Chapter 2
A Class of Robust Solution for Linear Bilevel Programming

Bo Liu, Bo Li and Yan Li

Abstract Under the way of the centralized decision-making, the linear bi-level programming (BLP) whose coefficients are supposed to be unknown but bounded in box disturbance set is studied. Accordingly, a class of robust solution for linear BLP is defined, and the original uncertain BLP was converted to the deterministic triple level programming, then a solving process is proposed for the robust solution. Finally, a numerical example is shown to demonstrate the effectiveness and feasibility of the algorithm.

Keywords Box disturbance · Linear bilevel programming · Robust optimization · Robust solution

2.1 Introduction

Bilevel programming (BLP) is the model with leader-follower hierarchical structure, which makes the parameter optimization problems as the constraints (Dempe 2002). In its decision framework, the upper level programming is connected with not only the decision variables in its level but also with the optimal solution in the lower level programming, while the optimal solution in the lower lever programming is affected by decision variables in the upper level

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programming. Due to the leader-follower hierarchical structure problems widely exist in the realistic decision-making environment, the scholars have been paying great attention to BLP and have brought about good results on the theory and algorithms (Bialas and Karwan 1982; Fortuny-Amat and McCarl 1981; Mathieu et al. 1994; Lai 1996). Some degree of uncertainty exists in realistic decision-making environment, such as the inevitable error of measuring instrument in data collection, incompleteness in data information, the approximate handle for the model and other factors; hence it is necessary to study on the uncertain Bilevel programming. For the uncertainty problem, the fuzzy optimization and stochastic optimization have been applied widely. However, it is difficult for decision-makers to give the precise distribution functions or membership functions which are required in above methods. Thus, the robust optimization become an important method, because it can seek for the best solution for the uncertain input without considering the parameter distribution of uncertain parameters and is immune from the uncertain data (Soyster 1973).

For the uncertain BLP, the definition of robust solution is influenced by the dependent degree of the upper and lower levels in the decision-making process. When the dependent degree is relative independence, the robust solution to the uncertain BLP is defined by the way of the decentralized decision-making (Li and Du 2011); when the dependent degree is relative dependence, the robust solution to the uncertain BLP is defined by the way of the centralized decision-making, that is, when the lower level seeks its own robust solution, it considers the influence to the robust solution of the upper level firstly. In the paper, the latter case will be discussed, and the coefficients of BLP are supposed to be unknown but bounded in box disturbance set. By the transform of the uncertain model, the robust solution of BLP is obtained. Finally, a numerical example is shown to demonstrate the effectiveness and feasibility of the algorithm.

2.2 The Definition of Robust BLP

2.2.1 The Model and the Definition

In this paper we consider Linear BLP formulated as follows:

$$
\begin{aligned}
&\min_x F(x,y) = c_1^T x + d_1^T y \\
&s.t. \; where \; y \; solves \\
&\quad \min_y f(x,y) = c_2^T x + d_2^T y \\
&s.t. \; Ax + By \geq h \\
&x, y \geq 0
\end{aligned}
\tag{2.1}
$$

In model (2.1),

$$x \in R^{m\times 1},\ y \in R^{n\times 1},\ c_l \in R^{m\times 1}, d_l \in R^{n\times 1},\ l \in \{1,2\},$$
$$A \in R^{r\times m},\ B \in R^{r\times n},\ h \in R^{r\times 1},$$

there is some uncertainty or variation in the parameters $c_1, d_1, c_2, d_2, A, B, h$. Let $(c_1, d_1, c_2, d_2, A, B, h) \in \mu$, μ is a given uncertainty set in Box disturbance as follows:

$$\mu := \left\{ (c_l, d_l, A, B, h) \left| \begin{array}{l} c_{li} = c^*_{li} + (u_{c_l})_i, -(u_{c_l})^*_i \le (u_{c_l})_i \le (u_{c_l})^*_i \\ d_{lj} = d^*_{lj} + (u_{d_l})_j, -(u_{d_l})^*_j \le (u_{d_l})_j \le (u_{d_l})^*_j \\ a_{ki} = a^*_{ki} + (u_A)_{ki}, -(u_A)^*_{ki} \le (u_A)_{ki} \le (u_A)^*_{ki} \\ b_{kj} = b^*_{kj} + (u_B)_{kj}, -(u_B)^*_{kj} \le (u_B)_{kj} \le (u_B)^*_{kj} \\ h_k = h^*_k + (u_h)_k, -(u_h)^*_k \le (u_h)_k \le (u_h)^*_k \\ l = \{1,2\}, \quad i \in \{1,\ldots,m\}, \\ j \in \{1,\ldots,n\}, \quad k = \{1,\ldots,r\}. \end{array} \right. \right\} \tag{2.2}$$

