Chapter 87
Optimal Enterprise Cash Management Under Uncertainty

Xiao-yun Wei and Li-yan Han

Abstract We present a dynamic model for enterprise cash management under uncertainty. The numerical method was used to obtain optimal level of cash holding. The results show that higher yield volatility of financial assets, liquidation cost of financial assets and coefficient of risk aversion will raise the demand for cash. It also shows that the optimal choice of inter-temporal model is different from that of single-period model. The former makes the manager choose to hold more cash. The reason is that long-horizon managers have an intrinsically larger need for cash to quell possible transaction and precautionary demand.

Keywords Cash management · Financial assets · Uncertainty

87.1 Introduction

Optimization models for cash management can be divided into two main groups based on objection function. The first deals with demand by cost-benefit or loss-benefit analysis, pioneered by Baumol–Tobin model (Baumol 1952; Tobin 1956), and extended, among others, by Frenkel and Jovanovic (1980, 1981), Bar-Ilan (1990), Dixit (1991), Ben-Bassat and Gottlieb (1992), Chang (1999) and Perry and Stadje (2000). In this approach the optimal demand for cash is decided by the trade-off between opportunity cost and benefits of cash holding. The second category of models concerns the demand by drift control theory, pioneered by Miller and Orr (1966), and extend by Bar-Ilan et al. (2004), Bar-Ilan and Lederman (2007).

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However, the authors above mainly consider cash only and they consider either single period or infinite horizons only. In this paper, we present a model to obtain the optimal allocation ration between cash and financial assets based on utility maximization for different horizons. The model departs from portfolio choice theory (Barberis 2000), Aizenman and Lee (2007) and multi-period newsboy model (Matsuyam 2006), and instead emphasizes the importance of cash in providing insurance for bankruptcy.

The rest of paper is organized as follows. In Sect. 87.2 we introduce the framework of cash management. A numerical example is presented particularly to calibrate our model in Sect. 87.3. Section 87.4 offers some concluding remarks.

87.2 The Model

We assume the manager, as a centralized decision maker in the enterprise, determines the split of the given level of assets W_t between cash and financial assets, where R_t and S_t denote two assets in the end of t period respectively. The enterprise has to meet the demand for payments such as transaction and debt repayment thus to reduce the probability of a possible bankruptcy. Then enterprise generally put its cash in commercial bank or purchase government bonds in order to seek greater security and liquidity but lower risk-free return. In contrast, the manager will typically pursue higher return by investing the remaining assets in risky long-term assets such as longer-term government bonds, stocks, corporate bonds, oil, minerals and real estate. The objective of the manager is to earn more profit on the basis of enterprise stability. The question, which is concerned, is to maximize the enterprise's utility function. In order to formulate this problem, the dynamics of the total assets W_t of enterprise will be introduced.

Consider we are at initial time 0 and want to write down the allocation problem for a manager with a horizon of t periods. We suppose the real annually interest rate on cash is $r_{f,t}$. The return on financial assets $r_{s,t}$ is assumed to follow an independently identical distribution, and $\mathrm{cov}\left(r_{s,s}, r_{s,t}\right) = 0,\ \forall s \neq t, s, t = 1, 2, 3\ldots$. Except the fundamental function of cash for transaction and debt payments, another function is to enhance confidence of investors, which is not considered by Bar-Ilan et al. (2004). This is the paradox of cash management—the more cash holdings, the lower may be the demand for them. Then the real demand for cash in period t, Y_t is given by

$$Y_t = g(X_t,\ R_{t-1}, S_{t-1}) \tag{87.1}$$

where X_t, with a corresponding density function $f(X_t)$, is the value of demands when the enterprise has no cash and financial assets. Then we have $g(X_t,\ 0,\ 0) = X_t$. And Y_t is a strictly increasing function of X_t and decreasing function of R_{t-1} and S_{t-1}. The joint density function for $(X_1, X_2, X_3, \ldots, X_t)$ is given by

$$f(X_1, X_2, X_3, \ldots, X_t) = \prod_{n=1}^{t} f(X_n) \tag{87.2}$$

In other words, the distributions of X_n and $X_m (n \neq m)$ are independent each other. We assume that L_t and C_t are lower and upper bound of demand X_t, and $l_t = L_t / W_{t-1}$, $c_t = C_t / W_{t-1}$.

