Chapter 7
A Rapid Data Processing and Assessing Model for "Scenario-Response" Types Natural Disaster Emergency Alternatives

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Abstract In the processes of assessing the emergency alternatives of "scenario-response" types natural disaster by Analytic Hierarchy (Network) Process (AHP/ANP), the elements or data of the scenario itself, the real-time data and the trend factors of the evolution of "scenario-response" types natural disaster emergencies etc. are usually inconsistent and intangible, which increase the difficulty of emergency alternatives assessment and delay the speed of emergency response. Therefore, in this paper, a logarithm mean induced bias matrix (LMIBM) model is proposed to quickly process the inconsistent data of "scenario-response" type's natural disaster when AHP/ANP is used to assess the natural disaster emergency alternatives and evolution trend factors of natural disaster emergency accidents. Two numerical examples are used to illustrate the proposed model, and the results show that LMIBM can quickly identify the inconsistent natural disaster data and improve the speed of emergency alternatives assessment and natural disaster response by AHP/ANP.

Keywords AHP/ANP · Data processing · Emergency alternatives assessment · Logarithm mean induced bias matrix · Natural disaster

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7.1 Introduction

In recent years, a number of studies on unexpected natural disaster emergency alternatives assessment from different perspectives have been conducted by many scholars since natural disasters have caused significant economic, social, financial, property and infrastructure damages as well as tragic loss of human lives worldwide (Xu and Lu 2009; Zhong et al. 2010; Wei et al. 2004; Bina and Edwards 2009). Quick assessment and scenario-response are very important for disaster managers to save lives and reduce the property losses in unexpected natural disaster emergency management (Tsai and Chen 2010, 2011). Therefore, "scenario-response" type's unexpected natural disaster emergency management is becoming a new hot research topic.

However, the involved attributes of "scenario-response" types natural disasters are different and usually intangible, which need to be quantified into quantitative values. The elements or data of the scenario itself, the real-data and the trend factors of the evolution of "scenario-response" type's natural disaster emergencies etc. are also usually inconsistent and intangible, which increase the difficulty of emergency alternatives assessment and delay the speed of emergency response. In addition, emergency management is regarded as a complex multi-objective optimization problem, e.g. selecting the best emergency response alternatives or emergency recovery alternatives and reasonably allocates relief resources etc., (Tufekci and Wallace 1998).

Furthermore, the unconventional characteristic of natural disaster evolution law increases the difficulty of quick assessment and response. The pairwise comparison technique, displayed as a positive reciprocal matrix in the Analytic Hierarchy Process (AHP) (Saaty 1994) and Analytic Network Process (ANP) (Levy and Taji 2007), is a well-established and widely used technique to deal with the intangible disaster attributes and assess the natural disaster emergency alternatives. However, the consistency issue in a positive reciprocal matrix is a big challenge that the emergency managers are facing, and it has been extensively studied over the past few decades, e.g. Koczkodaj and Szarek developed distance-based inconsistency reduction algorithms for pairwise comparisons (Koczkodaj and Szarek 2010; Cao et al. 2008) proposed a heuristic approach to modify the inconsistent comparisons in analytic hierarchy process, Li and Ma employed a Gower plot to detect ordinal and cardinal inconsistencies (Li and Ma 2007). Ergu et al. (2011b) proposed an induced bias matrix (IBM) model to improve the consistency ratio and further extended it to process data consistency in emergency management (Ergu et al. 2012) and in risk assessment and decision analysis (Ergu et al. 2011a). Based on the proposed induced bias matrix (IBM) model, in this paper, a logarithm mean induced bias matrix (LMIBM) model is proposed to process the inconsistent disaster data and simplify the observed numbers in order to improve the speed of disaster assessment and response when AHP/ANP is used.

The rest of this paper is organized as follows. The next section presents the theorems, corollary as well as the identifying processes of LMIBM. Two numerical examples are used to test the proposed model in Sect. 7.3. Section 7.4 concludes the paper.

