Chapter 7
Weighted and Constrained Consensus Control with Performance Optimization for Robust Distributed UAV Deployments with Dynamic Information Networks*

Le Yi Wang and George Yin

Abstract. A team of unmanned aerial vehicles (UAVs) in surveillance operations aims to achieve fast deployments, robustness against uncertainties and adversaries, and adaptability when the team expands or reduces. All these must be achieved under time-varying and local communication connections. This paper introduces a new framework for UAV control based on the emerging consensus control for networked systems. Due to unique features of UAV tasks, the consensus control problem becomes weighted and constrained, beyond the typical consensus formulation. Using only neighborhood communications among UAV members, the consensus control achieves global desired deployment. Algorithms are introduced and their convergence properties are established. It is shown that the algorithms achieve asymptotically the Cramér-Rao lower bound, and hence is asymptotically optimal among all algorithms. Examples and case studies demonstrate convergence, robustness, and scalability of the algorithms.

Keywords: UAV control, consensus control, team coordination, networked systems, stochastic approximation algorithm.

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7.1 Introduction

This study is motivated by the increasing demands of more advanced technology for information processing, control, coordination, and dynamic reconfiguration of networked unmanned aerial vehicles (UAVs). A cluster of UAVs for a coordinated task such as a surveillance mission forms a networked system; see Figure 7.1. Each subsystem is represented by a node which is a local dynamic system itself, and communication and connections of subsystems are represented by a network topology. The networked system aims to accomplish a joint mission, in the presence of uncertainties and attacks and under limited resources (such as the number of UAVs, communication data flow rates, and power consumptions).

A team of UAVs is subject constantly to uncertainties due to natural obstacles such as buildings, mountains, severe weather conditions that interrupt the network connections and observation capabilities, and to enemy attacks that may destroy some members and/or disrupt communications; see Figure 7.3. Consequently, the nodes and network topology switch during real-time operations.

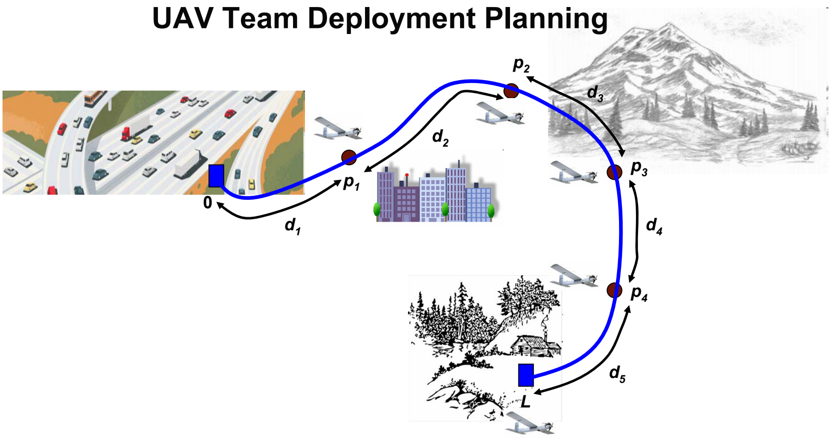

Fig. 7.1 UAVs on a deployment mission

This paper aims to introduce a new framework for UAV coordination and control, building on the emerging technology of network consensus control. The core target is to achieve suitable deployment of the team UAVs based on terrain conditions. In this paper, UAV deployment is formulated as a weighted and constrained consensus control problem that aims to coordinate all subsystems such that their formation converges to a desired distribution pattern. In UAV applications the desired pattern is that the weighted distances between consecutive UAVs are equal. Consensus control is an emerging

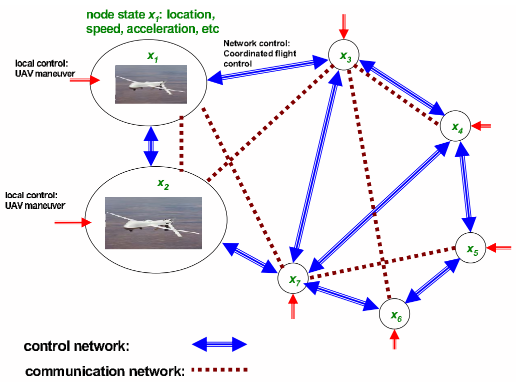

Fig. 7.2 Information and control network topology

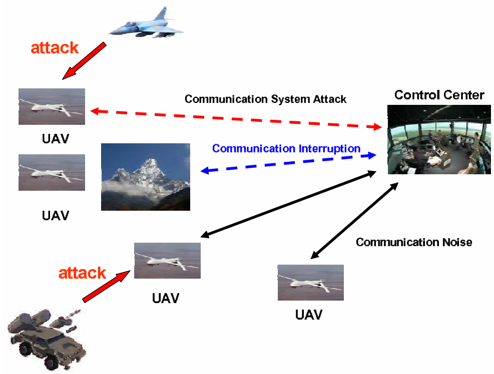

Fig. 7.3 Network Adversaries

field in networked control and remains an active research field. At present, most consensus controls are unconstrained and un-weighted. Nevertheless, UAV control is subject to terrain condition and deployment area constraints. As such, it commonly leads to a weighted and constrained consensus control problem.

Consensus control has drawn increased attention recently in a variety of application areas, including load balancing in parallel computing [21, 18], sensor networks [12], decentralized filtering, estimation, mobil agents [4], etc. The control methodologies developed up to date include deterministic control [4, 15], stochastic approximation algorithms [1, 2], switching network topologies [9, 11, 3, 5], etc.

Departing from the standard consensus control, the following four features are included in this paper.

