Relevance in Preference Structures

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Abstract Fuzzy preference and aversion relations allow measuring in a gradual manner the attitude of the individual regarding some pair of alternatives. Following the Preference-Aversion (P-A) model, previously introduced for identifying the subjective cognitive state for some decision situation; here, we explore a methodology for learning relevance degrees over the complete system of alternatives. In this way it is possible to identify in a quick way, the pieces of information that are more important for solving a given decision problem.

Keywords Preference structures · Preference-aversion · Relevance

1 Introduction

Preference relations express information obtained under natural conditions of subjectivity and uncertainty characteristic of human intelligence and rationality (see, e.g., [1–3]). Therefore, the decision process of an individual can be better described if the representation possibilities of the preference model take into consideration the rational capabilities of the individual.

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In this sense, the epistemic states of a decision problem can be explored and understood using the basic and general attributes of human rationality. Here, such rationality refers to the brain's capability of distinguishing between perceptions with positive (gains) or negative (losses) character (see e.g., [2, 4–6]). We do not refer only to monetary gains or losses, but in a more general way, to the positive and negative sides for each alternative when being compared with another one. Then, two *independent* evaluations for the positive and the negative perceptions are needed (as in [7]), following the general approach of the Preference-Aversion (P-A) model formally introduced in [8, 9].

In this paper, we focus on how the P-A model allows understanding the decision problem, where the general *attitude* (in the sense of [10, 11]) of the individual can be described considering *bipolar* (see [12–14], but also [15]) knowledge representation, and at the same time, allows identifying the *relevance* of each alternative, which can be measured according to the whole set of preference and aversion relations, i.e., the system of alternatives. Our main point is that relevance refers not only to alternatives that are strictly preferred, but also to alternatives that are strictly rejected, offering complementary information over the decision process of the individual and the complete system of alternatives.

2 Modeling Fuzzy Preference-Aversion Relations

Standard fuzzy preference models (see e.g. [16–19]) examine the subjective decision-making process using binary preference relations over a finite set of alternatives A. Such models explore preference relations as gradual predicates (in the sense of [20]), where some basic properties can be verified up to a certain degree.

From this standing point, the characterization of a binary fuzzy preference relation is given by (see e.g., [20, 21])

$$R(a,b) = \{\langle a, b, \mu_R(a,b)\rangle | a, b \in A\}, \tag{1}$$

where $\mu_R : A \times A \to [0,1]$ is the membership function for the fuzzy relation R, such that $\mu_R \in [0,1]$ is the membership intensity for every $(a,b) \in A \times A$, according to the verification of the property of "being at least as desired as."

Introducing an independently opposite counterpart for this preference predicate, the preference relation $R(a, b)$ can now be characterized in the following way (see e.g., [7, 15, 22, 23]),

$$R(a,b) = \{\langle a, b, \mu_R(a,b), \nu_R(a,b)\rangle | a, b \in A\}, \tag{2}$$

where $\mu_R, \nu_R : A \times A \to [0,1]$ are the membership and non-membership functions, respectively, such that $\mu_R, \nu_R \in [0,1]$ are the corresponding membership and non-membership intensities for every $(a,b) \in A \times A$, according to the verification of the opposite properties of being "at least as desired as" and "at least as rejected as," respectively.

3 The Fuzzy Preference-Aversion Model

The preference-aversion relation R is now composed by two input values or data parts, its positive part, the weak preference intensity $\mu_R(a,b)$, and its negative part, the weak aversion intensity $v_R(a,b)$. The inclusion of these two weak relations in the preference structure allows defining the basic P-A structure [8, 9],

$$R = \langle (P, I, J), (Z, G, H) \rangle \tag{3}$$

composed by six relations which are, strict preference P, indifference I, incomparability J, and strict aversion Z, negative indifference G, and incomparability on weak aversion H.

