Probabilistic Composite Rough Set and Attribute Reduction

Hongmei Chen, Tianrui Li, Junbo Zhang, Chuan Luo and Xinjiang Li

Abstract Composite rough set aims to deal with multiple binary relations simultaneously in an information system. In this paper, probabilistic composite rough set is presented by introducing the probabilistic method to composite rough set. Then, the distribution attribute reduction method under probabilistic composite rough set is investigated. Examples are given to illustrate the method.

1 Introduction

Rough Set Theory (RST) proposed by Pawlak is an important theory to deal with inconsistent and uncertain information [1, 2]. In Traditional Rough Set (TRS), the information system is complete, the data type in the information system is nominal and the relationship between objects is an equivalence relation. However, in real-

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H. Chen (✉) · T. Li · J. Zhang · C. Luo · X. Li
School of Information Science and Technology, Southwest Jiaotong University,
610031 Chengdu, China
e-mail: trli@swjtu.edu.cn

T. Li
e-mail: hmchen@swjtu.edu.cn

J. Zhang
e-mail: JunboZhang86@163.com

C. Luo
e-mail: luochuan@my.swjtu.edu.cn

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life applications, data missing may exist, data type may be various, and preference order in attributes' values may exist. Then, TRS has been extended to deal with different data types, data missing, and preference ordered data in Extended Rough Set (ERS) by replacing the equivalence relation with other binary relations [3–11]. Different binary relations, e.g., tolerance relations, partial order relations in interval valued information system, set-valued information system, and hybrid data information system have been defined in different conditions. In most ERS, there is a supposition that there exists only one data type and one relationship in an information system.

Some works have been done considering multiple relations or hybrid data types. When considering hybrid data types (nominal data and numerical data) in an information system, Hu et al. proposed Neighborhood Rough Set (NRS) to deal with homogeneous feature selection [12, 13]. Neighborhood relation and nearest neighborhood relation have been defined in NRS. Wei et al. studied the hybrid data in the framework of fuzzy rough set [14]. When considering multiple binary relations may exist in an information system, An and Tong proposed global binary relation and they further defined the approximation on upper and down union [15]. Abu-Donia proposed new types of rough set approximations using multi knowledge base, that is, a family of finite number of (reflexive, tolerance, dominance, equivalence) relations by two ways [16]. Zhang et al. defined composite relation and Composite Rough Set (CRS) model in [17]. Then, they investigated matrix-based rough set approach in CRS. However, the attributes reductions in an information system with multiple relations have not been investigated in the literatures.

Probabilistic rough set is an important extension of TRS by considering conditional probability between equivalence classes in TRS [18]. Yao and Wong proposed a Decision-Theoretic Rough Set model (DTRS) based on well established Bayesian decision procedure [21]. The parameters in DTRS are decided by loss function. Ziarko proposed Variable Precision Rough Set (VPRS) to deal with the errors in data [19, 20]. Slezak and Ziarko investigated Bayesian Rough Set (BRS) when considering the prior probability of an event [22]. Probabilistic rough set have been successfully applied to data mining [23].

In this paper, we investigate the extended model of CRS by introducing probabilistic method. Then, Probabilistic Composite Rough Set (PCRS) is proposed. Considering the arbitrary binary relation existing in PCRS, distribute attribute reduction in PCRS is discussed. Examples are given to illustrate the method presented in the paper.

The paper is organized as follows. In Sect. 1 , we review the basic concepts of CRS. In Sect. 2, PCRS is defined and its properties are discussed. In Sect. 3, we study the distribute reduction method in PCRS. In Sect. 4, we conclude the paper and outline the direction of future work.

Table 1 An Multi-data type decision information system

U	a_1	a_2	a_3	a_4	d
x_1	2	0.2	[2.17, 2.86]	{0, 1}	1
x_2	4	0.85	[3.37, 4.75]	{2}	2
x_3	3	0.31	[2.56, 4.10]	{1, 2}	0
x_4	1	0.74	[3.55, 5.45]	{1}	1
x_5	1	0.82	[3.46, 5.35]	{1}	0
x_6	2	0.72	[2.29, 3.43]	{1}	1
x_7	1	0.6	[2.22, 3.07]	{0, 2}	1
x_8	3	0.44	[2.51, 4.04]	{1, 2}	0

2 Composite Rough Set

In this section, we first review some concepts in CRS [17].

The definition of Multi-Data Type Decision Information system ($MDTDIS$) is given as follows:

Definition 21 A quadruple $MDTDIS = (U, A, V, f)$ is *an decision information system,* where U is a non-empty finite set of objects, called the universe. A is a non-empty finite set of attributes, $A = C \cup D, C \cap D = \emptyset$, where C and D denote the sets of condition attributes and decision attributes, respectively. $V = \bigcup_{C_i \in C} V_{C_i}$, V_{C_i} is domain of attributes set C_i, $V_{C_i} \cap V_{C_j} = \emptyset (i \neq j)$, $C = \cup C_i$, $C_i \cap C_j = \emptyset$. The data type of V_{C_i} and V_{C_j} is different. $f : U \times A \rightarrow V$ is an information function, which gives values to every object on each attribute, namely, $\forall a \in A$, $x \in U$, $f(x, a) \in V_a$.