1. In our recent work [23], a Markov model is used to treat a much larger class of systems, where the network graph is modulated by a discrete-time Markov chain. Our work in [23] also provides convergence and rates of convergence for the corresponding recursive algorithms.
2. Some of the useful features of [23] are extended to the weighted constrained consensus methodology in this paper.
3. In addition, the technique of post-iterate averaging is employed to enhance the power flow control algorithm. For detailed information on post-iterate averaging; see [13, 16] for its original introduction and [8, Chapter 11] for its extension to more general systems; see also [24]. With the iterate averaging, our algorithms provide the best convergence rate in terms of the best scaling factor and the smallest asymptotic covariance. Most significantly, they achieve asymptotically the well-known Cramér-Rao lower bounds [10], hence are best over all algorithms. This fast convergence feature is highly desirable for fast team formation.
4. Performance optimization is introduced to consensus control.

This framework offers several appealing features.

a. Local control to achieve a global deployment: Although a desired deployment is achieved for the entire team, each UAV only needs to communicate with its neighboring members as such communication costs and complexity remain minimal.
b. Scalability: Expanding and reduction of the team members does not complicate control strategies.
c. Robustness: Fluctuations in UAV positions, addition to or reduction of the UAVs can be readily accommodated. The weighted and constrained consensus control has applications in other areas such as platoon control of highway vehicles [20], power grid control [19], among others.

The rest of the paper is organized into the following sections. Section 7.2 describes how a typical UAV deployment problem can be formulated as a weighted and constrained consensus control problem. Algorithms for weighted and constrained consensus control are presented in Section 7.3. Their convergence properties and convergence rates are established. Section 7.4 further enhances the algorithms by post-iterate averaging. It is shown that consensus control for UAVs are subject to noises and their effect can be attenuated by the post-iterate averaging. Optimality of such modified algorithms in terms of convergence rates are established. Robustness and scalability of this framework are explained in Section 7.5 by showing its robustness against disturbances and adaptive capability when the network topology changes. Performance optimization is presented Section 7.6. It is shown that the control gain and step sizes can be selected separately. Consequently, the gain matrix can be selected to optimize a performance measure. The cases of optimal convergence rates and optimal robustness are presented. The feedback gains in consensus algorithms are selected to minimize a certain performance measure. Local implementation of such global optimization strategies is established. This leads a distributed optimization framework with the same network topology as the core consensus control. Finally, Section 7.7 points out future research directions.

7.2 UAV Coordinated Deployment Problems

7.2.1 Networked UAVs and Their Deployment

A team consists of r UAVs are to be deployed along a pathway of total length L. At time t, denote the total length of the surveillance range as $L(t)$. In the algorithm development, L is treated as a constant. Its changes will be viewed as a disturbance to the consensus control problem and d_i is the distance between UAV i and UAV $i-1$. We have the following constraint

$$\sum_{i=1}^{r} d_i(t) = L. \tag{7.1}$$

Due to terrain conditions, a desired coverage for an UAV differs at different locations. Each inter-vehicle distance has a terrain factor γ^i. The goal of the power flow control is to achieve consensus on weighted power d_i/γ^i, namely

$$\frac{d_i(t)}{\gamma^i} \to \beta, \quad i = 1, \ldots, r$$

for some constant β. The convergence is either with probability one (w.p.1.) or in means squares (MS). For notational convenience in the algorithm development, we use $x^i(t) = P^i(t)$ and denote the state vector $x(t) = [P^1(t), \ldots, P^r(t)]'$. The weighting coefficients are $\gamma = [\gamma^1, \ldots, \gamma^r]'$, and the state scaling matrix $\Psi = \text{diag}[1/\gamma^1, \ldots, 1/\gamma^r]$, where v' is the transpose of a vector or a matrix v. Let $\mathbb{1}$ be the column vector of all 1s. Together with the constraint (7.1), the target of the constrained and weighted consensus control is

$$\Psi x(t) \to \beta \mathbb{1} \ \text{ subject to } \ \mathbb{1}' x(t) = L.$$

It follows from $\gamma' \Psi = \mathbb{1}'$ that

$$\beta = \frac{L}{\gamma' \mathbb{1}} = \frac{L}{\gamma^1 + \cdots + \gamma^r}.$$

The UAVs are linked by an information network, represented by a directed graph $\mathcal{G}$ whose element (i, j) (called a directed edge from node i to node j) indicates an observation between UAV i on the distance d_j. This network defines the information network: $(i,j) \in \mathcal{G}$ indicates estimation of the state d^j by UAV i via a communication link. Also, the factor γ^j is known. For node i, $(i,j) \in \mathcal{G}$ is a departing edge and $(l,i) \in \mathcal{G}$ is an entering edge. Due to the nature of power lines, we assume that if $(i,j) \in \mathcal{G}$ then $(j,i) \in \mathcal{G}$. The total number of communication links in $\mathcal{G}$ is l_s. From its physical meaning, node i can always observe its own state, which will not be considered as a link in $\mathcal{G}$.

For a selected time interval T, the consensus control is performed at the discrete-time steps $nT, n = 1, 2, \ldots$. At the control step n, the value of x will be denoted by $x_n = [x_n^1, \ldots, x_n^r]'$. Power flow control updates x_n to x_{n+1} by the amount u_n

$$x_{n+1} = x_n + u_n \tag{7.2}$$

with $u_n = [u_n^1, \ldots, u_n^r]'$. In UAV deployment, a distance change d_n^{ij} (called link control) from UAV i to UAV j at the nth step is the decision variable. The control u_n^i is determined by the link control d_n^{ij} as i is

$$u_n^i = -\sum_{(i,j)\in\mathcal{G}} d_n^{ij} + \sum_{(j,i)\in\mathcal{G}} d_n^{ji}. \tag{7.3}$$

The most relevant implication in this control scheme is that for all n,

$$\sum_{i=1}^{r} x_n^i = \sum_{i=1}^{r} x_0^i = L \tag{7.4}$$

that is, the constraint (7.1) is always satisfied. Consensus control seeks control algorithms such that $\Psi x_n \to \beta\mathbb{1}$ under the constraint (7.4).

