueprint. *Ind Eng Manage* 4:46–50
- Wang Y (1998) *System engineering theory methodology and application*. Higher Education Press, Beijing, pp 35–38
- Wang Z (2000) *System engineering*. Shaanxi Science and Technology Press, Xi'an, pp 22–24
- Wang Q, Ge L (1996) The Warshall algorithm for computing reachability matrix. *J Xi'an Univ Technol* 12(1):80–82
- Wang S, Wei S, Wensheng J (1994) Comparison of the shortest path algorithms. *Syst Eng Electron* 1:43–49
- Yuan H, Wang D (2011) The reachable matrix approach for the shortest path. *Inform Control* 40(2):202–210
- Zhenkui W (1998) Notes about the methods of ISM, AHP and fuzzy appraisal. *J Tianjin Univ Comm* 5:45–46