Chapter 92
Research on Information Mining About Priority Weight of Linguistic Judgment Matrix

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Abstract The thesis makes mining information of decision maker as the breakthrough point, puts forward a ranking method involving parameter based on the linguistic judgment matrix, whose different value corresponds to different priority weight. It is necessary to add parameter in ranking method based on the linguistic judgment matrix, and there are three methods to select parameter, whose practicality and efficiency can be demonstrated by numerical examples.

Keywords Linguistic judgment matrix · Method to select parameter · Parameter · Preference

92.1 Introduction

In multi-attribution decision making, due to complexity and uncertainty of objective thing, and fuzziness of human thinking pattern, even for specialists, it is difficult to evaluate the attribute of any project, so it is convenient and reliable to make decision on some attributions with linguistic phrase. In accordance with existed research, there are two kinds of methods to make decision on linguistic information, one is ranking method based on the consistency of linguistic judgment matrix, and the other is applying operators to assemble decision-making information and ranking projects. Based on consistent linguistic judgment matrix or satisfactory linguistic judgment matrix, through shift formula, the first method which can rank transforms linguistic judgment matrix to real-value matrix. Literature (Chen and Hwang 1992) puts forward the shift scale method, and literature (Chen and Fan 2004) provides the method of transforming linguistic matrix to

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positive reciprocal matrix. The second method involves the induced ordered weighted averaging (IOWA) operator put forward by literature (Yager 2003), linguistic ordered weighted averaging (LOWA) operator in literature Herrera et al. (1996), linguistic weighted arithmetic averaging (LWAA) and extensive ordered weighted averaging (EOWA) operator in literature (Xu 1999),and other operators in literatures (Herrera et al. 1995, 1996, 2000; Herrera and Herrera-Viedma 2000; Umano et al. 1998; Wang and Fan 2002, 2003).

In ranking methods based on the consistency of linguistic judgment matrix, few literatures add parameters into ranking methods, although there is parameter in shift formula of literature (Chen and Fan 2004), it contains no active substance. The paper puts forward a ranking method of linguistic judgment matrix involving parameters, called parameter ranking method based on linguistic judgment matrix. The information of decision maker would be mined in methods, and then decision maker gets the better priority weights of linguistic judgment matrix.

92.2 Linguistic Judgment Matrix and its Consistence

Literature (Chen and Fan 2004; Fan and Jiang 2004) describes linguistic judgment matrix and its consistency. Assuming there is linguistic phrase set $S = \{S_\alpha | \alpha = -t, \ldots, -1, 0, 1, \ldots, t\}$, and decision-making problem is limited to finite set $A = \{a_1, a_2, \ldots, a_n\}$, where a_i denote the project i. Decision maker uses a matrix $P = (p_{ij})_{n\times n}$ to describe the information of the project set A, where p_{ij} evaluate project a_i and project a_j, when $p_{ij} = \{S_1, S_2, \ldots, S_t\}$, project a_i is better than project a_j, the more p_{ij} is, the greater project a_i is superior to project a_j, in contrast, $p_{ij} \in \{S_{-t}, \cdots, S_{-1}\}$, project a_j is better than project a_i, the smaller p_{ij} is, the greater project a_i is inferior to project a_j, while $p_{ij} = S_0$, project a_i is as good as project a_j, matrix P is called as linguistic judgment matrix.

Definition 1 (Herrera et al. 1995) Let $S = \{S_\alpha | \alpha = -t, \ldots, -1, 0, 1, \ldots, t\}$ denote natural language set, where S_i is the i-th natural language, the subscript i and the corresponding natural language can be obtained from following function I and I^{-1}:

$$I : S \to N, \quad I(S_i) = i, \quad S_i \in S$$

$$I^{-1} : N \to S, \quad I^{-1}(i) = S_i$$

Definition 2 (Chen and Fan 2004) With respect to $P = (P_{ij})_{n\times n}$, $\forall i, j, k \in J$, its elements satisfy with the following equation:

$$I(p_{ij}) + I(p_{jk}) + I(p_{ki}) = 0 \tag{92.1}$$

Then the linguistic judgment matrix is consistent.

$$I : S \to N, \quad I(S_i) = i, \quad S_i \in S$$

$$I^{-1} : N \to S, \quad I^{-1}(i) = S_i$$

92.3 Parameter Ranking Method Based on Linguistic Judgment Matrix

The logistic relation between linguistic judgment matrix and priority weight is put forward in this chapter, called as the parameter ranking method based on linguistic judgment matrix in the paper.