A link $(i,j) \in \mathcal{G}$ entails an estimate $\widehat{x}_n^{ij}$ of x_n^j by node i with observation noise d_n^{ij}. That is,

$$\widehat{x}_n^{ij} = x_n^j + d_n^{ij}. \tag{7.5}$$

Let $\widetilde{x}_n$ and d_n be the l_s-dimensional vectors that contain all $\widehat{x}_n^{ij}$ and d_n^{ij} in a selected order, respectively. Then, (7.5) can be written as

$$\widetilde{x}_n = H_1 x_n + d_n \tag{7.6}$$

where H_1 is an $l_s \times r$ matrix whose rows are elementary vectors such that if the ℓth element of $\widetilde{x}_n$ is $\widehat{x}^{ij}$ then the ℓth row in H_1 is the row vector of all zeros except for a "1" at the jth position. Each link in $\mathcal{G}$ provides information $\delta_n^{ij} = x_n^i/\gamma^i - \widehat{x}_n^{ij}/\gamma^j$, an estimated difference between weighted x_n^i and x_n^j. This information may be represented by a vector δ_n of size l_s containing all δ_n^{ij} in the same order as $\widetilde{x}_n$. δ_n can be written as

$$\delta_n = H_2 \Psi x_n - \widetilde{\Psi}\widetilde{x}_n = H_2\Psi x_n - \widetilde{\Psi} H_1 x_n - \widetilde{\Psi} d_n = H x_n - \widetilde{\Psi} d_n, \tag{7.7}$$

where the link scaling matrix $\widetilde{\Psi}$ is the $l_s \times l_s$ diagonal matrix whose k-th diagonal element is $1/\gamma^j$ if the k-th element of $\widetilde{x}_n$ is $\widehat{x}_n^{ij}$; H_2 is an $l_s \times r$ matrix whose rows are elementary vectors such that if the ℓth element of $\widetilde{x}(k)$ is $\widehat{x}^{ij}$ then the ℓth row in H_2 is the row vector of all zeros except for a "1" at the ith position, and $H = H_2\Psi - \widetilde{\Psi}H_1$.

Due to network constraints, the information δ_n^{ij} can only be used by nodes i and j. When the power control is linear, time invariant, and memoryless, we have $p_n^{ij} = \mu_n g_{ij}\delta_n^{ij}$ where g_{ij} is the link control gain and μ_n is a global time-varying scaling factor which will be used in state updating algorithms as the recursive step size. Let G be the $l_s \times l_s$ diagonal matrix that has g_{ij} as its diagonal element. In this case, the control becomes $u_n = -\mu_n J'G\delta_n$, where $J = H_2 - H_1$. For convergence analysis, we note that μ_n is a global control variable and we may represent u_n equivalently as

$$\begin{aligned} u_n &= -\mu_n J'G(Hx_n - \widetilde{\Psi}d_n) \\ &= -\mu_n(J'GHx_n - J'G\widetilde{\Psi}d_n) \\ &= \mu_n(Mx_n + Wd_n), \end{aligned} \tag{7.8}$$

with $M = -J'GH$ and $W = J'G\widetilde{\Psi}$. This, together with (7.2), leads to

$$x_{n+1} = x_n + \mu_n(Mx_n + Wd_n). \tag{7.9}$$

It can be directly verified that $\widetilde{\Psi}H_1\Psi^{-1} = H_1$, $H\Psi^{-1} = J$, $J\mathbb{1} = 0$, $\Psi^{-1}\mathbb{1} = \gamma$. These imply that $\mathbb{1}'M = 0$, $\mathbb{1}'W = 0$, $M\Psi^{-1}\mathbb{1} = M\gamma = 0$. The following assumption is imposed on the network.

(A0) The following conditions hold:

(1) All link gains are positive, $g_{ij} > 0$.
(2) $\mathcal{G}$ is strongly connected[1].

7.2.2 An Illustrative Example

We now use an example to illustrate the above concepts.

Example 1. A team of three UAVs must cover a surveillance line of length L, see Figure 7.4. UAV 1 controls the distance d_1, UAV 2 controls the distance d_2, and UAV 3 controls the distance d_3. Although d_4 is in fact a dependent variable since $d_1 + d_2 + d_3 + d_4 = L$, for systematic development, it is still formulated as a controlled variable. Then the condition $d_1 + d_2 + d_3 + d_4 = L$ is imposed as an additional constraint. The information topology is that in addition to observing their own controlled variables, UAV 1 observes also

[1] A directed graph is called strongly connected if there is a path from each node in the graph to every other node.

d_2, UAV 2 observes also d_1 and d_3, UAV 3 observes d_2 and d_4. the controller for d_4 obseerves d_3 also. The total length $L = 53.9$ km. Terrain factors $\gamma^1 = 12$, $\gamma^2 = 15$, $\gamma^3 = 20$, and $\gamma^4 = 28$. As a result,

$$\mathcal{G} = \{(1,2),(2,1),(2,3),(3,2),(3,4),(4,3)\}$$

$x = [d_1, d_2, d_3, d_4]'$, $\gamma = [12, 15, 20, 28]'$, $\Psi = \text{diag}[1/12, 1/15, 1/20, 1/28]$.