7.2 Logarithm Mean Induced Bias Matrix (LMIBM)

7.2.1 The Theorem of LMIBM

Definition 7.1 Matrix $A = [a_{ij}]_{n \times n}$ is said to be perfectly cardinal consistency if $a_{ij} = a_{ik}a_{kj}$ holds for all *i, j* and *k*, where $a_{ij} > 0$ *and* $a_{ij} = 1/a_{ji}$ for all *i*, *j*, and *k*.

Theorem 7.1 *The logarithm mean induced bias matrix* (*LMIBM*) *C should be a zero matrix if matrix A is perfectly consistent, that is*,

$$\begin{cases} C = \frac{1}{n}\log \prod A - \log A = 0 \\ (c_{ij}) = \left(\frac{1}{n}\log \prod\limits_{k=1}^{n} a_{ik}a_{kj} - \log a_{ij}\right) = 0 \end{cases} \tag{7.1}$$

Proof Since matrix A is perfectly consistent, namely, $a_{ik}a_{kj} = a_{ij}$ holds for all *i*, *j* and *k*, we have

$$\begin{aligned} c_{ij} &= \frac{1}{n}\log \prod_{k=1}^{n} a_{ik}a_{kj} - \log a_{ij} \\ &= \frac{1}{n}\log a_{ij}^{n} - \log a_{ij} = 0 \end{aligned}$$

Therefore, all values in matrix C are zeroes, and matrix C is a zero matrix if matrix A is perfectly consistent. □

Obviously, the above model can be transformed to the following models in terms of the properties of logarithm function:

$$\begin{cases} C = \log \sqrt[n]{\prod A} + \log A^{T} = 0 \\ (c_{ij}) = \left(\log \sqrt[n]{\prod\limits_{k=1}^{n} a_{ik}a_{kj}} + \log a_{ji}\right) = 0 \end{cases} \tag{7.2}$$

$$\begin{cases} C = \log \sqrt[n]{\prod A} \circ A^{T} = 0 \\ (c_{ij}) = \left(\log \sqrt[n]{\prod\limits_{k=1}^{n} a_{ik}a_{kj}} \cdot a_{ji}\right) = 0 \end{cases} \tag{7.3}$$

Theorem 7.2 *The logarithm mean induced bias matrix* (*LMIBM*) *C is an anti symmetric matrix if matrix A is inconsistent, that is,*

$$\begin{cases} C = -C^T \\ c_{ij} = -c_{ji} \end{cases} \tag{7.4}$$

Proof By formula (7.1) and the reciprocal property, we have

$$\begin{aligned} c_{ji} &= \frac{1}{n}\log\prod_{k=1}^{n} a_{jk}a_{ki} - \log a_{ji} = \frac{1}{n}\log\prod_{k=1}^{n}\frac{1}{a_{kj}a_{ik}} - \log\frac{1}{a_{ij}} \\ &= \frac{1}{n}\left(-\log\prod_{k=1}^{n} a_{kj}a_{ik} + \log a_{ij}\right) = -c_{ij} \end{aligned}$$

□

Corollary 7.1 *There must be some inconsistent entries in the logarithm mean induced bias matrix (LMIBM) C deviating from zero if the matrix A is inconsistent. Especially, any row or column of matrix C contains at least one non-zero entry.*

Proof by contradiction: Assume all entries in matrix C are zeroes even if the judgment matrix A is inconsistent, that is, $a_{ik}a_{kj} \neq a_{ij}$ holds for some i, j and k, but $c_{ij} = 0,\ (i,j = 1,\ldots,n)$, namely

$$c_{ij} = \frac{1}{n}\log\prod_{k=1}^{n} a_{ik}a_{kj} - \log a_{ij} = 0$$

We can get

$$a_{ij} = \sqrt[n]{\prod_{k=1}^{n} a_{ik}a_{kj}} = \sqrt[n]{\prod_{l=1}^{n} a_{il}a_{lj}}$$