If cash holding R_{t-1} at the end of period $t-1$ or at beginning of period t is higher than the demand Y_t in period t, the remaining cash $R_{t-1} - Y_t$ will earn $r_{f,t}$ rate of return and financial assets S_{t-1} earn $r_{s,t}$ rate of return. When R_{t-1} is lower than Y_t, the liquidation takes place and the level of cash reduces to zero. θ_t is the liquidation cost which must be paid for a unit of cash when the demand can't be complied with. Then $(Y_t - R_{t-1})(1 + \theta_t)$ units of financial assets must be liquidated to obtain $Y_t - R_{t-1}$ units of cash. We denote $\omega_t = R_{t-1}/W_{t-1}$ is the allocation ratio to the cash at the beginning of period t. $y_t = Y_t / W_{t-1}$ is the proportion of demand for cash to total assets W_{t-1} at the end of period $t-1$. Thus if t is larger than 1, ω_t is decided passively. Then W_t is given by the following expressions (87.1)–(87.3), where the first subscript of W_t and ω_t denotes the period and the second subscript denotes the scenario.

(1) t = 1

$$\begin{aligned} y_1 \leq \omega_1 &\Rightarrow W_{1,1} = W_0\left[(\omega_1 - y_1)\left(1 + r_{f,1}\right) + (1 - \omega_1)\left(1 + r_{s,1}\right) + y_1\right]; \\ y_1 > \omega_1 &\Rightarrow W_{1,2} = W_0\left\{[(1 - \omega_1) - (y_1 - \omega_1)(1 + \theta_1)]\left(1 + r_{s,1}\right) + y_1\right\}; \end{aligned}$$

(2) t = 2

$$\begin{aligned} & y_1 < \omega_1,\ y_2 < \omega_{2,1} \Rightarrow \\ & \omega_{2,1} = \frac{W_0(\omega_1 - y_1)\left(1 + r_{f,1}\right)}{W_{1,1}}; \\ & 1 - \omega_{2,1} = \frac{W_0(1 - \omega_1)\left(1 + r_{s,1}\right)}{W_{1,1}}; \\ & W_{2,2} = W_{1,1}\left\{\left[\left(1 - \omega_{2,1}\right) - \left(y_2 - \omega_{2,1}\right)(1 + \theta_2)\right]\left(1 + r_{s,2}\right) + y_2\right\}; \end{aligned}$$

$$\begin{aligned}
&y_1 > \omega_1,\ y_2 < \omega_{2,2} \Rightarrow \\
&\omega_{2,2} = 0; \\
&1 - \omega_{2,2} = \frac{W_0 \begin{bmatrix} (1-\omega_1)- \\ (y_1 - \ \omega_1)(1+\theta_1) \end{bmatrix} \left(1 + r_{s,1}\right)}{W_{1,2}}; \\
&W_{2,3} = W_{1,2} \begin{bmatrix} \left(\omega_{2,2} - y_2\right)\left(1 + r_{f,2}\right) \\ + \left(1 - \omega_{2,2}\right)\left(1 + r_{s,2}\right) + y_2 \end{bmatrix};
\end{aligned}$$

$$\begin{aligned}
&y_1 > \omega_1,\ y_2 > \omega_{2,1} \Rightarrow \\
&\omega_{2,2} = 0; \\
&1 - \omega_{2,2} = \frac{W_0 \begin{bmatrix} (1-\omega_1)- \\ (y_1 - \ \omega_1)(1+\theta_1) \end{bmatrix} \left(1 + r_{s,1}\right)}{W_{1,2}}; \\
&W_{2,4} = W_{1,2} \left\{ \begin{bmatrix} \left(1 - \omega_{2,2}\right)- \\ \left(y_2 - \omega_{2,2}\right)(1+\theta_2) \end{bmatrix} \left(1 + r_{s,2}\right) + y_2 \right\};
\end{aligned}$$

$$\begin{aligned}
&y_1 > \omega_1,\ y_2 > \omega_{2,1} \Rightarrow \\
&\omega_{2,1} = \frac{W_0(\omega_1 - y_1)\left(1 + r_{f,1}\right)}{W_{1,1}}; \\
&1 - \omega_{2,1} = \frac{W_0(1 - \omega_1)\left(1 + r_{s,1}\right)}{W_{1,1}}; \\
&W_{2,1} = W_{1,1}\left[(\omega_{21} - y_2)\left(1 + r_{f,2}\right) + \left(1 - \omega_{2,1}\right)\left(1 + r_{s,2}\right) + y_2\right];
\end{aligned}$$