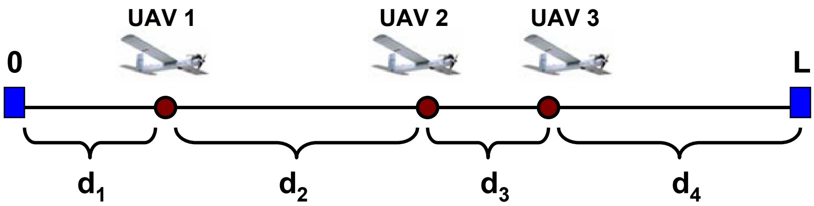

Fig. 7.4 A team of three UAVs

Since $L = 53.9$, we have

$$\beta = \frac{L}{\gamma^1 + \gamma^2 + \gamma^3 + \gamma^4} = 0.7187$$

and the weighted consensus is $\Psi x = 0.7187\mathbb{1}$ or

$$x = 0.7187\Psi^{-1}\mathbb{1} = [8.624, 10.781, 14.374, 20.124]'$$

By choosing the order for the links as $(1,2),(2,1),(2,3),(3,2),(3,4),(4,3)$, we have

$$\widetilde{x} = [\widehat{x}^{12}, \widehat{x}^{21}, \widehat{x}^{23}, \widehat{x}^{32}, \widehat{x}^{34}, \widehat{x}^{43}]'$$

and

$$H_1 = \begin{bmatrix} 0\,1\,0\,0 \\ 1\,0\,0\,0 \\ 0\,0\,1\,0 \\ 0\,1\,0\,0 \\ 0\,0\,0\,1 \\ 0\,0\,1\,0 \end{bmatrix}; H_2 = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \end{bmatrix}.$$

It follows that $\widetilde{\Psi} = \text{diag}[1/15, 1/12, 1/20, 1/15, 1/28, 1/20]$ and

$$H = H_2\Psi - \widetilde{\Psi}H_1$$

$$= \begin{bmatrix} 1/12 & 0 & 0 & 0 \\ 0 & 1/15 & 0 & 0 \\ 0 & 1/15 & 0 & 0 \\ 0 & 0 & 1/20 & 0 \\ 0 & 0 & 1/20 & 0 \\ 0 & 0 & 0 & 1/28 \end{bmatrix} - \begin{bmatrix} 0 & 1/15 & 0 & 0 \\ 1/12 & 0 & 0 & 0 \\ 0 & 0 & 1/20 & 0 \\ 0 & 1/15 & 0 & 0 \\ 0 & 0 & 0 & 1/28 \\ 0 & 0 & 1/20 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/12 & -1/15 & 0 & 0 \\ -1/12 & 1/15 & 0 & 0 \\ 0 & 1/15 & -1/20 & 0 \\ 0 & -1/15 & 1/20 & 0 \\ 0 & 0 & 1/20 & -1/28 \\ 0 & 0 & -1/20 & 1/28 \end{bmatrix},$$

$$J = H_2 - H_1 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Suppose the control gains on the links are selected as $g_{12} = g_{21} = 3$,$g_{23} = g_{32} = 7$, $g_{34} = g_{43} = 9$. Then $G = \text{diag}[3, 3, 7, 7, 9, 9]$. It follows that

$$M = -J'GH = \begin{bmatrix} -1/2 & 1/2.5 & 0 & 0 \\ 1/2 & -2/1.5 & 7/10 & 0 \\ 0 & 7/7.5 & -4/2.5 & 9/14 \\ 0 & 0 & 9/10 & -9/14 \end{bmatrix},$$

$$W = J'G\widetilde{\Psi} = \begin{bmatrix} 1/5 & -1/4 & 0 & 0 & 0 & 0 \\ -1/5 & 1/4 & 7/20 & -7/15 & 0 & 0 \\ 0 & 0 & -7/20 & 7/15 & 9/28 & -9/20 \\ 0 & 0 & 0 & 0 & -9/28 & 9/20 \end{bmatrix}.$$

Since $\mathbb{1}'J' = \mathbb{1}'(H_2 - H_1)' = 0$, we have $\mathbb{1}'M = 0$ and $\mathbb{1}'W = 0$. We can show that under Assumption (A0), M has rank $r-1$ and is negative semi-definite. The proof uses similar ideas as in [23] and hence is omitted here. Recall that a square matrix $\widetilde{Q} = (\widetilde{q}_{ij})$ is a generator of a continuous-time Markov chain if $\widetilde{q}_{ij} \geq 0$ for all $i \neq j$ and $\sum_j \widetilde{q}_{ij} = 0$ for each i. Note that a generator of the associated continuous-time Markov chain is irreducible if the system of equations

$$\begin{cases} \nu\widetilde{Q} = 0, \\ \nu\mathbb{1} = C \end{cases} \tag{7.10}$$

for a given constant $C > 0$ has a unique solution, where $\nu = [\nu_1, \ldots, \nu_r] \in \mathbb{R}^{1\times r}$ with $\nu_i/C > 0$ for each $i = 1, \ldots, r$. When $C = 1$, ν is the associated stationary distribution. Consequently, under Assumption (A0), M is a generator of a continuous-time irreducible Markov chain.

7.3 Weighted Consensus Control with Linear Constraints

7.3.1 Algorithms

We begin by considering the state updating algorithm (7.9)

$$x_{n+1} = x_n + \mu_n M x_n + \mu_n W d_n, \tag{7.11}$$

together with the constraint

$$\mathbb{1}' x_n = L, \tag{7.12}$$

where $\{\mu_n\}$ is a sequence of stepsizes, M is a generator of a continuous-time Markov chain (hence $\mathbb{1}'M = 0$), $\{d_n\}$ is a noise sequence.