Since $a_{lk} = 1/a_{kl}$, we can obtain

$$\begin{aligned} a_{ij} &= \sqrt[n]{\prod_{l=1}^{n} a_{il}a_{lk}a_{kl}a_{lj}} = \sqrt[n]{\prod_{l=1}^{n} a_{il}a_{lk}} \cdot \sqrt[n]{\prod_{l=1}^{n} a_{kl}a_{lj}} \\ &\Rightarrow a_{ij} = a_{ik}a_{kj} \end{aligned}$$

The result contradicts the previous assumption that $a_{ij} \neq a_{ik}a_{kj}$ for some j and k, and $c_{ij} = 0$. Therefore, any row or column of matrix C contains at least one non-zero entry. □

Based on Corollary 7.1, the most inconsistent data in matrix A can be identified by observing the largest value in the logarithm mean induced bias matrix C. According to *Theorem 7.2*, there are two equal absolute largest values in matrix C since the values are anti symmetric, therefore, one can either observe the absolute largest value above or below the main diagonal in the matrix C, which simplify the observing number of times to $n(n - 1)/2$ and improve the speed of disaster assessment.

7.2.2 The Processes of Inconsistency Identification by LMIBM

In this section, the processes of inconsistency identification by LMIBM are proposed based on above *Theorems* and *Corollary*, including three steps:

Step 1: Compute a column matrix L and a row matrix R, as shown in the two edges of matrix A.

$$
A = \begin{array}{cc}
 & L = \\
\begin{pmatrix}
a_{11} & \cdots & a_{1i} & \cdots & a_{1j} & \cdots & a_{1n} \\
 & \vdots & & \vdots & & \vdots & \\
a_{i1} & \cdots & a_{ii} & \cdots & a_{ij} & \cdots & a_{in} \\
 & \vdots & & & & \vdots & \\
a_{j1} & \cdots & a_{ji} & & a_{jj} & \vdots & a_{jn} \\
 & & & & & & \\
a_{n1} & \cdots & a_{ni} & \cdots & a_{nj} & \cdots & a_{nn}
\end{pmatrix} &
\begin{array}{l}
\prod_{k=1}^{n} a_{1k} \\
\prod_{k=1}^{n} a_{ik} \\
\prod_{k=1}^{n} a_{jk} \\
\prod_{k=1}^{n} a_{nk}
\end{array}
\end{array}
$$

$$
R = \prod_{k=1}^{n} a_{k1}, \cdots \prod_{k=1}^{n} a_{ki}, \cdots \prod_{k=1}^{n} a_{kj}, \cdots \prod_{k=1}^{n} a_{kn}
$$

where

$$
\begin{cases}
L = \left(\prod_{k=1}^{n} a_{1k}, \quad \cdots \quad \prod_{k=1}^{n} a_{ik}, \quad \cdots, \quad \prod_{k=1}^{n} a_{jk}, \quad \cdots, \quad \prod_{k=1}^{n} a_{nk} \right)^{T} \\
R = \left(\prod_{k=1}^{n} a_{k1}, \quad \cdots \quad \prod_{k=1}^{n} a_{ki}, \quad \cdots \quad \prod_{k=1}^{n} a_{kj}, \quad \cdots, \quad \prod_{k=1}^{n} a_{kn} \right)
\end{cases}
\tag{7.5}
$$

Step 2: Compute logarithm mean induced bias matrix (LMIBM) C by formula,

$$
\begin{cases}
C = \frac{1}{n} \log \prod A - \log A \\
\left(c_{ij} \right) = \left(\frac{1}{n} \log \prod_{k=1}^{n} a_{ik} a_{kj} - \log a_{ij} \right)
\end{cases}
\tag{7.6}
$$

where

$$
\prod A = L \times R = \left(\prod_{k=1}^{n} a_{ik} a_{kj} \right)_{n \times n}
$$

Step 3: Observe the absolute largest values above or below the main diagonal of matrix C, here denoted as $c_{ij}^{\max}$, then we can easily identify the corresponding a_{ij} in matrix A as the most inconsistent data. If there are other entries whose values are close to $c_{ij}^{\max}$, they can be identified as the most inconsistent entries.

