

Neutrality in Bipolar Structures

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Abstract In this paper, we want to stress that bipolar knowledge representation naturally allows a family of middle states which define as a consequence different kinds of bipolar structures. These bipolar structures are deeply related to the three types of bipolarity introduced by Dubois and Prade, but our approach offers a systematic explanation of how such bipolar structures appear and can be identified.

Keywords Bipolarity · Compatibility · Conflict · Ignorance · Imprecision · Indeterminacy · Neutrality · Symmetry

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1 Introduction

Dubois and Prade [8–10] distinguished between three types of bipolarity, named Type I, Type II, and Type III bipolarity, but in our opinion a unified approach for such a classification is missing.

Our approach in this paper focuses on how such bipolar models are being built in knowledge representation. Our main point is that bipolarity appears whenever two opposite arguments are taken into account, being the key issue how intermediate *neutral* stages are being generated from such an opposition. Such intermediate stages can be introduced from different circumstances and generate a different relationship with the basic two opposite arguments.

As pointed out in [17], different roles should be associated to different structures, and different structures justify different concepts. In particular, we claim that a semantic approach to bipolarity will allow different *neutral* stages between the two extremes under consideration, depending on the nature of these two extreme values. In this way, concepts such as *imprecision*, *indeterminacy*, *compatibility*, and *conflict* between poles will be distinguished. These *neutral* stages are somehow frequently understood as different forms of a heterogeneous *ignorance*. For example, ignorance—lack of information—is sometimes confused with symmetry in decision making—difficulties to choose between two poles despite clear available information. The lack of information can be based on simple imprecision or a deeper conceptual problem, whenever the two considered poles are not enough to fully explain reality. Similarly, a decision maker cannot be able to choose a unique pole when both poles simultaneously hold or when a conflict is being detected (random decision can be acceptable in the first case but not in the second place, where each pole can be rejected because of different arguments).

We should remind that Dubois and Prade [8] classify bipolarity in terms of the nature of the scales that are used and the relation between positive and negative information, differentiating as a consequence two types of bipolar scales: *univariate bipolar* and *bivariate unipolar* scales. Univariate bipolarity was associated in [8] and [14] to a linearly ordered set L in which the two ends are occupied by the poles $+$ and $-$, and certain *middle* value 0 might separate positive evaluations from the negative evaluations. An object is evaluated by means of a single value on L . On the other hand, bivariate unipolar scales admit in [8] positive and negative information to be measured separately by means of two unipolar scales, each one being occupied by a pole and a *neutral* state 0 that somehow appears in between both scales, but not as a middle value as in type I bipolarity. An object can receive two evaluations, which can be perceived as neither positive nor negative, as well as both positive and negative. This is the way type II and type III bipolarities are introduced in [8].

In the following sections, we shall offer an explanation to all those neutral stages between poles which will allow a systematic characterization of each different kind of bipolarity. In fact, we will see that the nature of the middle stage can be used to identify which bipolarity we are dealing with, although it should be

acknowledged that complex problems cannot be explained by means of a unique bipolar structure: different bipolarities will simultaneously appear in practice, showing that several semantics might coexist. Our approach will lead to the same differentiation proposed by Dubois and Prade, but giving a key role to the underlying structure-building process generated from the opposite poles and their associated semantic, that will produce in each circumstance a specific neutral middle stage.

2 Building Type I Bipolar Fuzzy Sets

Type I bipolarity assumes a scale that shows tension between a pole and its own negation. It suggests gradualness within a linear scale, and the middle intermediate value simply represents a scale *point of symmetry* (not that both poles hold). This linear structure is the characteristic property of type I bipolarity, and a single value in this scale simultaneously gives the distance to each pole. But such a symmetry value cannot be confused with compatibility or ignorance; it does not properly represent a new concept.

For example, meanwhile “tall” and “short” are viewed as two opposite grades of a unique “tallness” concept, such a “tallness” is being modeled as a type I bipolarity. In this case, we neither estimate “tallness” or “shortness”, but “height”.

Neutrality within two type I bipolar poles can more properly be associated to the unavoidable imprecision problem (uncertainty about the right value). *Imprecision* represents a very particular kind of *ignorance*, indeed an epistemic state different than both poles. When a decision maker has no information about the exact value, imprecision is maximum and the more information we get the more accurate we are.

Imprecision is the characteristic neutral state in type I bipolarity.

3 Building Type II Bipolar Fuzzy Sets

On the contrary, type II bipolarity requires the existence of two dual (perhaps antagonistic) concepts, related because they refer to a common concept but containing different information. Distance to one pole cannot be deduced from the distance to the other pole, so two separate evaluations are needed for each pole, although they share a common nature. Poles are not viewed as the extreme values of a unique gradualness scale like in type I bipolarity. Depending on the nature of such a duality, we may find different neutral intermediate states with differentiated meaning or semantics.

For example, two opposite concepts not necessarily cover the whole universe of discourse (as shown in [2] within a classification framework when compared to [19]). An object can neither fulfill a concept nor its opposite, a situation that by

definition cannot happen within type I bipolarity, where negation connects both poles. If classification poles are too distant, this situation suggests the need of some additional classification concept [1], and meanwhile we do not find such an additional concept, we have to declare an *indeterminacy* that is another kind of neutral *ignorance*, different in nature to previous *imprecision* (see [7]). This is the case when the dual of the concept is strictly contained in its negation (for example, “very short” is strictly contained in “not tall”). Poles do not cover the whole space of possibilities. There exists a region where none of both poles hold.

