Multirate Multisensor Data Fusion Algorithm for State Estimation with Cross-Correlated Noises

Yulei Liu, Liping Yan, Bo Xiao, Yuanqing Xia and Mengyin Fu

Abstract This paper is concerned with the optimal state estimation problem under linear dynamic systems when the sampling rates of different sensors are different. The noises of different sensors are cross-correlated and coupled with the system noise of the previous step. By use of the projection theory and induction hypothesis repeatedly, a sequential fusion estimation algorithm is derived. The algorithm is proven to be optimal in the sense of Linear Minimum Mean Square Error(LMMSE). Finally, a numerical example is presented to illustrate the effectiveness of the proposed algorithm.

Keywords State estimation · Data fusion · Cross-correlated noises · Asynchronous multirate multisensor

1 Introduction

Estimation fusion, or data fusion for estimation, is the problem of how to best utilize useful information contained in multiple sets of data for the purpose of estimating a quantity, e.g., a parameter or process [1]. It originated in the military field, and is now widely used in military and civilian fields, e.g., target tracking and localization, guidance and navigation, surveillance and monitoring, etc., due to its improved estimation accuracy, enhanced reliability, and survivability, etc.

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Most of the earlier works were based on the assumption of cross-independent sensor noises. Bar-Shalom [2], Chong et al. [3], Hashmipour et al. [4] proposed several optimal state estimation algorithms based on Kalman filtering, respectively, in which all sensor measurements are fused by use of centralized fusion structure. In the practical applications, most of multisensor systems often have the correlated noises when the dynamic process is observed in a common noisy environment [5]. Moreover, because most of the real systems are described in continuous forms, discretization is necessary when to get the state estimation on line, and in the process, the system noise and the measurement noises are shown to be coupled. Of course, the centralized filter can still be used for the systems with correlated noises as it is still optimal in the sense of LMMSE. However, the computation and power requirements are too huge to be practical.

Hence, a few pieces of work deal with coupled sensor noises. Duan proposed one systematic way to handle the distributed fusion problem based on a unified data model in which the measurement noises across sensors at the same time may be correlated [6]. Song also dealt with the state estimation problem with cross-correlated sensor noises, and proved that under a mild condition it is optimal [5]. A small amount of papers consider the coupled sensor noises and the correlation between sensor noises and system noises. Xiao et al. [7] considered the two kinds of correlations by augmentation and the computation is complex.

In all the papers mentioned above, the sampling rates of different sensors are the same. Based on the multi-sensor dynamic system in which different sensors observe the same target state with different sampling rates, Yan et al. [8] put forward a kind of optimal state estimation algorithm. The algorithm has stronger feasibility and practicality than the traditional state fusion algorithm, but it does not take the correlations of noises into account. Shi et al. [9] discussed the estimation when multisensors have multirate asynchronous sampling rates. However, it does not consider the sensor-correlations either.

In this paper, when the noises of different sensors are cross-correlated and when they are also coupled with the system noise of the previous step, also, when the sampling rates of different sensors are different, by use of the projection theory, a sequential algorithm is formulated. We analyzed the performance of the algorithm, and it is shown to be optimal in the sense of LMMSE.

The paper is organized as follows. In Sect. 2, the problem formulation is presented. Section 3 describes the optimal state estimation algorithm. Section 4 is the simulation results and Sect. 5 draws the conclusion.