Definition 22 $MDTDIS = (U, A, V, f)$ is an decision information system, $\forall x \in U$, $B = \cup B_k \subseteq C$, $B_k \subseteq C_k$, the composite relation CR_B is defined as

$$CR_B = \{(x, y) \Big| (x, y) \in \bigcap_{B_k \in B} R_{B_k}\} \tag{1}$$

where $R_{B_k} \subseteq U \times U$ is an binary relation defined by an attribute set B_k on U. Let $[x]_{CR_B} = \{y | y \in U, \forall B_k \in B, y R_{B_k} x\}$ denotes the composite class of x.

Definition 23 Let $M_{R_{B_i}} = [r_{ij}^{B_i}]_{n \times n} (B_i \subseteq C)$ is the relation matrix of binary relation R_{B_i}, where

$$r_{ij}^{B_i} = \begin{cases} r_{ij}^{B_i} = 1, & if\ x_i\ R_{B_i} x_j; \\ r_{ij}^{B_i} = 0, & if\ x_i\ \not R_{B_i} x_j. \end{cases} \tag{2}$$

Then, the relation matrix of composite relation CR_B is $M_{CR_B} = [z_{ij}]_{n \times n}$, where

$$z_{ij} = \bigwedge_{i=1}^{k} r_{ij}^{B_i}. \tag{3}$$

For convenience, three kinds of binary relations used in Example 21, e.g., neighborhood relation between numerical objects, partial relation between interval valued objects, and tolerance relation in between set-valued objects are introduced briefly as follows [6, 7, 11].

If $\forall v \in V_B$ $(B \subseteq C)$, v is set-valued, then the tolerance relation between set-valued objects is

$$R_B^{\cap} = \{(y,x) \in U \times U | f(y,a) \cap f(x,a) \neq \emptyset (\forall a \in B)\}$$

If $\forall v \in V_B$, v is a set of interval numbers, $f(x,a) = [f^L(x,a), f^U(x,a)]$, where $f^L(x,a)$, $f^U(x,a) \in R$, $f^L(x,a)$, and $f^U(x,a)$ are lower and upper limits of the interval number, respectively. Then the partial order relation between interval valued objects is

$$\begin{aligned} R_{I_B}^{\geq} = \{(y,x) \in U \times U \big| f^L(y,a) \geq f^L(x,a), \\ f^U(y,a) \geq f^U(x,a), f^L(y,b) \leq f^L(x,b), \\ f^U(y,b) \leq f^U(x,b), a \in A_1, b \in A_2\} \end{aligned} \tag{4}$$

If $\forall v \in V_B$ $(B \subseteq C)$, v is a numerical value, then $\forall x \in U$ and $B \subseteq C$, the neighborhood $\delta_B(x)$ of x in B is defined as:

$$\delta_B(x) = \{y \big| y \in U, \Delta^B(x,y) \leq \delta\} \tag{5}$$

where Δ is a distance function. The formula of Δ is

$$\Delta_P(x,y) = \left(\sum_{i=1}^{N} |f(x,a_i) - f(y,a_i)|^P \right)^{1/P} \tag{6}$$

In the following, we illustrate CRS by an example.

Example 21 Table 1 is an example of *MDTDIS*, where $U = \{x_i, 1 \leq i \leq 8\}$, $C = \{a_i, 1 \leq i \leq 4\}$, $D = \{d\}$, $V_{a_1} = \{1,2,3,4\}$ is nominal values, $V_{a_2} = \{0.2, 0.85, 0.31, 0.74, 0.82, 0.72, 0.6, 0.44\}$ is numerical values, $V_{a_3} = \{[2.17, 2.86], \cdots, [2.51, 4.04]\}$ is interval values, $V_{a_4} = \{\{0,1\}, \cdots, \{1,2\}\}$ is se-t valued. Suppose the relationship in different attributes are equivalence relation, neighborhood relation (δ= 0.26 , Euclidean distance) [11], partial relation [7], tolerance relation [6] on a_1 to a_6, respectively.

The relation matrixes of $R_{a_i} (1 \leq i \leq 4)$ are given as follows:

$$M_{R_{a_1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad M_{R_{a_2}} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

$$M_{R_{a_3}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad M_{R_{a_4}} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Then, $M_{CR_C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

Then, $[x_1]_{CR_C} = \{x_1\}, [x_2]_{CR_C} = \{x_2\}, [x_3]_{CR_C} = \{x_3\}, [x_4]_{CR_C} = \{x_4\}, [x_5]_{CR_C} = \{x_4, x_5\}, [x_6]_{CR_C} = \{x_6\}, [x_7]_{CR_C} = \{x_7\}, [x_8]_{CR_C} = \{x_3, x_8\}$.