For $l = \{1,2\}$, $i \in \{1,\ldots,m\}, j \in \{1,\ldots,n\}, k = \{1,\ldots,r\}$, $c^*_{li}, d^*_{lj}, a^*_{ki}, d^*_{kj}, h^*_k$ are the given data, and $(u_{c_l})^*_i, (u_{d_l})^*_j, (u_A)^*_{ki}, (u_B)^*_{kj}, (u_h)^*_k$ are the given nonnegative data.

Under the way of the centralized decision-making, the robust solution of uncertain BLP (1) is defined as follows:

Definition 1

(1) Constraint region of the linear BLP (1):

$$\Omega = \{(x,y) | Ax + By \ge h, x, y \ge 0, (A,B,h) \in \mu\}$$

(2) Feasible set for the follower for each fixed x

$$\Omega(x) = \{y | Ax + By \ge h, x, y \ge 0, (A,B,h) \in \mu\}$$

(3) Follower's rational reaction set for each fixed x

$$M(x) = \left\{ y \left| y \in \arg\min \left\{ \begin{array}{l} c_2^T x + d_2^T y,\ y \in \Omega(x), \\ (A,B,h) \in \mu \end{array} \right\} \right. \right\}$$

(4) Inducible region:

$$IR = \{(x,y) | (x,y) \in \Omega, y \in M(x)\}.$$

Definition 2 Let

$$F := \left\{ (x, y, t) \in R^m \times R^n \times R \left| \begin{array}{r} c_1^T x + d_1^T y \leq t \\ (x, y), (c_1,\ d_1) \in \mu \end{array} \right. \right\}$$

The programming

$$\min_{x,y,t} \{ t | (x, y, t) \in F \} \tag{2.3}$$

is defined as robust counterpart of uncertain linear BLP(1); F is defined as the robust feasible set of uncertain linear BLP(1).

2.2.2 The Transform of Uncertain BLP Model

Under the way of the centralized decision-making, based on the original idea of robust optimization that the objective function can get the optimal solution even in the worst and uncertain situation, the transform theorem can be described as followings:

Theorem *The robust linear BLP* (1) *with its coefficients unknown but bounded in box disturbance set* μ *is equivalent to Model* (2.4) *with certain coefficients as followings:*

$$\begin{aligned} & \min_x F(x, y) = \sum_{i=1}^m \left(c_{1i}^* + \left(\mu_{c_1} \right)_i^* \right) x_i + \sum_{j=1}^n \left(d_{1j}^* + \left(\mu_{d_1} \right)_j^* \right) y_j \\ & s.t.\ where\ d_2\ solves \\ & \quad \max_{d_2} \sum_{i=1}^m \left(c_{1i}^* + \left(\mu_{c_1} \right)_i^* \right) x_i + \sum_{j=1}^n \left(d_{1j}^* + \left(\mu_{d_1} \right)_j^* \right) y_j \\ & \quad s.t.\ d_{2j}^* - (u_{d_2})_j^* \leq d_{2j} \leq d_{2j}^* + (u_{d_2})_j^*, \quad j = \{1, \ldots, n\}; \\ & \quad where\ y\ solves \\ & \quad\quad \min_y f(x, y) = d_2^T y \\ & \quad s.t.\ \sum_{i=1}^m \left(a_{ki}^* - (\mu_A)_{ki}^* \right) x_i + \sum_{j=1}^n \left(b_{kj}^* - (\mu_B)_{kj}^* \right) y_j \geq h_k^* + (\mu_h)_k^*, \\ & \quad\quad k = \{1, \ldots, r\} \\ & \quad\quad x, y \geq 0. \end{aligned} \tag{2.4}$$