2 Problem Formulation

Consider the following generic linear dynamic system,

$$x(k+1) = A(k)x(k) + w(k), k = 0, 1, \ldots \tag{1}$$

$$z_i(k_i) = C_i(k_i)x(k_i) + v_i(k_i), i = 1, 2, \ldots, N \tag{2}$$

where, $x(k) \in R^n$ is the system state, $A(k) \in R^{n \times n}$ is the state transition matrix, and $w(k)$ is the system noise and is assumed to be Gaussian distributed with zero mean and variance being $Q(k)$, where $Q(k) \geq 0$. $z_i(k_i) \in R^{m_i}$ is the measurement of sensor i at time k_i. Assume the sampling rate of sensor i is S_i, and $S_i = S_1/n_i$, where n_i is known positive integers. Without loss of generality, let the sampling period of sensor 1 be the unit time, that is, $k_1 = k$. Then, sensor i has measurement at sampling point $n_i k$, in other words, $k_i = n_i k$. $C_i(k_i) \in R^{m_i \times n}$ is the measurement matrix. Measurement noise $v_i(k_i)$ is zero-mean and is Gaussian distributed with variance being $R(k)$, and

$$E\{w(k_i - 1)v_i^T(k_i)\} = S_i(k_i) \tag{3}$$

From the above equation, we can see that the measurement noises are coupled with the previous step system noise. Namely, $v_i(k_i)$ is correlated with $w(k_i - 1)$ at time $k = 0, 1, \ldots, i = 1, 2, \ldots, N$. If different sensors have measurements at the same time, their measurement noises are cross-correlated, i.e., $v_i(k_i)$ and $v_j(l_j)$ are coupled when $k_i = l_j$. That is, $E\{v_i(k_i)v_j^T(k_i)\} = R_{ij}(k_i) \neq 0$ where $i, j = 1, 2, \ldots, N$. For simplicity, denote $R_i(k_i) \stackrel{\Delta}{=} R_{ii}(k_i) > 0, i = 1, 2, \ldots, N$.

The initial state $x(0)$ is independent of $w(k)$ and $v_i(k_i)$, where $k = 1, 2, \ldots$, $i = 1, 2, \ldots, N$, and is assumed to be Gaussian distributed with

$$\begin{cases} E\{x(0)\} = x_0 \\ cov\{x(0)\} = E\{[x(0) - x_0][x(0) - x_0]^T\} = P_0 \end{cases} \tag{4}$$

where $cov\{x(0)\}$ means the covariance of $x(0)$.

It can be seen from the above description that sensor i will participate in the fusion process at time $n_i k$. Generally speaking, assume there are p sensors that have measurements at time k with measurements $z_{i_1}(k), z_{i_2}(k), \ldots, z_{i_p}(k)$. Then, to generate the optimal state estimate of x(k) in the measurement update step, the above p sensors shall be fused. The estimation of $x(k)$ is the information fusion of the above p sensors.

3 Optimal State Estimation Algorithm

Theorem 1 Based on the descriptions in Sect. 2, suppose we have known the optimal fusion estimation $\hat{x}(k-1|k-1)$ and its estimation error covariance $P(k-1|k-1)$ at time $k-1$, then the optimal state estimation of $x(k)$ at time k could be computed as follows,

$$\hat{x}_{i_j}(k|k) = \hat{x}_{i_{j-1}}(k|k) + K_{i_j}(k)[z_{i_j}(k) - C_{i_j}(k)\hat{x}_{i_{j-1}}(k|k)] \tag{5}$$

$$P_{i_j}(k|k) = P_{i_{j-1}}(k|k) - K_{i_j}(k)[C_{i_j}(k)P_{i_{j-1}}(k|k) + \Delta^T_{i_{j-1}}(k)] \quad (6)$$

$$\begin{aligned} K_{i_j}(k) = {} & [P_{i_{j-1}}(k|k)C^T_{i_j}(k) + \Delta_{i_{j-1}}(k)][C_{i_j}(k)P_{i_{j-1}}(k|k)C^T_{i_j}(k) \\ & + C_{i_j}(k)\Delta_{i_{j-1}}(k) + \Delta^T_{i_{j-1}}(k)C^T_{i_j}(k)]^{-1} + R_{i_j}(k) \end{aligned} \quad (7)$$