3 Probabilistic Composite Rough Set

In this section, we present a new extended rough set model, Probabilistic Composite Rough Set (PCRS), by introducing probability method in CRS.

$MDTDIS = (U, A, V, f)$ is an Multi-Data Decision Information system, $CR_B(x)$ is a composite class of x, for $\forall X \in U$, then $P(X, [x]_{CR_B}) = \frac{|X \cap [x]_{CR_B}|}{|[x]_{CR_B}|}$ is the conditional probability. Probabilistic Composite Rough Set (PCRS) is defined as follows:

Definition 31 Given a quadruple $MDTDIS = (U, A, V, f)$, $\forall X \in U$, $0 \le \mu \le l \le 1$,

$$\underline{apr}^{l}_{CR_B}(X) = \left\{x | P(X, [x]_{CR_B}) \ge l\right\} \tag{7}$$

$$\overline{apr}^{\mu}_{CR_B}(X) = \left\{x | P(X, [x]_{CR_B}) \ge \mu\right\} \tag{8}$$

Definition 32 For any $X \subseteq U$, the approximation accuracy of X is defined as follows:

$$\alpha^{l,\mu}_{CR_B}(X) = \frac{\left|\underline{apr}^{l}_{CR_B}(X)\right|}{\left|\overline{apr}^{\mu}_{CR_B}(X)\right|} \tag{9}$$

The roughness of X is defined below:

$$\rho^{l,\mu}_{CR_B}(X) = 1 - \alpha^{l,\mu}_{CR_B}(X) \tag{10}$$

Property 31 For $\underline{apr}^{l}_{CR_B}(X)$ and $\overline{apr}^{\mu}_{CR_B}(X)$, we have

1. If $l_1 \le l_2$, then $\underline{apr}^{l_1}_{CR_B}(X) \supseteq \underline{apr}^{l_2}_{CR_B}(X)$;
2. If $\mu_1 \le \mu_2$, then $\overline{apr}^{\mu_1}_{CR_B}(X) \supseteq \overline{apr}^{\mu_2}_{CR_B}(X)$;
3. If $l_1 \le l_2$, $\mu_1 \ge \mu_2$, then $\alpha^{l_1,\mu_1}_{CR_B}(X) \ge \alpha^{l_2,\mu_2}_{CR_B}(X)$,
4. If $l_1 \ge l_2$, $\mu_1 \le \mu_2$, then $\alpha^{l_1,\mu_1}_{CR_B}(X) \le \alpha^{l_2,\mu_2}_{CR_B}(X)$, $\rho^{l_1,\mu_1}_{CR_B}(X) \ge \rho^{l_2,\mu_2}_{CR_B}(X)$;

Example 31 For Table 1, R_d is an equivalence relation on decision attributes. $U/R_d = \{D_1, D_2, D_3\}, D_1 = \{x_1, x_4, x_6, x_7\}, D_2 = \{x_2\}, D_3 = \{x_3, x_5, x_8\}$.*Then*, $P(D_1, [x_1]_{CR_B}) = 1, P(D_1, [x_2]_{CR_B}) = 0, P(D_1, [x_3]_{CR_B}) = 0, P(D_1, [x_4]_{CR_B}) = 1,$ $P(D_1, [x_5]_{CR_B}) = 0.5, P(D_1, [x_6]_{CR_B}) = 1, P(D_1, [x_7]_{CR_B}) = 1, P(D_1, [x_8]_{CR_B}) = 0.$

If $l = 0.6$, $\mu = 0.3$, then $\underline{apr}^{l}_{CR_B}(D_1) = \{x_1, x_4, x_6, x_7\}$, $\overline{apr}^{\mu}_{CR_B}(D_1) = \{x_1, x_4, x_5, x_6, x_7\}$, $\alpha^{l,\mu}_{CR_B}(D_1) = 0.8$, $\rho^{l,\mu}_{CR_B}(D_1) = 0.2$.

4 Attribute Reduction in Probabilistic Composite Rough Set

Attribute reduction is an important task in data mining. RST has been applied successfully in attribute reduction [24]. In this section, distribution reduction method in [25] is extended to CRS. R_D is an equivalence relation on the decision attribute D. Then ${}^{U}/_{R_d} = \{D_1, D_2, \ldots, D_j, \ldots, D_r\}$ forms a partition on the universe U. $[x]_{R_D} = \{y | yR_D x, x, y \in U\}$. If $[x]_{CR_B} \subseteq [x]_{R_D}$, then *MDTDIS* is a consistent information system; If $[x]_{CR_B} \not\subset [x]_{R_D}$, then *MDTDIS* is an inconsistent information system.