Proof (1) Firstly, the constraint region Ω of the linear BLP (1) is transformed into the certain region. Consider the constraint region of the linear BLP (1):

$$\Omega = \{(x, y)|Ax + By \geq h, x, y \geq 0, \quad (A, B, h) \in \mu\}$$

According to the process of the transformation (Lobo et al. 1998), we can obtain

$$\begin{aligned}
& Ax + By \geq h, (A, B, h) \in \mu \\
& \Leftrightarrow 0 \leq \min_{\mu_A, \mu_B, \mu_h} \left\{ \sum_{i=1}^{m} a_{ki} x_i + \sum_{j=1}^{n} b_{kj} y_j - h_k \,\middle|\, \begin{array}{l} a_{ki} = a_{ki}^* + (u_A)_{ki}, \\ -(u_A)_{ki}^* \leq (u_A)_{ki} \leq (u_A)_{ki}^*; \\ b_{kj} = b_{kj}^* + (u_B)_{kj}, \\ -(u_B)_{kj}^* \leq (u_B)_{kj} \leq (u_B)_{kj}^*; \\ h_k = h_k^* + (u_h)_k, \\ -(u_h)_k^* \leq (u_h)_k \leq (u_h)_k^*; \\ i \in \{1, \ldots, m\}, \quad j \in \{1, \ldots, n\}, \\ k \in \{1, \ldots, r\} \end{array} \right\} \\
& \Leftrightarrow 0 \leq \sum_{i=1}^{m} a_{ki}^* x_i + \sum_{j=1}^{n} b_{kj}^* y_j - h_k^* \\
& \quad + \min_{\mu_A, \mu_B, \mu_h} \left\{ \sum_{i=1}^{m} (u_A)_{ki} x_i + \sum_{j=1}^{n} (u_B)_{kj} y_j - (u_h)_k \,\middle|\, \begin{array}{l} -(u_A)_{ki}^* \leq (u_A)_{ki} \leq (u_A)_{ki}^* \\ -(u_B)_{kj}^* \leq (u_B)_{kj} \leq (u_B)_{kj}^* \\ -(u_h)_k^* \leq (u_h)_k \leq (u_h)_k^* \\ i \in \{1, \ldots, m\}, \quad j \in \{1, \ldots, n\}, \\ k \in \{1, \ldots, r\} \end{array} \right\} \\
& \underset{\forall k \in \{1, \ldots, r\}}{\overset{x, y \geq 0}{\Leftrightarrow}} 0 \leq \sum_{i=1}^{m} \left(a_{ki}^* - (u_A)_{ki}^*\right) x_i + \sum_{j=1}^{n} \left(b_{kj}^* - (u_B)_{kj}^*\right) y_j - \left(h_k^* + (u_h)_k^*\right) \\
& \Leftrightarrow \sum_{i=1}^{m} \left(a_{ki}^* - (u_A)_{ki}^*\right) x_i + \sum_{j=1}^{n} \left(b_{kj}^* - (u_B)_{kj}^*\right) y_j \geq h_k^* + (u_h)_k^*, \quad k \in \{1, \ldots, r\}
\end{aligned} \tag{2.5}$$

So the linear BLP (1) is transformed into the model (2.6) as followings:

$$\begin{aligned}
&\min_{x} F(x,y) = c_1^T x + d_1^T y \\
&s.t.\ where\ y\ solves \\
&\min_{y} f(x,y) = c_2^T x + d_2^T y \\
&s.t. \sum_{i=1}^{m} \left(a_{ki}^* - (u_A)_{ki}^*\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^* - (u_B)_{kj}^*\right)y_j \geq h_k^* + (u_h)_k^* \\
&k = \{1, \ldots, r\} \\
&x, y \geq 0
\end{aligned} \tag{2.6}$$

(2) Next, according the equivalent form (Lobo et al. 1998)

$$\begin{array}{c} \min_{x} f(x) \\ s.t.\ x \in D \end{array} \Leftrightarrow \begin{array}{c} \min_{x,t}\ t \\ s.t. f(x) \leq t \\ x \in D \end{array}$$

and the K-T method, the model (2.6) can be transform-ed into the model (2.7) (Li and Du 2011):

$$\begin{aligned}
&\min_{x,t} F(x,y) = t \\
&s.t.\ c_1^T x + d_1^T y \leq t \\
&where\ y\ solves \\
&\quad \min_{y} f(x,y) = c_2^T x + d_2^T y \\
&s.t. \sum_{i=1}^{m} \left(a_{ki}^* - (u_A)_{ki}^*\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^* - (u_B)_{kj}^*\right)y_j \geq h_k^* + (u_h)_k^* \\
&\quad k = \{1, \ldots, r\} \\
&\quad x, y \geq 0
\end{aligned} \tag{2.7}$$