(3) t

Let a set R^+ be defined by

$$R^+ = \{a | a \geq 0, a \in R\}$$

Where R denotes a set of all real numbers. Moreover, A_t and B_t, $t = 1, 2, 3\ldots$, are defined by

$$A_t = \{y_t | y_t \leq \omega_t, y_t \in R\},\ B_t = R^+ - A_t$$

Then we denotes

$$\begin{aligned}
&\Omega_1 = A_1 \times A_2 \times \cdots \times A_t\ ,\\
&\Omega_2 = A_1 \times A_2 \times \cdots \times A_{t-1} \times B_t\ ,\\
&\cdots\\
&\Omega_{2^t-1} = B_1 \times B_2 \times \cdots \times B_{t-1} \times A_t\ ,\\
&\Omega_{2^t} = B_1 \times B_2 \times \cdots \times B_t.
\end{aligned}$$

$$y = (\mathbf{y}_1, y_2, \ldots, y_t)^T$$

$$\begin{aligned}
&\mathbf{y} \in \Omega_1 \Rightarrow\\
&\omega_{t,1} = \frac{W_{t-2,1}\left(\omega_{t-1,1} - y_{t-1}\right)\left(1 + r_{f,t-1}\right)}{W_{t-1,1}};\\
&1 - \omega_{t,1} = \frac{W_{t-2,1}\left(1 - \omega_{t-1,1}\right)\left(1 + r_{s,t-1}\right)}{W_{t-1,1}};\\
&W_{t,1} = W_{t-1,1}\left[\begin{array}{l}\left(\omega_{t,1} - y_t\right)\left(1 + r_{f,t}\right)\\ +\left(1 - \omega_{t,1}\right)\left(1 + r_{s,t}\right) + y_t\end{array}\right];
\end{aligned}$$

$$\begin{aligned}
&y \in \Omega_2 \Rightarrow\\
&\omega_{t,1} = \frac{W_{t-2,1}\left(\omega_{t-1,1} - y_{t-1}\right)\left(1 + r_{f,t-1}\right)}{W_{t-1,1}};\\
&1 - \omega_{t,1} = \frac{W_{t-2,1}\left(1 - \omega_{t-1,1}\right)\left(1 + r_{s,t-1}\right)}{W_{t-1,1}};\\
&W_{t,2} = W_{t-1,1}\left\{\left[\begin{array}{l}\left(1 - \omega_{t,1}\right) -\\ \left(y_t - \omega_{t,1}\right)\left(1 + \theta_t\right)\end{array}\right]\left(1 + r_{s,t}\right) + y_t\right\};
\end{aligned}$$

……

$$\begin{aligned}
&y \in \Omega_{t-1} \Rightarrow\\
&\omega_{t,2^{t-1}} = 0;\\
&1 - \omega_{t,2^{t-1}} = \frac{W_{t-2,2^{t-2}}\left[\begin{array}{l}\left(1 - \omega_{t-1,2^{t-1}}\right) -\\ \left(y_{t-1} - \omega_{t-1,2^{t-1}}\right)\left(1 + \theta_{t-1}\right)\end{array}\right]\left(1 + r_{s,t-1}\right)}{W_{t-1,2^{t-1}}};\\
&W_{t,2^t-1} = W_{t-1,2^{t-1}}\left[\begin{array}{c}\left(\omega_{t,2^{t-1}} - y_t\right)\left(1 + r_{f,t}\right)\\ +\left(1 - \omega_{t,2^{t-1}}\right)\left(1 + r_{s,t}\right) + y_t\end{array}\right];
\end{aligned}$$

$$
\begin{aligned}
& y \in \Omega_t \Rightarrow \\
& \omega_{t,2^{t-1}} = 0; \\
& 1 - \omega_{t,2^{t-1}} = \frac{W_{t-2,2^{t-2}} \left[\begin{array}{c} \left(1 - \omega_{t-1,2^{t-1}}\right) - \\ \left(y_{t-1} - \omega_{t-1,2^{t-1}}\right)\left(1 + \theta_{t-1}\right) \end{array} \right] \left(1 + r_{s,t-1}\right)}{W_{t-1,2^{t-1}}} = 1; \\
& W_{t,2^t} = W_{t-1,2^{t-1}} \left\{ \left[\begin{array}{c} \left(1 - \omega_{t,2^{t-1}}\right) - \\ \left(y_t - \omega_{t,2^{t-1}}\right)\left(1 + \theta_t\right) \end{array} \right] \left(1 + r_{s,t}\right) + y_t \right\};
\end{aligned}
$$