Theorem 1 A sufficient and necessary condition of the consistent linguistic judgment matrix $P = (p_{ij})_{n\times n}$ is that there exist a positive normalized vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ and θ, which satisfy the following formula:

$$I(p_{ij}) = \log_\theta \frac{\omega_i}{\omega_j},\ i,j \in J,\ where\ \theta > 1. \tag{92.2}$$

Proof: Necessary condition Assume $\theta > 1$, let

$$\omega_i = \theta^{\frac{1}{n}\sum_{k=1}^{n} I(pik)} / \sum_{i=1}^{n} \theta^{\frac{1}{n}\sum_{k=1}^{n} I(pik)},\ i \in J \tag{92.3}$$

then $\forall i \in J,\ \omega_i > 0$, it is obvious to exist $\sum_{i=1}^{n} \omega_i = 1$.

Because p_{ij} is related to ω_i and ω_j, it is reasonable to assume $I(p_{ij}) = \eta(\omega_i) - \eta(\omega_j)$, where $\eta(\omega_i) = (i \in J)$is monotonously increasing function. If the linguistic judgment matrix $P = (p_{ij})_{n\times n}$ is consistent, $\forall i,j \in J$, $I(p_{ij}) = \eta(\omega_i) - \eta(\omega_j)$, $I(p_{ij}) = -I(p_{ji})$ from Eq. (92.1), $I(p_{ij}) = I(p_{ik}) - I(p_{jk})$, Therefore, $\omega_i/\omega_j = \theta^{\frac{1}{n}\sum_{k=1}^{n} I(p_{ik})} / \theta^{\frac{1}{n}\sum_{j=1}^{n} I(p_{jk})} = \theta^{I(p_{ij})}$, also $I(p_{ij}) = \log_\theta \frac{\omega_i}{\omega_j}, i,j \in J$.

Sufficient condition

If p_{ij} of linguistic judgment matrix satisfies with $I(p_{ij}) = \log_\theta \frac{\omega_i}{\omega_j}, i,j \in J$, where $\theta > 1,\ \omega_i > 0,\ \omega_i > 0$, and $\sum_{i=1}^{n} \omega_i = 1$, it is easy to draw the following conclusion: $I(p_{IJ}) + I(p_{jk}) + I(p_{ki}) = 0$, $\log_\theta \frac{\omega_i}{\omega_j} + \log_\theta \frac{\omega_i}{\omega_k} + \log_\theta \frac{\omega_k}{\omega_i} = \log_\theta \left(\frac{\omega_i}{\omega_j} \cdot \frac{\omega_i}{\omega_k} \cdot \frac{\omega_k}{\omega_i} \right) = \log_\theta 1 = 0$. So, the linguistic judgment matrix $P = (p_{ij})_{n\times n}$ is consistent. From formula (92.2) and $\theta > 1$, it is not difficult to draw the following conclusion: $p_{ij} \in \{s_1, s_2, \ldots, s_t\} \Leftrightarrow I(p_{ij}) > 0 \Leftrightarrow \frac{\omega_i}{\omega_j} > 1 \Leftrightarrow \omega_i > \omega_j$, the more p_{ij} is, the more $\frac{\omega_i}{\omega_j}$ is, in other word, the project a_i is prior to the project a_j to greater extent; $p_{ij} \in \{s_{-t}, \cdots, s_{-1}\} \Leftrightarrow I(p_{ij}) < 0 \Leftrightarrow \frac{\omega_i}{\omega_j} < 1 \Leftrightarrow \omega_i < \omega_j$, the smaller p_{ij} is, the smaller $\frac{\omega_i}{\omega_j}$ is, in other word, the project a_i is inferior to the project a_j to greater extent; $p_{ij} = s_0 \Leftrightarrow I(p_{ij}) = 0 \Leftrightarrow \frac{\omega_i}{\omega_j} = 1 \Leftrightarrow \omega_i = \omega_j$ which demonstrate that it is reasonable to regard the positive normalized vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ as the priority weighty of evaluation the project. From theorem 1 and formula (92.3), it is easy to draw the following conclusion: under the condition of the same linguistic scale and consistency, the priority vectors are a family of ranking vectors involving parameter, the priority should changed with parameter, which provides some suggestions how to establish the priority vector of project from linguistic judgment matrix, meanwhile, which puts forward a new method of defining the priority vector of project.