(3) Similar to the transformation (2.5),

$$c_1^T x + d_1^T y \leq t, (c_1, d_1) \in \mu \overset{x,y \geq 0}{\Leftrightarrow} \sum_{i=1}^{m} \left(c_{1i}^* + \left(\mu_{c_1}\right)_i^*\right)x_i + \sum_{j=1}^{n} \left(d_{1j}^* + \left(\mu_{d_1}\right)_j^*\right)y_j \leq t$$

So the model (2.7) can be transformed to the model (2.8) as follows:

$$\begin{aligned}
&\min_{x,t} F(x,y) = \sum_{i=1}^{m} \left(c_{1i}^{*} + \left(\mu_{c_1}\right)_i^{*}\right)x_i + \sum_{j=1}^{n} \left(d_{1j}^{*} + \left(\mu_{d_1}\right)_j^{*}\right)y_j \\
&s.t.\, where\ y\ solves \\
&\quad \min_{y} f(x,y) = c_2^T x + d_2^T y \\
&s.t.\ \sum_{i=1}^{m} \left(a_{ki}^{*} - (u_A)_{ki}^{*}\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^{*} - (u_B)_{kj}^{*}\right)y_j \geq h_k^{*} + (u_h)_k^{*}, \\
&\quad k \in \{1,\ldots,r\}; \\
&\quad x, y \geq 0
\end{aligned} \tag{2.8}$$

(4) Next, because the optimal solution of BLP (1) is not influenced by the value of c_2, we only consider how to choose the value of d_2. Based on the original idea of robust optimization, the model (2.8) is transformed into the model (2.4) above.

2.3 Solving Process of the Model

The deterministic triple level programming (2.4) can be written as the following programming (2.9) by the K-T method.

$$\begin{aligned}
&\min_{x} \max_{d_2,y,u,v} F(x,y) = \sum_{i=1}^{m} \left(c_{1i}^{*} + \left(\mu_{c_1}\right)_i^{*}\right)x_i + \sum_{j=1}^{n} \left(d_{1j}^{*} + \left(\mu_{d_1}\right)_j^{*}\right)y_j \\
&s.t.\, d_{2j} = \sum_{k=1}^{r} u_k \left(b_{kj}^{*} - (u_B)_{kj}^{*}\right) + v_j \\
&\quad d_{2j}^{*} - (u_{d_2})_j^{*} \leq d_{2j} \leq d_{2j}^{*} + (u_{d_2})_j^{*} \\
&\quad u_k \left[\sum_{i=j}^{m} \left(a_{kj}^{*} - (\mu_A)^{*}\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^{*} - (\mu_B)_{kj}^{*}\right)y_j - \left(h_k^{*} + (\mu_h)_k^{*}\right)\right] = 0 \\
&\quad v_j y_j = 0 \\
&\quad \sum_{i=1}^{m} \left(a_{ki}^{*} - (\mu_A)_{ki}^{*}\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^{*} - (\mu_B)_{kj}^{*}\right)y_j \geq h_k^{*} + (\mu_h)_k^{*}, \\
&\quad j = \{1,\ldots,n\}, k = \{1,\ldots,r\}; \\
&\quad x, y, u, v \geq 0.
\end{aligned} \tag{2.9}$$

According to the literature (Wang 2010), the model (2.9) can be transformed into the model (2.10) as follows

$$
\begin{aligned}
&\min_{x,d_2,y,u,v} \ t \\
&s.t. \sum_{i=1}^{m} \left(c_{1i}^{*} + \left(\mu_{c_1}\right)_i^{*}\right)x_i + \sum_{j=1}^{n} \left(d_{1j}^{*} + \left(\mu_{d_1}\right)_j^{*}\right)y_j \le t \\
&\quad d_{2j} = \sum_{k=1}^{r} u_k\left(b_{kj}^{*} - \left(u_B\right)_{kj}^{*}\right) + v_j, \\
&\quad d_{2j}^{*} - \left(u_{d_2}\right)_j^{*} \le d_{2j} \le d_{2j}^{*} + \left(u_{d_2}\right)_j^{*}, \\
&\quad u_k\left[\sum_{i=1}^{m} \left(a_{ki}^{*} - \left(\mu_A\right)_{ki}^{*}\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^{*} - \left(\mu_B\right)_{kj}^{*}\right)y_j - \left(h_k^{*} + \left(\mu_h\right)_k^{*}\right)\right] = 0, \\
&\quad v_j y_j = 0, \\
&\quad \sum_{i=1}^{m} \left(a_{ki}^{*} - \left(\mu_A\right)_{ki}^{*}\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^{*} - \left(\mu_B\right)_{kj}^{*}\right)y_j \ge h_k^{*} + \left(\mu_h\right)_k^{*}, \\
&\quad j = \{1, \ldots, n\}, k = \{1, \ldots, r\}, \\
&\quad x, y, u, v \ge 0.
\end{aligned} \tag{2.10}
$$