The manager's preferences over terminal wealth are described by constant relative risk-aversion utility functions of the form

$$
u(W_t) = \frac{W_t^{1-A}}{1-A} \tag{87.3}
$$

The manager's problem is to solve equation

$$
\begin{aligned}
V(W_t) &= \max_{\omega_1} E\left(E_0\left(u(W_t)\middle| r_{s,1}, r_{s,2}, \ldots, r_{s,t}\right)\right) \\
&= \max_{\omega_1} E \left\{ \left[\begin{array}{c} \int_{\Omega_1} u\left(W_{t,1}\right) h(y) dy \\ + \int_{\Omega 2} u\left(W_{t,2}\right) h(y) dy \ldots \\ + \int_{\Omega_{2^t}} u\left(W_{t,2^t}\right) h(y) dy \end{array} \right] | r_{s,1}, r_{s,2}, \ldots, r_{s,t} \right\}
\end{aligned}
\tag{87.4}
$$

Where $\max\limits_{\omega_1}$ denotes the problem is solving the optimal ω_1^*, and denotes the fact that the manager calculates the expected return from the beginning of period 1 on. E is the expectation operator of $r_{s,t}$. $h(y)$ is the joint density function for $(y_1, y_2, y_3, \ldots, y_t)$.

When budget constraint binds, that is, $(1 - \omega_t) - (y_t - \omega_t)(1 + \theta_t) < 0$, the final wealth in period t is 0. Now even though the manager liquidates all financial assets, they could not satisfy the payments, the bankruptcy is likely to occur.

87.3 Numerical Example

The numerical solution is solved as follows. We assume

$$
\begin{aligned}
& P(X_t = i) = 0.25,\ i = 1, 2, 3, 4\ , \\
& P\left(r_{s,t} = 0.03\right) = P\left(r_{s,t} = 0.08\right) = 0.5,\ L_t = 1,\ H_t = 4\ , \\
& Y_t = X_t - 0.005 R_{t-1} - 0.002 S_{t-1} \text{ and } W_0 = 12\ , \\
& r_{f,t} = 0.04,\ \theta_t = 0.5,\ A = 5,\ \ t = 1, 2, 3, 4, 5, 6.
\end{aligned}
$$

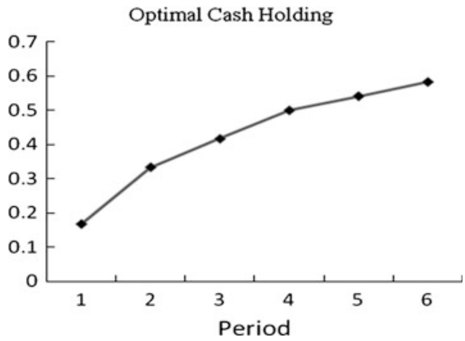

Fig. 87.1 Optimal allocation to cash for different horizons

The simulation results are reported in Fig. 87.1. The optimal ω_1 maximizes the function (87.4).

The results show that the optimal choice of inter-temporal model is different from that of single-period model. The former makes the manager choose to hold more cash. The reason is long-horizon managers has an intrinsically larger need for cash to quell possible transaction and precautionary demand. And we also conclude that higher yield volatility of financial assets, liquidation cost of financial and coefficient of risk aversion will raise the demand for cash. For saving space, we omitted the figure in the paper.

87.4 Concluding Remarks

This paper presents a dynamic model for enterprise cash management under uncertainty. The numerical method was used to obtain optimal level of cash holdings. The results show that higher yield volatility of financial assets, liquidation cost of financial assets and coefficient of risk aversion will raise the demand for cash. It also shows that the optimal choice of inter-temporal model is different from that of single-period model. The former makes the manager choose to hold more cash. The reason is long-horizon managers has an intrinsically larger need for cash to quell possible transaction and precautionary demand.

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