92.4 The Necessity of Selecting Parameter

The following example 1 demonstrates that different parameter may induce different ranking project.

Example 1 There are two selectable projects with two attributes u_1, u_2. After a decision maker grades every attribute from 0 to 100, the decision-making matrix B can be obtained, whose normalized matrix is R. The decision maker constructs the linguistic judgment matrix H through pairwise comparison in accordance with linguistic scale $S = \{S_\alpha | \alpha = -5, \ldots, -1, 0, 1, \ldots, 5\}$, $H = \begin{bmatrix} S_0 & S_4 \\ S_{-4} & S_0 \end{bmatrix}$, it is obvious that the judgment H is consistent.

Next, we consider the following two situations. First, assuming $\theta = 1.4953$, from formula (92.3), $\omega = (0.8333,\ 0.1667)$ can be obtained, through utilizing simply weighing method, the evaluation of above two projects is $Z = (0.5056,\ 0.4944)$, so $a_1 \succ a_2$ (Tables 92.1, 92.2).

Secondly, assuming $\theta = 1.6818$, from formula (92.3), $\omega = (0.8889,\ 0.1111)$ can be obtained, through utilizing simply weighing method, the evaluation of above two projects is $Z = (0.4926,\ 0.5074)$, so $a_2 \succ a_1$. Example 1 demonstrates that different parameter may induce different ranking result under multiple standards, so it is necessary to select reasonable parameter and to put forward ranking method by introducing parameter in the decision based on judgment matrix.

Table 92.1 Decision-making matrix B

	u_1	u_2
a_1	70	70
a_2	80	30

Table 92.2 Normalized matrix of R from B

	u_1	u_2
a_1	0.4667	0.7
a_2	0.5333	0.3

92.5 The Method to Select Parameter

In accordance with above analysis, in order to have reasonable weight, it is essential to obtain suitable parameter, the paper puts forward following three methods.

92.5.1 *The First Comprehensive Weight Method Based on Linguistic Judgment Matrix*

When there are less than 5 selectable projects, it is considerable to apply the first comprehensive weight method, whose stages are as follows. First, the decision maker selects two projects from projects $A_1, A_2, \ldots, A_n$, such as A_1, A_2. Secondly, the decision maker gives the two projects real-valued weight $\omega_1', \omega_2' \left(\sum_{i=1}^{2} \omega_i' = 1\right)$. Thirdly, insert ω_1', ω_2' into the formula (92.2), and obtain the following formula:

$$I(p_{12}) = \log_\theta \frac{\omega_1'}{\omega_2'} \tag{92.4}$$

Fourthly, from formula (92.4), it is not difficult to solve the parameter θ which embodies the preference of the decision maker. Finally, it is important to insert the value of θ into the formula (92.3) to solve the priority weight which embodies the preference of decision maker to greater extent.