$$\begin{aligned} \Delta_{i_j}(k) = {} & \prod_{u=j}^{1}[I - K_{i_u}(k)C_{i_u}(k_{i_u})]S_{i_{j+1}}(k) - K_{i_j}(k)R_{i_j,i_{j+1}}(k) \\ & - \sum_{q=2}^{j}\prod_{u=j}^{q}[I - K_{i_u}(k)C_{i_u}(k_{i_u})]K_{i_{q-1}}(k)R_{i_{q-1},i_{j+1}}(k) \end{aligned} \quad (8)$$

where $j = 1, 2, \ldots, p$. For $j = 0$,

$$\hat{x}_{i_0}(k|k) = \hat{x}(k|k-1) = A(k-1)\hat{x}(k-1|k-1) \quad (9)$$

$$\begin{aligned} P_{i_0}(k|k) & = P(k|k-1) \\ & = A(k-1)P(k-1|k-1)A^T(k-1) + Q(k-1) \end{aligned} \quad (10)$$

$$\Delta_{i_0}(k) = S_{i_1}(k) \quad (11)$$

The above $\hat{x}_{i_j}(k|k)$ and $P_{i_j}(k|k)$ denote the state estimation of $x(k)$ and the corresponding estimation error covariance based on observations of sensors $i_1, i_2, \ldots, i_j$ respectively. When $j = p$, we have $\hat{x}_s(k|k) = \hat{x}_{i_p}(k|k)$ and $P_s(k|k) = P_{i_p}(k|k)$, which are the optimal state fusion estimation and the corresponding estimation error covariance, where subscript 's' means the sequential fusion.

In addition, from (5), we can see that the sensor with the highest sampling rate is the first sensor whose sampling period is assumed to be the unit time, so sensor 1 is sensor i_1. That is, $\hat{x}_{i_1}(k|k) = \hat{x}_1(k|k)$, $P_{i_1}(k|k) = P_1(k|k)$.

Proof The theorem derives from gradually use of the projection theorem. For $i = 1, 2, \ldots, N$, denote

$$Z_i(k) = \{z_i(1), z_i(2), \ldots, z_i(k)\} \quad (12)$$

$$Z_1^i(k) = \{z_1(k), z_2(k), \ldots, z_i(k)\} \quad (13)$$

$$\bar{Z}_1^i(k) = \{Z_1^i(l)\}_{l=1}^k \quad (14)$$

where $Z_i(k)$ is the measurements of sensor i up to time k. If sensor i has no measurement at time l, we denote $z_i(l) = 0$, therefore, the above descriptions are meaningful. $Z_1^i(k)$ is the measurement of sensors $1, 2, \ldots, i$ at time k. $\bar{Z}_1^i(k)$ is the measurements of all sensors at time k and before.

In the sequel, we will prove Theorem 1 deductively by applying the projection theorem. Suppose, we have obtained $\hat{x}_{i_{j-1}}(k|k)$ and the corresponding estimation error covariance $P_{i_{j-1}}(k|k)$, next we will show how to get $\hat{x}_{i_j}(k|k)$ and $P_{i_j}(k|k)$.

Applying the projection theorem, we have

$$\begin{aligned}\hat{x}_{i_j}(k|k) &= E\{x(k)|\bar{Z}_1^N(k-1), Z_1^{i_j}(k)\} \\ &= E\{x(k)|\bar{Z}_1^N(k-1), Z_1^{i_{j-1}}(k), z_{i_j}(k)\} \\ &= \hat{x}_{i_{j-1}}(k|k) + cov\{\tilde{x}_{i_{j-1}}(k|k), \tilde{z}_{i_{j-1}}(k)\} \cdot var\{\tilde{z}_{i_j}(k)\}^{-1}\tilde{z}_{i_j}(k)\end{aligned} \tag{15}$$

where $\tilde{z}_{i_j}(k) = z_{i_j}(k) - \hat{z}_{i_j}(k)$, and

$$\begin{aligned}&\hat{z}_{i_j}(k) = E\{z_{i_j}(k)|\bar{Z}_1^N(k-1), Z_1^{i_{j-1}}(k)\} \\ &= E\{C_{i_j}(k)x(k) + v_{i_j}(k)|\bar{Z}_1^N(k-1), Z_1^{i_{j-1}}(k)\} \\ &= C_{i_j}(k)\hat{x}_{i_{j-1}}(k|k)\end{aligned} \tag{16}$$