7.2.3 The Processes of Inconsistency Adjustment by LMIBM

In above section, the most inconsistent data is identified. In the following, the estimating formula of identified inconsistent data is derived to adjust the identified inconsistent data. Assume a_{ij} is the identified inconsistent data, which is corresponding to the $c_{ij}^{\max}$ in matrix C, we have

$$\begin{aligned} c_{ij}^{\max} &= \frac{1}{n}\log\prod_{k=1}^{n} a_{ik}a_{kj} - \log a_{ij} \\ &= \frac{1}{n}\log a_{ij}^2 \prod_{lk=1\neq i,j}^{n} a_{ik}a_{kj} - \log a_{ij} \\ &= \frac{1}{n}\log a_{ij}^2 \tilde{a}_{ij}^{n-2} - \log a_{ij} \end{aligned}$$

We can get

$$\begin{aligned} nc_{ij}^{\max} &= \log a_{ij}^2 \tilde{a}_{ij}^{n-2}/a_{ij} \\ \Rightarrow \tilde{a}_{ij} &= a_{ij}\sqrt[n-2]{10^{nc_{ij}^{\max}}} = a_{ij}10^{\frac{nc_{ij}^{\max}}{n-2}} \end{aligned} \tag{7.7}$$

Therefore, the identified inconsistent can be estimated by formula (7.7).

7.3 Illustrative Examples

Assume a decision maker needs to quickly assess the disaster degree of four places attacked by earthquake with respect to the indirect economic loss to make a "scenario-response" types relief resource allocation, the 4×4 matrix A with $CR = 0.173 > 0.1$ used in Ergu et al. (2011b) and Liu (1999) is assumed to be the collected judgment matrix by emergency expert in this paper, that is,

$$A = \begin{pmatrix} 1 & 1/9 & 3 & 1/5 \\ 9 & 1 & 5 & 2 \\ 1/3 & 1/5 & 1 & 1/2 \\ 5 & 1/2 & 2 & 1 \end{pmatrix}$$

Apply the LMIBM to this matrix.

7.3.1 Step I: Inconsistency Identification

Step 1: Compute the column matrix L and the row matrix R by formula (7.5),

$$L = \begin{pmatrix} 0.0667 & 90 & 0.0333 & 5 \end{pmatrix}^T$$

,

$$R = \begin{pmatrix} 15 & 0.0111 & 30 & 0.2 \end{pmatrix}$$

Step 2: Compute LMIBM C by formula (7.6), we get

$$C = \frac{1}{4}\log(L \times R) - \log A$$
$$= \begin{pmatrix} 0 & 0.1717 & -0.4019 & 0.2302 \\ -0.1717 & 0 & 0.1589 & 0.0128 \\ 0.4019 & -0.1589 & 0 & -0.2430 \\ -0.2302 & -0.0128 & 0.2430 & 0 \end{pmatrix}$$

Step 3: Identify the absolute largest value $c_{ij}^{\max}$ either above the main diagonal in matrix C or below it. Here the data below the main diagonal are used, and we obtain that $c_{ij}^{\max} = c_{31}^{\max} = 0.4019$, then the corresponding element a_{31} in matrix A is regarded as the most inconsistent entry.

7.3.2 Step II: Inconsistency Adjustment

Applying formula (7.7) to estimate the possible proper value of a_{31}, we get

$$\tilde{a}_{31} = a_{31} 10^{\frac{4}{4-2}c_{31}} = \frac{1}{3} 10^{2 \times 0.4019} = 2.1217 \approx 2$$

Therefore, the identified inconsistent data and its estimated value are the same as the ones in (Ergu et al. 2011b) and (Liu 1999), whose $CR = 0.0028 < 0.1$, but the number of observed entries is reduced to 6 entries.