In this way, there exist six functions [8, 9],

$$p, i, j, z, g, h : [0,1]^2 \rightarrow [0,1], \tag{4}$$

such that

$$P(a,b) = p(\mu_R(a,b),\ \mu_R(b,a)), \tag{5}$$

$$I(a,b) = i(\mu_R(a,b),\ \mu_R(b,a)), \tag{6}$$

$$J(a,b) = j(\mu_R(a,b),\ \mu_R(b,a)), \tag{7}$$

$$Z(a,b) = z(v_R(a,b),\ v_R(b,a)), \tag{8}$$

$$G(a,b) = g(v_R(a,b),\ v_R(b,a)), \tag{9}$$

$$H(a,b) = h(v_R(a,b),\ v_R(b,a)). \tag{10}$$

Using standard fuzzy logic operators [24], where the valued union or disjunction can be represented by a continuous t-norm S, the valued intersection or conjunction by a continuous t-norm T, and n is a strict negation, the following system of equations holds (following [9, 16]),

$$S(p(\mu_R, \mu_{R^{-1}}),\ i(\mu_R, \mu_{R^{-1}})) = \mu_R, \tag{11}$$

$$S(p(\mu_R, \mu_{R^{-1}}),\ i(\mu_R, \mu_{R^{-1}}),\ p(\mu_{R^{-1}}, \mu_R)) = S(\mu_R, \mu_{R^{-1}}), \tag{12}$$

$$S(p(\mu_{R^{-1}}, \mu_R),\ j(\mu_R, \mu_{R^{-1}})) = n(\mu_R), \tag{13}$$

$$S(z(v_R, v_{R^{-1}}),\ g(v_R, v_{R^{-1}})) = v_R, \tag{14}$$

$$S(z(v_R, v_{R^{-1}}),\ g(v_R, v_{R^{-1}}),\ z(v_{R^{-1}}, v_R,)) = S(v_R, v_{R^{-1}}), \tag{15}$$

$$S(z(v_{R^{-1}}, v_R),\ h(v_R, v_{R^{-1}})) = n(v_R). \tag{16}$$

Therefore, the P-A model takes into account a type of rationality that frames alternatives in terms of gains and losses, assigning two different and separate values for expressing weak preference and weak aversion. In this way, different

Table 1 The complete P-A cognitive space

R_{PA}	z	z^{-1}	g	h
P	pz	pa	pg	ph
p^{-1}	pa^{-1}	pz^{-1}	pg^{-1}	ph^{-1}
I	iz	iz^{-1}	ig	ih
J	jz	jz^{-1}	jg	jh

pieces of information can be ordered according to the strength of the positive and negative sides of things.

The conjunctive meaning of the basic P-A structure can now be examined, as there are ten different situations for describing the cognitive state of the individual in the face of decision. Hence, the complete P-A structure, defined by [8, 9],

$$R_{PA} = \langle \mu_R, \nu_R \rangle = T(\langle p, i, j \rangle,\ \langle z, g, h \rangle), \tag{17}$$

represents the system of gradual situations that arise from the independent reasoning over gains and losses. Such states constitute a complete semantic space for characterizing the subjective perceptions of a *gains-losses rational* individual.

Such valuation space is represented in Table 1, and it is composed by [8, 9],

- Ambivalence: $pz = T(p, z)$,
- Strong preference: $pa = T(p, z^{-1})$,
- Pseudo-preference: $pg = T(p, g)$,
- Semi-strong preference: $ph = T(p, h)$,
- Pseudo-aversion: $iz = T(i, z)$,
- Strong indifference: $ig = T(i, g)$,
- Positive indifference: $ih = T(i, h)$,
- Semi-strong aversion: $jz = T(j, z)$,
- Negative indifference: $jg = T(j, g)$,
- Strong incomparability: $jh = T(j, h)$.

Notice the different characteristics of the epistemic states represented by each one of these relations. For example, it can be the case that a decision is seen clearer by valuing the negative aspects of alternatives, as in *strong preference* (pa), where a is better than b and b is worse than a. Or it can be the case that there exists certain uneasiness over the available options in the face of decision, like in *pseudo-preference* (pg), where a is better than b but the two alternatives are considered equally bad. But it can also be the case of *negative indifference* (jg), where the two possibilities are just as bad, i.e., full discomfort on the options is expressed in this way. Or it can also be the case of *ambivalence* (pz), where one alternative is considered at the same time better and worse, a major conflict typical of real-life decision problems.

The relation of conflict toward a decision can then be carefully examined by the different aspects of ambivalence, which rises from the joint consideration of aversion and preference. In this sense, the P-A model offers a formal methodology for representing ambivalence in a non-symmetric and expressive way (for more details see [8, 9]).

4 Relevance Degrees for Fuzzy Preference-Aversion Relations

The determination of which part of the available information is most relevant for solving some decision problem relies on our objective and knowledge of the problem at hand (following the general insights presented in [25–27]). In this context, it is necessary to take into consideration the concept of relevance for the representation of the individual's subjective perceptions and judgments. Here we propose relevance degrees over the individual's preference-aversion relations, making reference to the relative importance of valued arguments. Our objective is to obtain a balanced evaluation between the amount of available information and the existing knowledge and ignorance over the complete system of alternatives.