Since the algorithm (7.11) is a stochastic approximation procedure, we can use the general framework in Kushner and Yin [8] to analyze the asymptotic properties. Since $\mathbb{1}'M = 0$ and $\mathbb{1}'W = 0$, starting from the initial condition with $\mathbb{1}'x_0 = L$, the constraint $\mathbb{1}'x_n = L$ is always satisfied by the algorithm structure.

(A1)

1. The stepsize satisfies the following conditions: $\mu_n \geq 0$, $\mu_n \to 0$ as $n \to \infty$, and $\sum_n \mu_n = \infty$.
2. The noise $\{d_n\}$ is a stationary ϕ-mixing sequence such that $Ed_n = 0$, $E|d_n|^{2+\Delta} < \infty$ for some $\Delta > 0$, and that the mixing measure $\widetilde{\phi}_n$ satisfies

$$\sum_{k=0}^{\infty} \widetilde{\phi}_n^{\Delta/(1+\Delta)} < \infty, \tag{7.13}$$

where

$$\begin{aligned} &\widetilde{\phi}_n = \sup_{A\in\mathcal{F}^{n+m}} E^{(1+\Delta)/(2+\Delta)} |P(A|\mathcal{F}_m) - P(A)|^{(2+\Delta)/(1+\Delta)}, \\ &\mathcal{F}_n = \sigma\{d_k; k < n\}, \quad \mathcal{F}^n = \sigma\{d_k; k \geq n\}. \end{aligned}$$

Under Assumption (A0), M has an eigenvalue 0 of multiplicity 1 and all other eigenvalues are in the left complex plan (i.e., the real parts of the eigenvalues are negative). The null space of M is spanned by the vector $\gamma = [\gamma^1, \ldots, \gamma^r]'$.

Some commonly used stepsize sequences includes $\mu_n = a/n^\alpha$ for $1/2 < \alpha \le 1$. In such cases, $\sum_{n=1}^{\infty} \mu_n = \infty$ but $\sum_{n=1}^{\infty} \mu_n^2 < \infty$.

Note that ϕ-mixing sequences contain independent noises as a special case. However, they can represent a much larger class of noises to accommodate communication uncertainties such as signal interference, signal fading, latency, etc. As a consequence of (A1), the ϕ-mixing implies that the noise sequence $\{d_n\}$ is strongly ergodic [6, p. 488] in that for any m

$$\frac{1}{n}\sum_{j=m}^{m+n-1} d_j \to 0, \quad \text{w.p.1 as } n \to \infty. \tag{7.14}$$

7.3.2 Convergence of Algorithms

To study the convergence of the algorithm (7.11), we employ the stochastic approximation methods developed in [8]. Instead of working with the discrete-time iterations, we examine sequences defined in an appropriate function space. This will enable us to get a limit ordinary differential equation (ODE). The significance of the ODE is that the stationary point is exactly the true value of the desired weighted consensus. Then, convergence becomes a stability issue. We define

$$t_n = \sum_{j=0}^{n-1} \mu_j, \;\; m(t) = \max\{n : t_n \le t\}, \tag{7.15}$$

the piecewise constant interpolation $x^0(t) = x_n$ for $t \in [t_n, t_{n+1})$, and the shift sequence $x^n(t) = x^0(t + t_n)$. Due to the page limitation, we shall only outline the main steps involved in the proof. We can first derive a preliminary estimate on the second moments of x_n.

Lemma 1. *Under Assumption* (A1)*, for any* $0 < T < \infty$,

$$\sup_{n \le m(T)} E|x_n|^2 \le K \;\textit{and}\; \sup_{0 \le t \le T} E|x^n(t)|^2 \le K, \tag{7.16}$$

for some $K > 0$*, where* $m(\cdot)$ *is defined in* (*7.15*).

Proof. We only indicate the main ideas and leave most of the details out. Concerning the first estimate, because of the boundedness of the second moment $E|d_n|^2$, the condition $\sum_{j=1}^{\infty} \mu_j^2 < \infty$, we can derive

$$E|x_n| \le K + K\sum_{j=1}^{n} \mu_j E|x_j|^2. \tag{7.17}$$

Here and henceforth, K is used as a generic positive constant, whose values may change for different usage. Applying the Grownwall's inequality to

(7.17), and then taking sup over $n \leq m(T)$, the first error bound is obtained. Likewise, we can obtain the second estimate. □

Theorem 1. *Under Assumption* (A1)*, the iterates generated by the stochastic approximation algorithm (7.11) satisfies $\Psi x_n \to \beta \mathbb{1}$ w.p.1 as $n \to \infty$.*

Ideas of Proof. We only present the main ideas below. We show that $\{x^n(\cdot)\}$ is equicontinuous in the extended sense (see [8, p. 102] for a definition) w.p.1. To verify this, we note that by the argument in the first part of the proof in [22, Theorem 3.1],

$$\sum_{j=1}^{\infty} \mu_j d_j \text{ converges w.p.1.}$$

Define $\Phi^0(t) = \sum_{j=1}^{m(t)-1} \mu_j d_j$ and $\Phi^n(t) = \Phi^0(t_n + t)$, where $m(\cdot)$ is defined in (7.15). Then we can show that for each $T > 0$ and $\varepsilon > 0$, there is a $\delta > 0$ such that

$$\limsup_n \sup_{0 \leq |t-s| \leq \delta} |\Phi^n(t) - \Phi^n(s)| \leq \varepsilon \quad \text{w.p.1.}$$