By introducing a large constant M, the model (2.10) above can be transformed into a mixed integer programming as follows (Fortuny-Amat and McCarl 1981):

$$
\begin{aligned}
&\min_{x,d_2,y,u,v,t,w} \ t \\
&s.t. \sum_{i=1}^{m} \left(c_{1i}^{*} + \left(\mu_{c_1}\right)_i^{*}\right)x_i + \sum_{j=1}^{n} \left(d_{1j}^{*} + \left(\mu_{d_1}\right)_j^{*}\right)y_j \le t \\
&\quad d_{2j} = \sum_{k=1}^{r} u_k\left(b_{kj}^{*} - \left(u_B\right)_{kj}^{*}\right) + v_j, \\
&\quad d_{2j}^{*} - \left(u_{d_2}\right)_j^{*} \le d_{2j} \le d_{2j}^{*} + \left(u_{d_2}\right)_j^{*}, \\
&\quad y_j \le M t_j, \\
&\quad v_j \le M\left(1 - t_j\right), \\
&\quad u_k \le M w_k, \\
&\quad \sum_{i=1}^{m} \left(a_{ki}^{*} - \left(\mu_A\right)_{ki}^{*}\right)x_i + \sum_{j=1}^{n} \left(b_{kj}^{*} - \left(\mu_B\right)_{kj}^{*}\right)y_j - \left(h_k^{*} + \left(\mu_h\right)_k^{*}\right) \le M(1 - w_k) \\
&\quad \sum_{i=1}^{m} \left[a_{ki}^{*} - \left(\mu_A\right)_{ki}^{*}\right] \cdot x_i + \sum_{j=1}^{n} \left[b_{kj}^{*} - \left(\mu_B\right)_{kj}^{*}\right] \cdot y_j \ge h_k^{*} + \left(\mu_h\right)_k^{*}, \\
&\quad j = \{1, \ldots, n\}, k = \{1, \ldots, r\}, t_j \in \{0, 1\}, w_k \in \{0, 1\}, \\
&\quad x, y, u, v \ge 0.
\end{aligned} \tag{2.11}
$$

The model (2.11) can be solved by the software Lingo 9.0

2.4 A Numerical Example

We give a numerical example to demonstrate the proposed approach as follows:

$$\begin{aligned}
&\min_{x} F = c_{11}x + d_{11}y_1 + d_{12}y_2 \\
&s.t.\ 2.5 \le x \le 8 \\
&\quad where\ y_1,\ y_2\ solve \\
&\quad \min_{y_1,y_2} f = d_{21}y_1 + d_{22}y_2 \\
&\quad s.t.\ a_{11}x_1 + b_{11}y_1 + b_{12}y_2 \ge h_1 \\
&\qquad a_{21}x_1 + b_{21}y_1 + b_{22}y_2 \ge h_2 \\
&\qquad a_{31}x_1 + b_{31}y_1 + b_{32}y_2 \ge h_3 \\
&\qquad a_{41}x_1 + b_{41}y_1 + b_{42}y_2 \ge h_4 \\
&\qquad y_1 \ge 0, \\
&\qquad y_2 \ge 0
\end{aligned}$$

where $a_{11} = 0,\ b_{21} = 1,\ a_{31} = 0,\ a_{41} = 0,\ b_{41} = 0,\ b_{42} = -1.$
And the others are the uncertain data, the given variables and disturbances are

$$c_{11}^{*} = 1.5,\ d_{11}^{*} = -1.5,\ d_{12}^{*} = -2,$$

$$d_{21}^{*} = 1.5,\ d_{22}^{*} = -3.5,$$

$$b_{11}^{*} = 1.75,\ b_{12}^{*} = -1.15,\ h_{1}^{*} = 2.5,$$

$$a_{21}^{*} = -3.5,\ b_{22}^{*} = -1.5,\ h_4 = 5.75.$$

$$h_{2}^{*} = -11, b_{31}^{*} = -3.5, b_{32}^{*} = -1.25, h_{3}^{*} = -23,$$

$$(u_{c_1})_1^{*} = 0.5, (u_{d_1})_1^{*} = 0.5, (u_{d_1})_2^{*} = 3, (u_{d_2})_1^{*} = 0.5, (u_{d_2})_2^{*} = 0.5,$$