Therefore, relevance degrees have to take into consideration the relative importance of a given alternative. Here we make a proposal for obtaining such degrees using the independent methodology of the P-A model, where a positive order, O^+, and a negative one, O^-, are separately induced over the alternatives in A. Following [28] where the concept of dimension for a simple order is examined (not to be confused with the dimension approach used in [29]), the dimension $d[a]$ of an element $a \in A$ can be understood as the maximum length d of chains $c \prec b \prec \ldots \prec a$ in O^+ having a for greatest element, where $b \prec a$ holds if $p(\mu_R, \mu_{R^{-1}}) > \varepsilon$ holds for certain threshold $\varepsilon > 0$.

Depending on the objective of the decision maker, it may be of interest to identify not only the strict preference chain, but also the indifference or the incomparability ones. Then, we say that $d[a]$, as it has been just defined, makes reference to the preference chain $d_p[a]$, and that the dimension $d_i[a]$ or $d_j[a]$ of an element $a \in A$ can also be understood as the maximum length of chains $c \sim b \sim \ldots \sim a$ or $c \not\prec b \not\prec \ldots \not\prec a$ in O^+, respectively, having a for greatest element, where $b \sim a$ holds if $i(\mu_R, \mu_{R^{-1}}) > \varepsilon$ holds for certain threshold $\varepsilon > 0$ and $b \not\prec a$ holds if $j(\mu_R, \mu_{R^{-1}}) > \varepsilon$ holds for certain threshold $\varepsilon > 0$.

Similarly, by the aversion dimension $d_{z,g,h}[a]$ of an element $a \in A$ it can be understood the maximum length d of chains $c \lhd b \lhd \ldots \lhd a$, $c \approx b \approx \ldots \approx a$, or $c \not\approx b \not\approx \ldots \not\approx a$ in O^- having a for greatest element, where $b \lhd a$ holds if $z(\nu_R, \nu_{R^{-1}}) > \delta$ holds for certain threshold, $\delta > 0$, $b \approx a$ holds if $g(\nu_R, \nu_{R^{-1}}) > \delta$ holds for certain threshold $\delta > 0$, and $b \not\approx a$ holds if $h(\nu_R, \nu_{R^{-1}}) > \delta$ holds for certain threshold $\delta > 0$. Finally, by the dimension of O^+, $d_{p,i,j}[O^+]$, and the dimension of O^-, $d_{z,g,h}[O^-]$, it is meant the maximum length of a chain in O^+ and O^-, respectively.

Definition 1 For every alternative $a \in A$, the relevance degree of a regarding the order given by $\langle p, i, j \rangle$ is defined as,

$$\lambda_a^{p,i,j} = \frac{d_{p,i,j}[a]}{d_{p,i,j}[O^+]}, \quad (18)$$

and the relevance degree of a regarding the order given by $\langle z, g, h \rangle$ is defined as,

$$\lambda_a^{z,g,h} = \frac{d_{z,g,h}[a]}{d_{z,g,h}[O^-]}. \tag{19}$$

In this way, for a given alternative a and for every $a, b \in A \times A$, the positive relevance degree of a, $\lambda_a^{p.i.j}$, can be built by counting over how many alternatives a is, up to a certain degree greater than ε, strictly preferred, indifferent, or incomparable, while the negative one, $\lambda_a^{z,g,h}$, can be built by counting over how many alternatives a is, up to a certain degree greater than δ, strictly worse, just as bad, or incomparable on aversion attributes. Hence, these relevance degrees make use of all of the available information about some alternative a regarding all of the other alternatives in A. In this sense, such relevance degrees measure the relative importance of one alternative with respect to the complete system of alternatives.

5 Construction of Relevance Degrees Under the Preference-Aversion Model: An Example

As we have seen, relevance is a concept that deals jointly with the importance and the meaning of things. In this approach, we make a first attempt to treat the concept of relevance over a given set of alternatives, focusing on some degree for its quantification. Then, we assume that relevance refers to the pieces of information that attracts with more intensity our attention.

Taking these observations into consideration, we illustrate now the use of relevance degrees over the P-A model. For example, consider a set of alternatives $A = \{a, b, c, d\}$ and an individual with the preference intensities $(\mu_R, \mu_{R^{-1}})$ of Table 2, where, e.g., $\mu_R(a, b) = 0.70$, and the aversion intensities $(\nu_R, \nu_{R^{-1}})$ given in Table 3.