Alternatively, negation can be implied by its dual concept, in such a way that both poles overlap (as “not very tall” overlaps “not very short”, see [6]). In this case, type II bipolarity generates a different specific intermediate concept that cannot be associated to *indeterminacy*, but to a different kind of *neutrality*. We are talking about a simultaneous verification of both poles which is not a point of symmetry. In type I bipolarity, symmetry refers to a situation *in between* poles; meanwhile, in this type II bipolarity, *compatibility* means that both poles simultaneously hold.

In this way, we find that type II bipolarity may generate two different neutral intermediate concepts, depending on the semantic relation between poles. While in type I bipolarity one of the poles is precisely the negation of the other, in type II bipolarity, two alternative type II bipolarities may appear (but they will not simultaneously appear).

4 Building Type III Bipolar Fuzzy Sets

Type III bipolarity refers to negative and positive pieces of information (see [11]), implying the existence of two families of arguments, that should be somehow aggregated. Poles here represent like bags of arguments. Poles are not direct arguments like in type I and type II bipolarity.

Type III bipolarity suggests a different construction than the one used for type I or type II bipolarity. In such type I or type II bipolarity, we start from the opposition between two extreme values within a single characteristic or between two dual poles, but in both cases assuming one single common concept. A concept generates its negation or some kind of dual concept, and their description may need one single value or two values, which can be directly estimated.

Type III bipolarity appears like a second degree bipolar type II structure, being both poles complex concepts like *positive-negative* or *good-bad*, for example, each pole needing a specific description or decomposition to be understood.

Type III bipolarity is essentially more complex than type II bipolarity, for example when we are asked to list on one side “positive” arguments and “negative” arguments on the other side.

This is the standard situation in multi-criteria decision making. It is in this context where *conflict* can naturally appear as another neutrality stage, besides *imprecision*, *compatibility*, and *indeterminacy*. In a complex problem, of course we can find at the same time strong arguments supporting both poles (see, e.g., [18]).

Such a *conflict* stage is natural within type III bipolarity, but of course it may happen that those arguments as a whole do not suggest a complete description of reality, suggesting *indeterminacy*, similarly to type II *indeterminacy*. Notice that in type II bipolarity *compatibility* can appear, but *conflict* should not be expected when dealing with a pair of extreme values coming from simple (1-dimensional) argument. Such a *conflictive* argument belongs to the type III bipolarity context.

It is worth noticing that meanwhile type I and type II bipolarity are built up from symmetrical poles, type III bipolarity is built from asymmetrical poles (see [8, 9]).

The key issue is the different semantic relation between poles. A different semantic relation produces a different intermediate *neutral* state.

Finally, it must be pointed out that in this third more complex problem all previous situations might be implied. Underlying criteria can define a conflict, but they can also define *indeterminacy* situation, resembling the type II *indeterminacy* as already pointed out (too poor descriptions suggesting *ignorance*), and in addition, each underlying criteria is subject to a symmetrical bipolarity framework, allowing *imprecision* or *compatibility* (besides type II *indeterminacy*). For example, the semantic relation between “very good” and “very bad” is the same relation as “very tall” and “very short”, allowing when something is neither “very good” or “very bad” a similar *indeterminacy* to the one that appears when something is neither “very tall” or “very short”.

But *conflict* is essentially a type III bipolarity performance.

5 Building General Bipolarities

From the above comments it is clear that some bipolar problems require a quite complex structure.

Type I bipolarity use to imply the existence of an *imprecision* state besides a possible *point of symmetry*.

Type II bipolar implies two potential different intermediate states (*indeterminacy* and *compatibility*).

Type III bipolarity implies the possibility of a *conflict* stage, but previous intermediate stages can also appear associated to each underlying criteria, if not directly.

A general bipolarity model should allow all those four semantic neutral states between poles beside the non semantic *points of symmetry* that may appear within type I bipolarity. In principle, a general bipolar representation should be prepared to simultaneously deal with and evaluate all these states.

The semantic argument is anyway needed to distinguish between different bipolarities, and to produce structured type-2 fuzzy sets [16], as proposed in [17].

It is also interesting to realize that this semantic approach allows an alternative explanation to Atanassov’s intuitionistic fuzzy sets (see [3–5]), realizing how different examples given by this author can be explained by means of different semantics and therefore different bipolarities.

6 Conclusion

When poles are defined within a binary context, in terms of a single concept and its *negation* (e.g., “tall” and “not tall”), a linear order of gradation states is allowed around the *point of symmetry*, simply meaning equal distance to both concepts within a linear order, like and in Probability Theory [15] or Fuzzy Sets [20] (see also [12]). An object is associated *to some extent* to both poles by means of a unique family of intermediate *gradation* states. This is the framework for type I bipolarity.

In type II bipolarity, we have dual concepts as poles. These two dual poles can overlap (like “more or less tall” and “more or less short”) or they can create a region that is far away from poles (like “very tall” and “very short”). Anyway, opposite poles are not complementary, so two different situations can be generated. If a pole is much smaller than the negation of the opposite pole, *indeterminacy* will be natural. If a pole is much bigger than the negation of the opposite pole, *compatibility* will naturally appear.

Type III bipolarity is the natural framework for multicriteria decision making (see [13]). Poles are complex concepts and need a multicriteria description, and of course different criteria can produce a *conflict*. But also each one of the other situations can appear within each simple criterion. In addition, such a multicriteria description can be so poor that a strict *ignorance* can appear.

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