The above estimates together with the form of the recursion imply that $x^n(\cdot)$ is equicontinuous in the extended sense. Next, we can extract a convergent subsequence, which will be denoted by $x^{n_\ell}(\cdot)$. Then the Arzela-Ascoli theorem concludes that $x^{n_\ell}(\cdot)$ converges to a function $x(\cdot)$ which is the unique solution (since the recursion is linear in x) of the ordinary differential equation (ODE)

$$\dot{x}(t) = Mx(t). \tag{7.18}$$

Moreover, from basic properties of Markov chains (see [25, Appendix A.1]), as $t \to \infty$, the solution $x(t)$ to (7.18) satisfies that $x(t)$ converges to the set Γ. That is, $\mathrm{dist}(x(t), \Gamma) \to 0$ as $t \to \infty$, where $\mathrm{dist}(\cdot,\cdot)$ is the usual distance function defined by $\mathrm{dist}(x, \Gamma) = \inf_{y \in \Gamma} |x - y|$. Consequently, as $n \to \infty$ and $q(n_\ell) \to \infty$, $x^{n_\ell}(\cdot + q(n_\ell)) \to \Gamma$.

Furthermore, the algorithm (7.11) together with $x_n' \mathbb{1} = L$ leads to the desired weighted consensus. The equilibria of the limit ODE (7.18) and this constraint lead to the following system of equations

$$\begin{cases} Mx = 0 \\ \mathbb{1}' x = L. \end{cases} \tag{7.19}$$

The irreducibility of M then implies that (7.19) has a unique solution $x_* = \beta \Psi^{-1} \mathbb{1} = \beta \gamma$, which is precisely the weighted consensus.

Example 2. We now use the system in Example 1 to demonstrate the weighted consensus control. As in Example 1, the total distance is 53.9 km. Suppose that the initial distance distribution from the three UAVs are $d_0^1 = 12$ km; $d_0^2 = 14$ km; $d_0^3 = 10.9$ km; $d_0^4 = 17$ km. Weighted consensus for UAV control aims to distribute distances according to the terrain

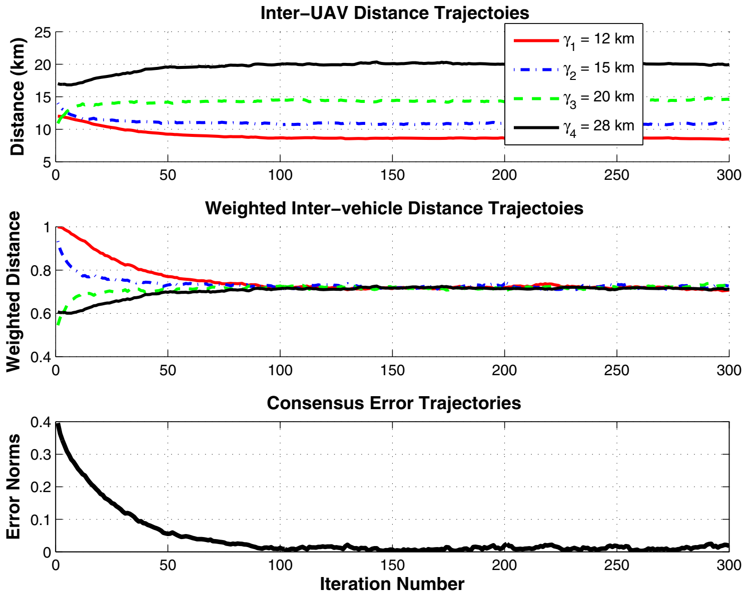

Fig. 7.5 UAV distance control with weighted consensus

conditions defined by $\gamma^1 = 12$, $\gamma^2 = 15$, $\gamma^3 = 20$, $\gamma^4 = 28$, with the total $\mathbb{1}'\gamma = 75$. The target percentage distance distribution over the whole length is $[12/75, 15/75, 20/75, 28/75] = [0.1600, 0.2000, 0.2667, 0.3733]$. From the total length of 53.9 km, the goal of weighted consensus is $d^1 = 8.624$ km; $d^2 = 10.780$ km; $d^3 = 14.373$ km; $d^4 = 20.123$ km.

Suppose that the link observation noises are i.i.d sequences of Gaussian noises with mean zero and variance 1. Figure 7.5 shows the inter-UAV distance trajectories. Staring from a large disparity in distance distribution, the top plot shows how distances are gradually distributed according to the terrain conditions. The middle plot illustrates that the weighted distances converge to a constant. The weighted consensus error trajectories are plotted in the bottom figure.

7.4 Post-Iterate Averaging for Improved Convergence under Large Observation Noise

The basic stochastic approximation algorithm (7.11) demonstrates desirable convergence properties under relatively small observation noises. However, its convergence rate is not optimal. Especially when noises are large, its convergence may not be sufficiently fast and its states show fluctuations. For example, for the same system as in Example 2, if the noise standard deviation is