So, we have to find an overall preference-aversion order reflecting the individual's attitude toward the available alternatives in A. For all (a, b) in $A \times A$, a person is supposed to perceive preference of a over b if a is strictly preferred to b, i.e., $p(\mu_R, \mu_{R^{-1}}) > \varepsilon$ holds, and similarly to support the aversion for a over b if a is strictly worse than b, i.e., $z(\nu_R, \nu_{R^{-1}}) > \delta$ holds.

Table 2 Preference intensities

$\mu_R, \mu_{R^{-1}}$	a	b	c	d
a	1.00	0.70	0.40	0.30
b	0.30	1.00	0.60	0.70
c	0.70	0.90	1.00	0.50
d	0.70	0.90	0.50	1.00

Table 3 Aversion intensities

$\nu_R, \nu_{R^{-1}}$	a	b	c	d
a	1.00	0.80	0.60	0.40
b	0.80	1.00	0.90	0.90
c	0.40	0.10	1.00	0.00
d	0.10	0.80	0.20	1.00

Table 4 Strict preference intensities

P	a	b	c	d
a	0.00	0.40	0.00	0.00
b	0.00	0.00	0.00	0.00
c	0.30	0.30	0.00	0.00
d	0.40	0.20	0.00	0.00

Then (see e.g., [8, 9, 16]), knowing that p has a lower bound in

$$p(\mu_R, \mu_{R^{-1}}) = T^L(\mu_R, n(\mu_{R^{-1}})), \tag{20}$$

where T^L is the Lukasiewicz t-norm, and in an analogous way, z has an upper bound in

$$z(\nu_R, \nu_{R^{-1}}) = T^m(\nu_R, n(\nu_{R^{-1}})), \tag{21}$$

where T^m is the t-norm of the minimum, we obtain the strict preference intensities shown in Table 4 and the strict aversion intensities of Table 5.

Notice that we have followed the basic intuition of Cummulative Prospect Theory [4, 30], where it is argued that losses loom larger than gains in decision under uncertainty, so aversion is valued by a greater t-norm than preference.

So, taking $\varepsilon = \delta = 0.01$, we identify the maximum length of a chain in P, such that $d_p[O^+] = 2$. Hence, $\lambda_a^p = 1/2$, $\lambda_b^p = 0$, $\lambda_c^p = 1$, and $\lambda_d^p = 1$. In the same way, we can see that $d_z[O^-] = 3$, and $\lambda_a^z = 1$, $\lambda_b^z = 1$, $\lambda_c^z = 2/3$, and $\lambda_d^z = 1$.

Therefore, we can see that alternatives c and d are the most relevant ones weighing only the strict preference intensities, while c is the less relevant one regarding the strict aversion ones. Following a basic decision rule where negative or harmful aspects have to be avoided in order to reach a satisfactory outcome (see [9]), alternative c stands out because of its positive and non-negative relevance.

Applying the P-A model over the two most relevant alternatives $\{c, d\}$, using the t-norm T^m of the minimum for the construction of R_{PA}, we find that the decision situation between c and d can be described by positive indifference and

Table 5 Strict preference intensities

Z	a	b	c	d
a	0.00	0.20	0.60	0.40
b	0.20	0.00	0.90	0.20
c	0.40	0.10	0.00	0.00
d	0.10	0.10	0.20	0.00

Table 6 The P-A description of the decision problem between alternatives $\{c, d\}$

$R(c,d)$	z	z^{-1}	g	h
p	0.00	0.00	0.00	0.00
p^{-1}	0.00	0.00	0.00	0.00
i	0.00	0.20	0.00	0.50
j	0.00	0.20	0.00	0.50

strong incomparability, each one with intensities 0.5, and by pseudo-aversion and semi-strong aversion of d over c, with intensities of 0.2 (see Table 6). These results help us explain that there is a strong conflict between both alternatives (existence of strong incomparability).

Combining the Preference-Aversion model with relevance degrees, we obtain a decision support system that recommends choosing alternative c over d, given that its positive aspects are always stronger than the negative ones. In this way, by obtaining the relevance degrees and organizing the information according to the P-A model, the final results can be understood and explained, arriving to a descriptively satisfactory answer.

6 Conclusion

The P-A model is a natural framework for understanding the cognitive process behind decision making, where relevance degrees can be introduced for identifying the pieces of information that are most important in a quick and direct way. Relevance is a complex concept that needs to be addressed with more detail, following the general insights of [26, 27], under conditions of abundance of information and decision making.