Let $P(D_j, [x]_{CR_B}) = \frac{|D_j \cap [x]_{CR_B}|}{|[x]_{CR_B}|}$ be the conditional probability of $[x]_{CR_B}$ on D. Let $\mu_B(x) = (P(D_1, [x]_{CR_B}), P(D_2, [x]_{CR_B}), \ldots, P(D_r, [x]_{CR_B}))$ denote the distribution function of x on B. Then the distribution reduction on *MDTDIS* is given.

Definition 41 In $MDTDIS = (U, C \cup D, V, f)$, $B \subseteq C$, if $\forall x \in U$, $\mu_B(x) = \mu_C(x)$, and $\neg\exists E \subset B$, $\mu_E(x) = \mu_C(x)$, then B is the distribution reduction of C.

Definition 42 The discernibility matrix of distribution reduction $M_{Dis} = [m_{ij}]_{n \times n}$ is

$$m_{ij} = \begin{cases} DisAtr_{ij}\Big([x_i]_{CR_B}, [x_j]_{CR_B}\Big) \in D^* \\ C\Big([x_i]_{CR_B}, [x_j]_{CR_B}\Big) \notin D^* \end{cases} \tag{11}$$

where $D^* = \Big\{([x_i]_{CR_B}, [x_j]_{CR_B}) : \mu_C(x_i) \neq \mu_C(x_j)\Big\}$, $DisAtr_{ij} = a_k$, $a_k \in B_i : x_i R_{B_i} x_j$.

Definition 43 The discernibility formula of distribution reduction is

$$M = \wedge\{\vee\{a_k : a_k \in DisAtrij\}\}(i, j \leq n) \tag{12}$$

The minimum disjunction form is

$$M_{min} = \bigvee_{k=1}^{p} (\bigwedge_{i=1}^{q_k} a_i) \tag{13}$$

Let $B_k = \{a_s : s = 1, 2, \cdots, q_k\}$. Then $RED = \{B_k : k = 1, 2, \cdots, p\}$ is the set of distribution reduction.

Example 41 (Continuation of Examples 2.1 and 3.1) We compute the reducts of the information system.

1. Firstly, the distribution function of $x_i (1 \leq i \leq 8)$ is $\mu_C(x_1) = \{1, 0, 0\}$, $\mu_C(x_2) = \{0, 1, 0\}$, $\mu_C(x_3) = \{0, 0, 1\}$, $\mu_C(x_4) = \{1, 0, 0\}$, $\mu_C(x_5) = \{0.5, 0, 0.5\}$, $\mu_C(x_6) = \{1, 0, 0\}$, $\mu_C(x_7) = \{1, 0, 0\}$, $\mu_C(x_8) = \{0, 0, 1\}$.
2. Next, by Definition 4.3, the discernibility matrix M_{Dis} is
3. Then, $M = \{a_1\} \wedge \{a_3\}$. $M_{min} = \{a_1, a_3\}$.
4. There is a reduct for the information system, i.e., $B_1 = \{a_1, a_3\}$.

$$M_{Dis} = \begin{bmatrix} C & \{a1,a2,a4\} & \{a1\} & C & \{a1,a2\} & C & C & \{a1\} \\ \{a1,a2,a3,a4\} & C & \{a2,a3\} & \{a1,a4\} & \{a1,a4\} & \{a1,a3,a4\} & \{a1,a3\} & \{a1,a3\} \\ \{a1,a3\} & \{a1,a2\} & C & \{a1,a2\} & \{a1,a2\} & \{a1,a2,a3\} & \{a1,a2,a3\} & C \\ C & \{a1,a2,a3,a4\} & \{a1,a3\} & C & \{a3\} & C & C & \{a1,a2,a3\} \\ \{a1,a2,a3\} & \{a1,a3,a4\} & \{a1,a2,a3\} & C & C & \{a1,a3\} & \{a3,a4\} & \{a1,a2,a3\} \\ C & \{a1,a4\} & \{a1,a2\} & C & \{a1\} & C & C & \{a1,a2\} \\ C & \{a1\} & \{a1,a2\} & C & \{a1\} & C & C & \{a1\} \\ \{a1,a3,a4\} & \{a1,a2\} & C & \{a1,a2\} & \{a1,a2\} & \{a1,a2\} & \{a1,a3,a4\} & C \end{bmatrix}$$

5 Conclusions

In this paper, we proposed the PCRS which aims to deal with multiple binary relations in a *MDTDIS* in the framework of probabilistic method. Then, the distribution attribute reduct is investigated. Examples are given to illustrate the method proposed in the paper. In the future work, we will develop algorithms to verify the efficiency of the approach in real-life applications.

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