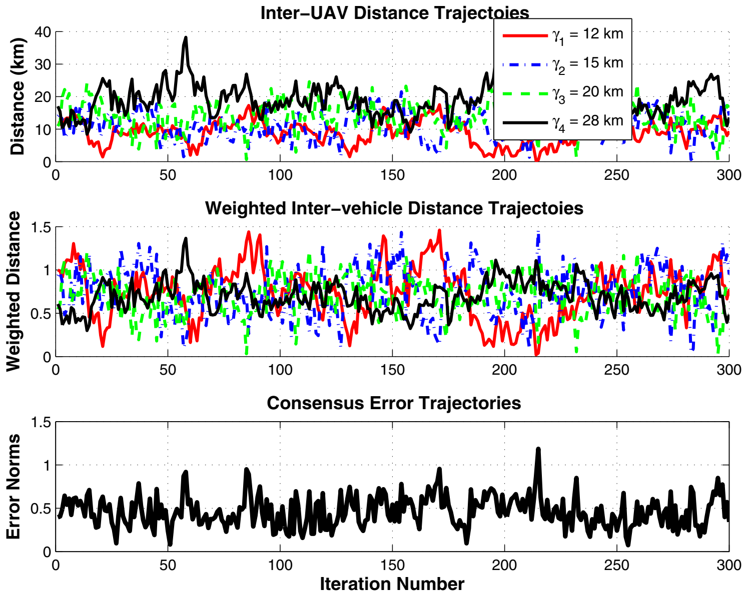

Fig. 7.6 UAV distance control with weighted consensus under large observation noise

increased from 1 to 20, its state trajectories demonstrate large variations, as shown in Figure 7.6.

To improve the efficiency, we take a post-iterate averaging, resulting in a two-stage stochastic approximation algorithm. For definiteness and simplicity, we take $\mu_n = c/n^\alpha$ for some $(1/2) < \alpha < 1$ and $c > 0$. The algorithm is modified to

$$\begin{aligned} x_{n+1} &= x_n + \frac{c}{n^\alpha} M x_n + \frac{c}{n^\alpha} W d_n \\ \overline{x}_{n+1} &= \overline{x}_n - \frac{1}{n+1}\overline{x}_n + \frac{1}{n+1} x_{n+1}. \end{aligned} \tag{7.20}$$

In what follows, for simplicity, we take $c = 1$ henceforth. Since $\mathbb{1}'M = 0$ and $\mathbb{1}'W = 0$, we have $\mathbb{1}'\overline{x}_n = L$. As a result, the constraint (7.1) remains satisfies after the post-iterate averaging. For some of the detailed analysis, we refer the reader to [24].

7.4.1 Asymptotic Efficiency

Strong convergence of the averaged $\overline{x}_n$ follows from that of x_n. This is stated in the following theorem with its proof omitted.

Theorem 2. *Suppose the conditions of Theorem* 1 *are satisfied. For iterates generated by algorithm (7.20)* (*together with the constraint* $\mathbb{1}' x_n = L$), $x_n \to \beta \Psi^{-1} \mathbb{1}$ *w.p.1 as* $n \to \infty$.

To proceed, we define

$$\overline{B}_n(t) = \frac{\lfloor nt \rfloor}{\sqrt{n}} (\Theta_{\lfloor nt \rfloor + 1} - \widetilde{x}_*). \tag{7.21}$$

We next show that asymptotically, the "effective" term of the normalized error above is given by $-\Gamma^{-1} B_n(t)$.

Lemma 2. *In addition to the assumptions* (A1)–(A3), *assume* Γ *is a stable matrix* (*all of its eigenvalues have negative real parts*). *Then for* $t \in [0, 1]$,

$$\overline{B}_n(t) = -\Gamma^{-1} B_n(t) + o(1), \quad \textit{where} \quad o(1) \to 0$$

in probability uniformly in t *as* $n \to \infty$.

Remark 1. In the absence of the nonadditive noise, Γ becomes $\widetilde{M}$. The stability of $\widetilde{M}$ is verified by using the irreducibility of the generator M.

We are now ready to present the following theorem.

Theorem 3. *Under the conditions of Lemma* 2, *the following assertions hold:*

- $\overline{B}_n(\cdot)$ *converges weakly to* $\overline{B}(\cdot)$ *a Brownian motion whose covariance is given by* $\Gamma^{-1} \Sigma_0 (\Gamma^{-1})' t$;
- $\widetilde{x}_n - \widetilde{x}_*$ *is asymptotically normal with mean 0 and asymptotic covariance* $\Gamma^{-1} \Sigma_0 (\Gamma^{-1})' / n$.

Outline of Proof. To prove the first part of the theorem, we need only evaluate its covariance. This in turn follows from the well-known Slutsky theorem. To obtain the second part, set $t = 1$ in part one. Using Lemma 2 and part of the theorem, the desired result follows. □

We now establish the optimality of the algorithms. For clarity, we will include the dimension of the vector $\mathbb{1}$ in notation in the following derivations when needed. Also, the detailed proofs of the theorems are omitted. The reader is referred to our recent work [23] for details.

Partition the matrix M as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \tag{7.22}$$

where $M_{11} \in \mathbb{R}^{(n-1) \times (n-1)}$, $M_{12} \in \mathbb{R}^{(n-1) \times 1}$, $M_{21} \in \mathbb{R}^{(n-1) \times 1}$, and $M_{22} \in \mathbb{R}^{1 \times 1}$. Accordingly, we also partition $\bar{x}_n$, x_n, and W as

$$\overline{x}_n = \begin{bmatrix} \widetilde{\overline{x}}_n \\ \overline{x}_n^r \end{bmatrix}; \; x_n = \begin{bmatrix} \widetilde{x}_n \\ x_n^r \end{bmatrix}; \; W = \begin{bmatrix} \widetilde{W} \\ W_1 \end{bmatrix}, \tag{7.23}$$

respectively, with compatible dimensions with those of M.