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References

1. Amo A, Montero J, Biging G, Cutello V (2004) Fuzzy classification systems. Eur J Oper Res 156:495–507
2. Cacioppo J, Gardner W, Berntson G (1997) Beyond bipolar conceptualizations and measures: the case of attitudes and evaluative space. Pers Soc Psychol Rev 1:3–25
3. Montero J, Gómez D, Bustince H (2007) On the relevance of some families of fuzzy sets. Fuzzy Sets Syst 158:2429–2442
4. Kahneman D, Tversky A (1979) Prospect theory: an analysis of decision under risk. Econometrica 47:263–291
5. Keil A, Stolarova M, Moratti S, Ray W (2007) Adaptation in human visual cortex as a mechanism for rapid discrimination of aversive stimuli. NeuroImage 36:472–479

6. O'Doherty J, Kringelback M, Rolls E, Hornak J, Andrews C (2001) Abstract reward and punishment representations in the human orbitofrontal cortex. Nat Neurosci 4:95–102
7. Atanassov K (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
8. Franco C, Montero J, Rodríguez JT (2011) On partial comparability and fuzzy preference-aversion models. In: Proceedings of the ISKE conference, Shangai, paper 1448 15–17 December
9. Franco C, Montero J, Rodríguez JT (2012) A fuzzy and bipolar approach to preference modeling with application to need and desire. Fuzzy Sets Syst 214:20–34. doi:10.1016/j.fss.2012.06.006
10. Kaplan K (1972) On the ambivalence-indifference problem in attitude theory and measurement: a suggested modification of the semantic differential technique. Psychol Bull 77:361–372
11. Osgood Ch, Suci G, Tannenbaum P (1958) The measurement of meaning. University of Illinois Press, Urbana
12. Benferhat S, Dubois D, Kaci S, Prade H (2006) Bipolar possibility theory in preference modeling: representation, fusion and optimal solutions. Inform Fus 7:135–150
13. Bonnefon JF, Dubois D, Fargier H, Leblois S (2008) Qualitative heuristics for balancing the pros and the cons. Theor Decis 65:71–95
14. Dubois D, Prade H (2008) An introduction to bipolar representations of information and preference. Int J Intell Syst 23:866–877
15. Rodríguez JT, Franco C, Montero J (2011) On the relationship between bipolarity and fuzziness. In: Proceedings of the EUROFUSE conference, Régua, 193–205, September 21–23
16. Fodor J, Roubens M (1994) Fuzzy preference modelling and multicriteria decision support. Kluwer Academic, Dordrecht
17. Montero J, Tejada J, Cutello C (1997) A general model for deriving preference structures from data. Eur J Oper Res 98:98–110
18. Roubens M, Vincke Ph (1985) Preference modeling. Springer, Berlin
19. Van der Walle B, de Baets B, Kerre E (1998) Characterizable fuzzy preference structures. Ann Oper Res 80:105–136
20. Goguen J (1969) The logic of inexact concepts. Synthese 19:325–373
21. Zadeh L (1975) The concept of a linguistic variable and its application to approximate reasoning-I. Inf Sci 8:199–249
22. Franco C, Montero J (2010) Organizing information by fuzzy preference structures—Fuzzy preference semantics. In: Proceedings of the ISKE conference, Hangzhou, 135–140, 15–16 November
23. Franco C, Rodríguez JT, Montero J (2010) Information measures over intuitionistic four valued fuzzy preferences. In: Proceedings of the IEEE-WCCI, Barcelona, 18–23 July, 1971–1978
24. Scheweizer B, Sklar A (1983) Probabilistic metric spaces. North-Holland, Amsterdam
25. Keynes J (1963) A treatise on probability. MacMillan, London
26. Yager R (2007) Relevance in systems having a fuzzy-set-based semantics. Int J Intell Syst 22:385–396
27. Yager R, Petry F (2005) A framework for linguistic relevance feedback in content-based image retrieval using fuzzy logic. Inf Sci 173:337–352
28. Birkhoff G (1964) Lattice theory. American Mathematical Society, Providence
29. Gonzalez-Pachón J, Gómez D, Montero J, Yáñez J (2003) Soft dimension theory. Fuzzy Sets Syst 137:137–149
30. Tversky A, Kahneman D (1992) Advances in prospect theory: cumulative representation of uncertainty. J Risk Uncertain 5:297–323