In the following, an 8 × 8 pair-wise comparison matrix A introduced in Cao et al. (2008), Ergu et al. (2011b) and Xu and We (1999) is introduced to test the proposed model for matrix with high order.

$$A = \begin{bmatrix} 1 & 5 & 3 & 7 & 6 & 6 & 1/3 & 1/4 \\ 1/5 & 1 & 1/3 & 5 & 3 & 3 & 1/5 & 1/7 \\ 1/3 & 3 & 1 & 6 & 3 & 4 & 6 & 1/5 \\ 1/7 & 1/5 & 1/6 & 1 & 1/3 & 1/4 & 1/7 & 1/8 \\ 1/6 & 1/3 & 1/3 & 3 & 1 & 1/2 & 1/5 & 1/6 \\ 1/6 & 1/3 & 1/4 & 4 & 2 & 1 & 1/5 & 1/6 \\ 3 & 5 & 1/6 & 7 & 5 & 5 & 1 & 1/2 \\ 4 & 7 & 5 & 8 & 6 & 6 & 2 & 1 \end{bmatrix}$$

Applying the LMIBM to this matrix.

7.3.3 Step I: Inconsistency Identification

Step 1: The column matrix L and the row matrix R by formula (7.5) are,

$$L = \begin{pmatrix} 315 & 0.0857 & 86.4 & 0 & 0.0009 & 0.0037 & 218.75 & 80640 \end{pmatrix}$$

$$R = \begin{pmatrix} 0.0032 & 11.6667 & 0.0116 & 141120 & 1080 & 270 & 0.0046 & 0 \end{pmatrix}$$

Step 2: Compute LMIBM C by formula (7.6), we get

$$C = \frac{1}{8}\log(L \times R) - \log A$$
$$= \begin{pmatrix} 0 & -0.2533 & -0.4069 & 0.1109 & -0.0867 & -0.1619 & 0.4969 & 0.3010 \\ 0.2533 & 0 & 0.1017 & -0.1886 & -0.2313 & -0.3066 & 0.2731 & 0.0984 \\ 0.4069 & -0.1017 & 0 & 0.1076 & 0.1441 & -0.0561 & -0.8286 & 0.3277 \\ -0.1109 & 0.1886 & -0.1076 & 0 & 0.2126 & 0.2623 & -0.0911 & -0.3539 \\ 0.0867 & 0.2313 & -0.1441 & -0.2126 & 0 & 0.2258 & 0.0273 & -0.2143 \\ 0.1619 & 0.3066 & 0.0561 & -0.2623 & -0.2258 & 0 & 0.1026 & -0.1391 \\ -0.4969 & -0.2731 & 0.8286 & 0.0911 & -0.0273 & -0.1026 & 0 & -0.0198 \\ -0.3010 & -0.0984 & -0.3277 & 0.3539 & 0.2143 & 0.1391 & 0.0198 & 0 \end{pmatrix}$$

Step 3: The absolute largest value $c_{ij}^{\max}$ above the main diagonal in matrix C is $c_{37}^{\max} = -0.8286$, then the corresponding element a_{37} in matrix A is regarded as the most inconsistent entry.

7.3.4 Step II: Inconsistency Adjustment

Applying formula (7.7) to estimate the possible proper value of a_{37}, we get

$$\tilde{a}_{37} = a_{37}10^{\frac{8}{8-2}c_{37}} = 610^{8/6\times(-0.8286)} = 0.4714 \approx 1/2$$

The identified inconsistent data are the same as the ones in Cao et al. (2008), Ergu et al. (2011b) and Xu and We (1999), whose $CR = 0.0828 < 0.1$, but the number of observed entries is reduced to 28 entries instead of 56.

7.4 Conclusion

In this paper, a logarithm mean induced bias matrix (LMIBM) is proposed to quickly process the inconsistent disaster data when AHP and ANP are used to assess the “scenario-response” type’s natural disaster emergency alternatives. The processes of inconsistency identification of LMIBM and estimating formula are proposed and derived. Two numerical examples are used to illustrate the proposed model. Since LMIBM is not only based on the original matrix and independent to the way of deriving the priority weights, but also can reduce the observed number of the induced bias data, therefore, the proposed model can speed up the processes of inconsistent disaster data identification in a judgment matrix and improve the speed of “scenario-response” types disaster emergency alternatives assessment and response.

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