So

$$\begin{aligned}&\tilde{z}_{i_j}(k) = z_{i_j}(k) - \hat{z}_{i_j}(k) \\ &= z_{i_j}(k) - C_{i_j}(k)\hat{x}_{i_{j-1}}(k|k) \\ &= C_{i_j}(k)x_{i_j}(k) + v_{i_j}(k) - C_{i_j}(k)\hat{x}_{i_{j-1}}(k|k) \\ &= C_{i_j}(k)\tilde{x}_{i_{j-1}}(k|k) + v_{i_j}(k)\end{aligned} \tag{17}$$

Therefore

$$\begin{aligned}&cov\{\tilde{x}_{i_{j-1}}(k|k), \tilde{z}_{i_j}(k)\} = E\{\tilde{x}_{i_{j-1}}(k|k)\tilde{z}_{i_j}^T(k)\} \\ &= E\{\tilde{x}_{i_{j-1}}(k|k)[C_{i_j}(k)\tilde{x}_{i_{j-1}}(k|k) + v_{i_j}(k)]^T\} \\ &= P_{i_{j-1}}(k|k)C_{i_j}^T(k) + \Delta_{i_{j-1}}(k)\end{aligned} \tag{18}$$

and

$$\begin{aligned}var\{\tilde{z}_{i_j}(k)\} &= E\{\tilde{z}_{i_j}(k)\tilde{z}_{i_j}^T(k)\} \\ &= E\{[C_{i_j}(k)\tilde{x}_{i_{j-1}}(k|k) + v_{i_j}(k)] \cdot [C_{i_j}(k)\tilde{x}_{i_{j-1}}(k|k) + v_{i_j}(k)]^T\} \\ &= C_{i_j}(k)P_{i_{j-1}}(k|k)C_{i_j}^T(k) + R_{i_j}(k) + C_{i_j}(k)\Delta_{i_{j-1}}(k) + \Delta_{i_{j-1}}^T(k)C_{i_j}^T(k)\end{aligned} \tag{19}$$

where

$$\Delta_{i_{j-1}}(k) = E\{\tilde{x}_{i_{j-1}}(k|k)v_{i_j}^T(k)\} \tag{20}$$

By use of the inductive assumption, we have

$$\begin{aligned}\tilde{x}_{i_{j-1}}(k|k) &= x(k) - \hat{x}_{i_{j-1}}(k|k) \\ &= x(k) - \hat{x}_{i_{j-2}}(k|k) - K_{i_{j-1}}(k)[z_{i_{j-1}}(k) - C_{i_{j-1}}(k)\hat{x}_{i_{j-2}}(k|k)] \\ &= \tilde{x}_{i_{j-2}}(k|k) - K_{i_{j-1}}(k)[C_{i_{j-1}}(k)x(k) + v_{i_{j-1}}(k) - C_{i_{j-1}}(k)\hat{x}_{i_{j-2}}(k|k)] \\ &= [I - K_{i_{j-1}}(k)C_{i_{j-1}}(k)]\tilde{x}_{i_{j-2}}(k|k) - K_{i_{j-1}}(k)v_{i_{j-1}}(k)\end{aligned} \tag{21}$$