Example 2 If decision maker gives following linguistic judgment matrix A and real-value matrix A' induced from A.

$$A = \begin{vmatrix} S_0 & S_{-1} & S_0 & S_0 \\ S_1 & S_0 & S_1 & S_1 \\ S_0 & S_{-1} & S_0 & S_0 \\ S_0 & S_{-1} & S_0 & S_0 \end{vmatrix}, \quad A' = \begin{vmatrix} 1 & \theta^{-1} & 1 & 1 \\ \theta & 1 & \theta & \theta \\ 1 & \theta^{-1} & 1 & 1 \\ 1 & \theta & 1 & 1 \end{vmatrix}$$

The decision maker gives the projects A_1, A_2 the weight $(\omega_1', \omega_2') = (0.4, 0.6)$, it is obvious that $A_{2\times 2}$ is consistent, principle submatrices of A_1, A_2also are consistent.

From formula (92.4), $I(p_{12}) = \log_\theta \frac{\omega_1'}{\omega_2'}$, $-1 = \log_\theta \frac{0.4}{0.6}$, obtain parameter $\theta = 1.5$, then insert $\theta = 1.5$ into formula(92.3): $\omega_i = \theta^{\frac{1}{n}\sum_{k=1}^{n} I(p_{ik})} / \sum_{i=1}^{n} \theta^{\frac{1}{n}\sum_{k=1}^{n} I(p_{ik})}$, so

$$\omega_1 = \frac{\theta^{-1/4}}{\theta^{-1/4} + \theta^{3/4} + \theta^{-1/4} + \theta^{-1/4}} = \frac{1.5^{-1/4}}{1.5^{-1/4} + 1.5^{3/4} + 1.5^{-1/4} + 1.5^{-1/4}}$$
$$\omega_2 = \frac{\theta^{3/4}}{\theta^{-1/4} + \theta^{3/4} + \theta^{-1/4} + \theta^{-1/4}} = \frac{1.5^{3/4}}{1.5^{-1/4} + 1.5^{3/4} + 1.5^{-1/4} + 1.5^{-1/4}}$$
$$\omega_3 = \frac{\theta^{-1/4}}{\theta^{-1/4} + \theta^{3/4} + \theta^{-1/4} + \theta^{-1/4}} = \frac{1.5^{-1/4}}{1.5^{-1/4} + 1.5^{3/4} + 1.5^{-1/4} + 1.5^{-1/4}}$$
$$\omega_4 = \frac{\theta^{-1/4}}{\theta^{-1/4} + \theta^{3/4} + \theta^{-1/4} + \theta^{-1/4}} = \frac{1.5^{-1/4}}{1.5^{-1/4} + 1.5^{3/4} + 1.5^{-1/4} + 1.5^{-1/4}}$$

The priority vector of the project is

$$\omega = (0.2222,\ 0.3333,\ 0.2222,\ 0.2222).$$

92.5.2 The Second Comprehensive Weight Method Based on Linguistic Judgment Matrix

If there are more selectable projects whose number is between 5 and 9, considering the complexity and diversity of decision making and human thinking, it is possible to have deviation, so the paper puts forward the second comprehensive weight method based on the consideration that the weight obtained from formula (92.3) and the subjective weight of decision maker should be smaller. The stages of the above method are as follows: First, every decision maker gives the subjective weight to arbitrary three projects in order to obtain more preference information. Secondly, the optimization model should satisfy with the following equation:

$$\min f(\theta) = \sum_{i=1}^{3} \left(\theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{ik})} - \omega_i' \sum_{i=1}^{3} \theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{ik})} \right)^2,$$
$$s.t\, \theta \geq 1$$

where $\omega_1', \omega_2', \omega_3'$ is the subjective weight, and $\sum_{i=1}^{3} \omega_i' = 1$

Thirdly, the parameter can be obtained from above model, and is inserted in the formula (92.3), the priority weight of project can be found.