$$(u_b)_{11}^{*} = 0.25, (u_b)_{12}^{*} = 0.15, (u_h)_1^{*} = 0.5,$$

$$(u_a)_{21}^{*} = 0.5, (u_b)_{22}^{*} = 0.5, (u_h)_2^{*} = 1,$$

$$(u_b)_{31}^{*} = 0.5, (u_b)_{32}^{*} = 0.25, (u_h)_3^{*} = 1, (u_h)_4^{*} = 0.25.$$

According to the theorem and these data above, robust model transformed is demonstrated as

$$\min_{x,y,u,v,\eta,z} \; t$$
$$
\begin{aligned}
s.t.\; & 2x - y_1 + y_2 \le t \\
& 2.5 \le x \le 8, \\
& y_2 \le 5.5, \\
& 1.5y_1 - 1.3y_2 \ge 3, \\
& -4x + y_1 - 2y_2 \ge -10, \\
& -4y_1 - 1.5y_2 \ge -22, \\
& -1 \le 1.5u_1 + u_2 - 4u_3 + v_1 \le 2, \\
& -4 \le -1.3u_1 - 2u_2 - 1.5u_3 - u_4 + v_2 \le -3, \\
& y_1 \le M\eta_1, y_2 \le M\eta_2, \\
& v_1 \le M(1-\eta_1),\; v_2 \le M(1-\eta_2), \\
& u_k \le Mz_k, \quad k = 1,2,3,4. \\
& 1.5y_1 - 1.3y_2 - 3 \le (1-z_1), \\
& -4x + y_1 - 2y_2 + 10 \le (1-z_2),
\end{aligned}
$$

$$
\begin{aligned}
& -4y_1 - 1.5y_2 + 22 \le (1-z_3), \\
& -y_2 + 5.5 \le (1-z_4), \\
& x, y, u, v, \eta, z \ge 0, \\
& \eta_1, \eta_2 \in \{0.1\}, z_k \in \{0.1\}, \quad k = 1,2,3,4.
\end{aligned}
$$

By the software Lingo 9.0, the robust solution is obtained as follows:

$$(x, y_1, y_2) = (2.5201, 4.6443, 2.2819),$$

The robust optimal value is $F_{\min} = 2.6779$.

2.5 Conclusion and Future Work

Under the way of the centralized decision-making, a class of robust solution for uncertain linear BLP is defined, which expands further the application of BLP in different circumstances. And based on the original idea of robust optimization, the uncertain BLP was converted to the deterministic triple level programming. The solving process is proposed to obtain the robust solution of uncertain linear BLP. Finally, a numerical example is shown to demonstrate the effectiveness and feasibility of the algorithm.

References

Bialas WF, Karwan MH (1982) On two-level optimization. IEEE Trans Autom control AC-27(1):211–214

Dempe S (2002) Foundations of bilevel programming. Kluwer Academic Publisher, Boston

Fortuny-Amat J, McCarl B (1981). A representation and economic interpretation of two-level programming problem. J Oper Res Soc 32:783–792

Fortuny-Amat J, McCarl BA (1981) Representation and economic interpretation of two-level programming problem. J Oper Res Soc 32(7): 83–792

Lai YJ (1996) Hierarchical optimization: a satisfactory solution. Fuzzy Sets and Syst 77:321–335

Li Y, Du G (2011) Robust linear bilevel programming under ellipsoidal uncertainty. Syst Eng 11:96–100

Lobo MS, Vandenberghe L, Boyd S, Lebret H (1998) Application of second-order cone programming. Linear Algebra Appl 284:193–228

Mathieu R, Pittard L, Anandalingam G (1994) Genetic algorithm based approach to bi-level linear programming. Oper Res 28:1–21

Soyster AL (1973) Convex programming with set-inclusive constraints and applications to inexact linear programming. Oper Res 21:1154–1157

Wang J (2010) Research on the methods of interval linear bi-level programming, pp 54–55. Tianjin University, Tianjin