Substitute (21) into (20), and by use of the inductive hypothesis, we have

$$\begin{aligned}
\Delta_{i_{j-1}}(k) &= E\{\tilde{x}_{i_{j-1}}(k|k)v_{i_j}^T(k)\} \\
&= [I - K_{i_{j-1}}(k)C_{i_{j-1}}(k)]E\{\tilde{x}_{i_{j-2}}(k|k)v_{i_j}^T(k)\} - K_{i_{j-1}}(k)E\{v_{i_{j-1}}(k)v_{i_j}^T(k)\} \\
&= \prod_{u=j-1}^{1}[I - K_{i_u}(k)C_{i_u}(k)]S_{i_j}(k) - K_{i_{j-1}}(k)R_{i_{j-1},i_j}(k) \\
&\quad - \sum_{q=2}^{j-1}\prod_{u=j-1}^{q}[I - K_{i_u}(k)C_{i_u}(k)]K_{i_{q-1}}(k)R_{i_{q-1},i_j}(k)
\end{aligned} \tag{22}$$

Substitute (18), (19) and the second equation of (17) into (15), we have

$$\hat{x}_{i_j}(k|k) = \hat{x}_{i_{j-1}}(k|k) + K_{i_j}(k)[z_{i_j}(k) - C_{i_j}(k)\hat{x}_{i_{j-1}}(k|k)] \tag{23}$$

where

$$\begin{aligned}
K_{i_j}(k) &= cov\{\tilde{x}_{i_{j-1}}(k|k), \tilde{z}_{i_{j-1}}(k)\}var\{\tilde{z}_{i_{j-1}}(k)\} \\
&= [P_{i_{j-1}}(k|k)C_{i_j}^T(k) + \Delta_{i_{j-1}}(k)] \cdot [C_{i_j}(k)P_{i_{j-1}}(k|k)C_{i_j}^T(k) + R_{i_j}(k) \\
&\quad + C_{i_j}(k)\Delta_{i_{j-1}}(k) + \Delta_{i_{j-1}}^T(k)C_{i_j}^T(k)]^{-1}
\end{aligned} \tag{24}$$

The estimation error covariance should be computed by

$$\begin{aligned}
P_{i_j}(k|k) &= E\{\tilde{x}_{i_j}(k|k)\tilde{x}_{i_j}^T(k|k)\} \\
&= E\{[x(k) - \hat{x}_{i_j}(k|k)][x(k) - \hat{x}_{i_j}(k|k)]^T\} \\
&= E\{[(I - K_{i_j}(k)C_{i_j}(k))\tilde{x}_{i_{j-1}}(k|k) - K_{i_j}(k)v_{i_j}(k)] \\
&\quad \cdot [(I - K_{i_j}(k)C_{i_j}(k))\tilde{x}_{i_{j-1}}(k|k) - K_{i_j}(k)v_{i_j}(k)]^T\} \\
&= [I - K_{i_j}(k)C_{i_j}(k)]P_{i_{j-1}}(k|k)[I - K_{i_j}(k)C_{i_j}(k)]^T \\
&\quad + K_{i_j}(k)R_{i_j}(k)K_{i_j}^T(k) - [I - K_{i_j}(k)C_{i_j}(k)]\Delta_{i_{j-1}}(k)K_{i_j}^T(k) \\
&\quad - K_{i_j}(k)\Delta_{i_{j-1}}^T(k)[I - K_{i_j}(k)C_{i_j}(k)]^T \\
&= P_{i_{j-1}}(k|k) - K_{i_j}(k)[C_{i_j}(k)P_{i_{j-1}}(k|k) + \Delta_{i_{j-1}}^T(k)
\end{aligned} \tag{25}$$

where Eq. (24) is used.