Let $d_i = \frac{1}{3}\sum_{k=1}^{3} I(p_{ik}),\ i = 1, 2, 3,$ then the above model can be simplified.

$$\min f(\theta) = \sum_{i=1}^{3} \left(\theta^{d_i} - \omega_i' \sum_{i=1}^{3} \theta^{d_i} \right)^2$$
$$s.t\, \theta \geq 1$$

Example 3 If decision maker gives following linguistic judgment matrix A and real-value matrix A' induced from A.

$$A = \begin{vmatrix} S_0 & S_{-1} & S_0 & S_0 & S_{-3} \\ S_1 & S_0 & S_1 & S_1 & S_{-2} \\ S_0 & S_{-1} & S_0 & S_0 & S_{-3} \\ S_0 & S_{-1} & S_0 & S_0 & S_{-3} \\ S_3 & S_2 & S_3 & S_3 & S_0 \end{vmatrix}, \quad A' = \begin{vmatrix} 1 & \theta^{-1} & 1 & 1 & \theta^{-3} \\ \theta & 1 & \theta & \theta & \theta^2 \\ 1 & \theta^{-1} & 1 & 1 & \theta^{-3} \\ 1 & \theta & 1 & 1 & \theta^{-3} \\ \theta^3 & \theta^{-2} & \theta^3 & \theta^3 & 1 \end{vmatrix}$$

If one specialist gives the projects A_1, A_2, A_3 the subjective weight $(w'_1, w'_2, \omega'_3)^T = (0.25, 0.5, 0.25)$, it is reasonable to minimize the difference between the weight obtained from formula (92.3) and subjective weight $\omega'_1, \omega_2,' \omega'_3$, and to construct mathematical model:

$$\min f(\theta) = \sum_{i=1}^{3} \left(\theta^{d'_i} - \omega'_i \sum_{i=1}^{3} \theta^{d'_i} \right)^2$$
$$s.t\, \theta \geq 1$$

where $d'_i = \frac{1}{3} \sum_{k=1}^{3} I(p_{ik}),\ i = 1, 2, 3,$

$$\min f(\theta) = 2\left[\theta^{-1/3} - 0.25(\theta^{-1/3} + \theta^{2/3} + \theta^{-1/32})\right]^2 + \left[\theta^{2/3} - 0.5(\theta^{-1/3} + \theta^{2/3} + \theta^{-1/32})\right]^2, \quad s.t\, \theta \geq 1$$

$\theta = 1.833$, insert $\theta = 1.833$ into formula (92.3), and obtain the priorityvector $\omega = (0.124, 0.3047, 0.124, 0.124, 0.3882)$.

92.5.3 *The Third Comprehensive Weight Method Based on Linguistic Judgment Matrix*

If there are more selectable projects whose number is between 5 and 9, considering the deviation of decision maker understanding the scale, the paper puts forward the third comprehensive weight method based on the linguistic judgment matrix, whose stages are as follows. First, every decision maker gives the subjective weight to arbitrary three projects. Secondly, insert the weight into formula (92.3) to obtain three equations, and solve the unknown parameters $\theta_1, \theta_2, \theta_3$. Thirdly, compute the average θ of $\theta_1, \theta_2, \theta_3$, $\theta = \frac{1}{3}(\theta_1 + \theta_2 + \theta_3)$. Finally, after inserting θ into the formula (92.3), the priority weight of project can be found. If the decision maker gives the subjective weight w'_1, w'_2, ω'_3 to the projects A_1, A_2, A_3, the following three equations can be obtained from formula (92.3).

$$\omega_1 = \theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{1k})} \Big/ \sum_{i=1}^{3} \theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{ik})} \quad (92.5)$$

$$\omega_2 = \theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{2k})} \Big/ \sum_{i=1}^{3} \theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{ik})} \quad (92.6)$$

$$\omega_3 = \theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{3k})} \Big/ \sum_{i=1}^{3} \theta^{\frac{1}{3}\sum_{k=1}^{3} I(p_{ik})} \quad (92.7)$$

Through applying the third method, the example 3 also can be solved.

92.6 Conclusion

The paper discusses the problem about parameter of priority of linguistic judgment matrix, demonstrates the necessity of adding parameter in the sorting method based on linguistic judgment matrices, puts forward the conclusion of obtain the parameter value through mining information of decision maker, and some methods of selecting parameter through making full use of the preference information which can be reflected by the subjective weight.Project supported by the Scientific Research Foundation of the Higher Education Institutions of Guangxi Zhuang Autonomous Region (Grant No. 201204LX394).

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