Combine (22), (23), (24), and (25), we obtain

$$\hat{x}_{i_j}(k|k) = \hat{x}_{i_{j-1}}(k|k) + K_{i_j}(k)[z_{i_j}(k) - C_{i_j}(k)\hat{x}_{i_{j-1}}(k|k)] \tag{26}$$

$$P_{i_j}(k|k) = P_{i_{j-1}}(k|k) - K_{i_j}(k) \cdot [C_{i_j}(k)P_{i_{j-1}}(k|k) + \Delta_{i_{j-1}}^T(k)] \tag{27}$$

$$K_{i_j}(k) = [P_{i_{j-1}}(k|k)C_{i_j}^T(k) + \Delta_{i_{j-1}}(k)] \cdot [C_{i_j}(k)P_{i_{j-1}}(k|k)C_{i_j}^T(k) + R_{i_j}(k) + C_{i_j}(k)\Delta_{i_{j-1}}(k) + \Delta_{i_{j-1}}^T(k)C_{i_j}^T(k)]^{-1} \tag{28}$$

$$\Delta_{i_j}(k) = \prod_{u=j}^{1}[I - K_{i_u}(k)C_{i_u}(k_{i_u})]S_{i_{j+1}}(k) - K_{i_j}(k)R_{i_j,i_{j+1}}(k) - \sum_{q=2}^{j}\prod_{u=j}^{q}[I - K_{i_u}(k)C_{i_u}(k_{i_u})]K_{i_{q-1}}(k)R_{i_{q-1},i_{j+1}}(k) \tag{29}$$

Let $\hat{x}_s(k|k) = \hat{x}_{i_p}(k|k)$ and $P_s(k|k) = P_{i_p}(k|k)$, then we obtain the state estimation of sequential fusion $\hat{x}_s(k|k)$ and $P_s(k|k)$, and the proof is completed.

4 Simulation

To illustrate the effectiveness of the proposed algorithm, a numerical example is provided in this section.

A target is observed by three sensors, which could be described by Eqs. (1) and (2). Sensor 1 has the highest sampling rate S_1, and the sampling rates of sensor 2 and sensor 3 are S_2 and S_3, respectively, which meet $S_1 = 2S_2 = 3S_3$. And

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, C_1 = [\,1 \quad 0\,], C_2 = [\,1 \quad 0\,], C_3 = [\,0 \quad 1\,]$$

Sensor 1 and sensor 2 observe the position, and sensor 3 observes the velocity.

At time k, the correlations of measurement noises covariance are given by

$$R_1(k) = cov(v_1(k)) = 0.048, R_2(k) = cov(v_2(k)) = 0.064$$
$$R_3(k) = cov(v_3(k)) = 0.064, R_{12}(k) = E[v_1(k)v_2^T(k)] = 0.032$$
$$R_{13}(k) = E[v_1(k)v_3^T(k)] = 0.016, R_{23}(k) = E[v_2(k)v_3^T(k)] = 0.016$$

So, the measurement noises covariance is

$$R(k) = \begin{bmatrix} 0.048 & 0.032 & 0.016 \\ 0.032 & 0.064 & 0.016 \\ 0.016 & 0.016 & 0.064 \end{bmatrix}.$$

and $Q(k) = cov(w_k) = \begin{bmatrix} 0.02 & 0.01 \\ 0.01 & 0.04 \end{bmatrix}$. The covariances between the system noise and the measurement noises are given by

$$S_1(k) = E\{w(k-1)v_1^T(k)\} = \begin{bmatrix} 0.0050 \\ 0.0025 \end{bmatrix}$$

$$S_2(k) = E\{w(k-1)v_2^T(k)\} = \begin{bmatrix} 0.0050 \\ 0.0025 \end{bmatrix}$$

$$S_3(k) = E\{w(k-1)v_3^T(k)\} = \begin{bmatrix} 0.0025 \\ 0.0100 \end{bmatrix}$$

To derive $\hat{x}_s(k|k)$, in the measurement update step, if k could be divided by 2 but could not be divided by 3, then we will use the observations of sensor 1 and sensor 2. Similarly, if k could be divided by 3 but could not be divided by 2, then the observations of sensor 1 and sensor 3 should be fused to generate the estimate of $x(k)$. However, if k could be divided by both 2 and 3, that is, k is a multiple of 6, the observations of sensor 1, sensor 2, and sensor 3 should be used. Otherwise, we only use the observations of sensor 1.

The initial conditions are $x_0 = \begin{bmatrix} 10 \\ 0.1 \end{bmatrix}$, $P_0 = 3 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

The Monte Carlo simulation results are shown in Figs. 1, 2, 3, and 4. Using only the measurements of sensor 1, "KF" denotes the Kalman filtering when the sensor noises are cross-correlated and are coupled with the previous step system noise, and "NKF" denotes Kalman filtering when the noises are all independent of each other. Using the measurements of all three sensors, "SFKF" denotes the algorithm given in Theorem 1, and "NSFKF" denotes the sequential fusion algorithm when the noises are treated as independent.

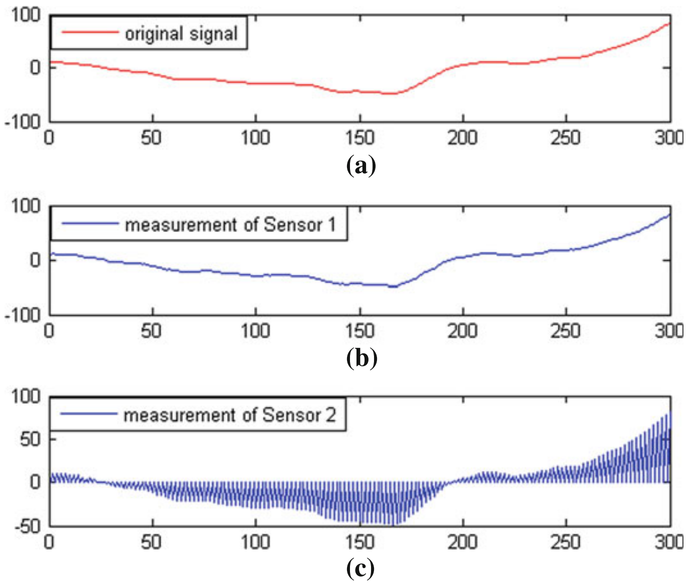


Fig. 1 Position and measurements of sensor 1 and 2

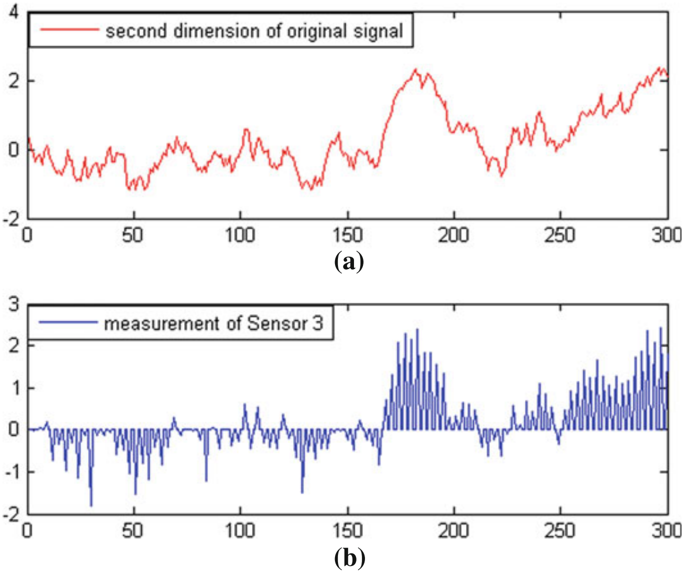


Fig. 2 Velocity and measurements of sensor 3

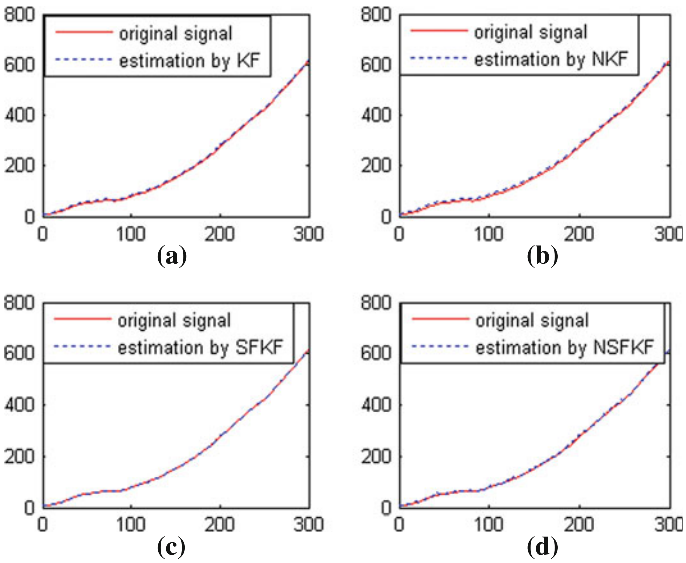
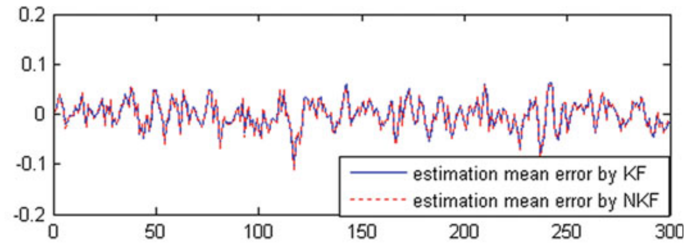
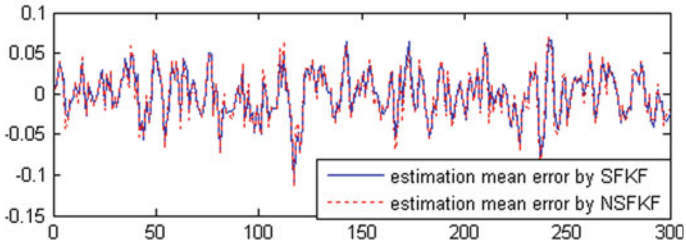


Fig. 3 Position estimations

In Fig. 1, from (a) to (c) are the first dimension of the original signal, the measurement of sensor 1 and the measurements of sensor 2. From the figure, we can see that sensor 2 only has measurements in the even number points and in odd



(a)



(b)

Fig. 4 The statistical position estimation errors of 100 simulations

number points the measurements are zero. In Fig. 2, from (a) to (b) are the second dimension of the original signal and the measurements of sensor 3. Sensor 3 has measurements only when the sampling points are multiple of 3. It can be seen that the measurements are corrupted by noises.

Figure 3 shows the state estimations of the position dimension. From Fig. 3 (a) through (d) are the estimations generated by use of KF, NKF, SFKF, and NSFKF, respectively. For comparison, the estimations are shown in blue dotted line, while the original signal is shown in the red real line. From this figure, it can be seen that all algorithms generate good estimations, whereas the presented algorithm (SFKF) shows the best performance.

In Fig. 4, the statistical estimation errors of 100 simulation runs are shown, where, the position estimation errors of KF and NKF are shown in (a) by the lines of real-blue and dotted-red, respectively. It can be seen from the figure that the errors of KF are slightly less than that of NKF. And (b) shows the position estimation errors of SFKF and NSFKF in blue real lines and red dotted lines, respectively. Also, we can see that the errors of SFKF are less than that of NSFKF.

Briefly, the simulation results in this section illustrate the effectiveness of the presented algorithm.

5 Conclusion

When the sampling rates of different sensors are different and when the measurement noises are cross-correlated and are also coupled with the system noise of the previous step, by use of the projection theory and induction hypothesis repeatedly, a sequential fusion algorithm is generated. The algorithm is proven to be optimal in the sense of Linear Minimum Mean Square Error (LMMSE) mathematically and is applicable to more general cases compared to the existed algorithms.

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