Lemma 3. *Under Assumption A0, M_{11} is full rank.*

This result indicates that we can concentrate on $r-1$ components of $\overline{x}_n$. We can show that the asymptotic rate of convergence is independent of the choice of the $r-1$ state variables. To study the rates of convergence of $\overline{x}_n$, without loss of generality we need only examine that of $\overline{\widetilde{x}}_n$. It follows from that

$$\begin{cases} \widetilde{x}_{n+1} = \widetilde{x}_n + \mu_n (M_{11}\widetilde{x}_n + M_{12} x_n^r + \widetilde{W} d_n) \\ \qquad = \widetilde{x}_n + \mu_n (\widetilde{M}\widetilde{x}_n + \widetilde{W} d_n), \\ \overline{\widetilde{x}}_{n+1} = \overline{\widetilde{x}}_n - \dfrac{1}{n+1}\overline{\widetilde{x}}_n + \dfrac{1}{n+1}\widetilde{x}_{n+1}, \end{cases} \tag{7.24}$$

where

$$\widetilde{M} = M_{11} - M_{12} 1\!\!1'_{r-1}.$$

Note that the noise is now reduced also to $\widetilde{W} d_n$, which is $r-1$ dimensional but is a function of l_s dimensional link noise d_n. Let $D = I_{r-1} + 1\!\!1_{r-1} 1\!\!1'_{r-1}$.

Lemma 4. *Assume* (A0)*. Then $\widetilde{M} = M_{11} D$ and is full rank.*

For convergence speed analysis, let

$$e_n = \overline{x}_n - \beta \Psi^{-1} 1\!\!1_n.$$

Decompose $e_n = [\widetilde{e}'_n, e_n^r]'$.

Theorem 4. *Suppose that $\{d_n\}$ is a sequence of i.i.d. random variables with mean zero and covariance $E d_n d'_n = \Sigma$. Under Assumption* (A0)*, the weighted consensus errors $\widetilde{e}_n$ satisfies that $\sqrt{n}\widetilde{e}_n$ converges in distribution to a normal random variable with mean* 0 *and covariance given by*

$$\widetilde{M}^{-1} \widetilde{W} \Sigma \widetilde{W}' (\widetilde{M}^{-1})'.$$

Note that the above result does not require any distributional information on the noise $\{\varepsilon(k)\}$ other than the zero mean and finite second moments. We now state the optimality of the algorithm when the density function is smooth.

Theorem 5. *Suppose that the noise $\{d_n\}$ is a sequence of i.i.d. noise with a density $f(\cdot)$ that is continuously differentiable. Then the recursive sequence $\overline{\widetilde{x}}_n$ is asymptotically efficient in the sense of the Cramér-Rao lower bound on $E\widetilde{e}'_n \widetilde{e}_n$ being asymptotically attained,*

$$nE\widetilde{e}'_n \widetilde{e}_n \to \mathrm{tr}(\widetilde{M}^{-1} \widetilde{W} \Sigma \widetilde{W}' (\widetilde{M}^{-1})'). \tag{7.25}$$

The convergence speed and optimality of e_n is directly related to these of $\widetilde{e}_n$.

Corollary 1. *Under the conditions of Theorem 5, the sequence $\{\overline{x}_n\}$ is asymptotically efficient in the sense of the Cramér-Rao lower bound on $Ee'_n e_n$*

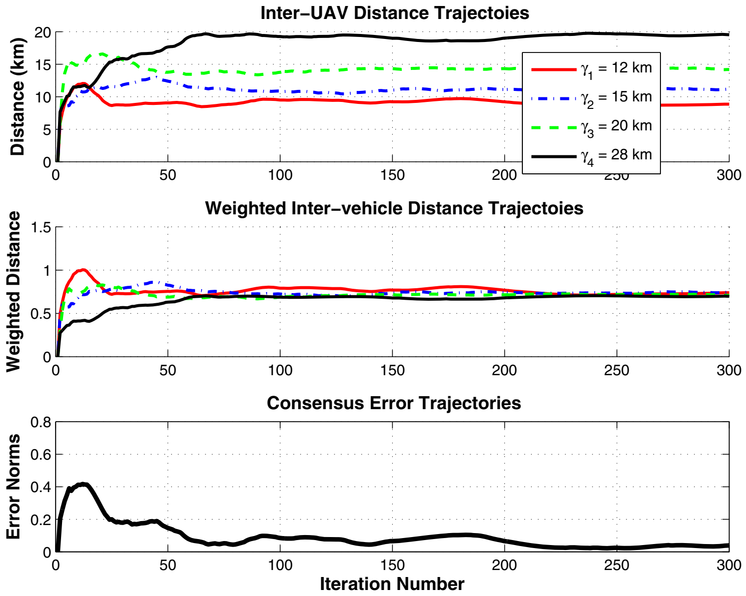

Fig. 7.7 UAV distance control with post-iterate averaging on weighted consensus algorithms

being asymptotically attained. The asymptotically optimal convergence speed is

$$nEe_n'e_n \to \operatorname{tr}(D\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})') \tag{7.26}$$

where $D = I_{r-1} + \mathbb{1}_{r-1}\mathbb{1}_{r-1}'$.

Example 3. We now use the system in Example 2 to illustrate the effectiveness of post-iterate averaging. Suppose that the link observation noises are i.i.d sequences of Gaussian noises of mean zero and standard deviation 20. Now, the consensus control is expanded with post-iterate averaging. Figure 7.7 shows the distance trajectories. The distance distributions converge to the weighted consensus faster with much less fluctuations.

7.5 Robustness and Scalability

7.5.1 Robustness to Disruption of Terrain Conditions and Mission Goals

When a mission changes its tasks, say by modifying its length L of the surveillance range, they are represented by a sudden change in L. UAV coordination will re-distribute the inter-vehicle distances by the weighted consensus to reach a new equilibrium of consensus.

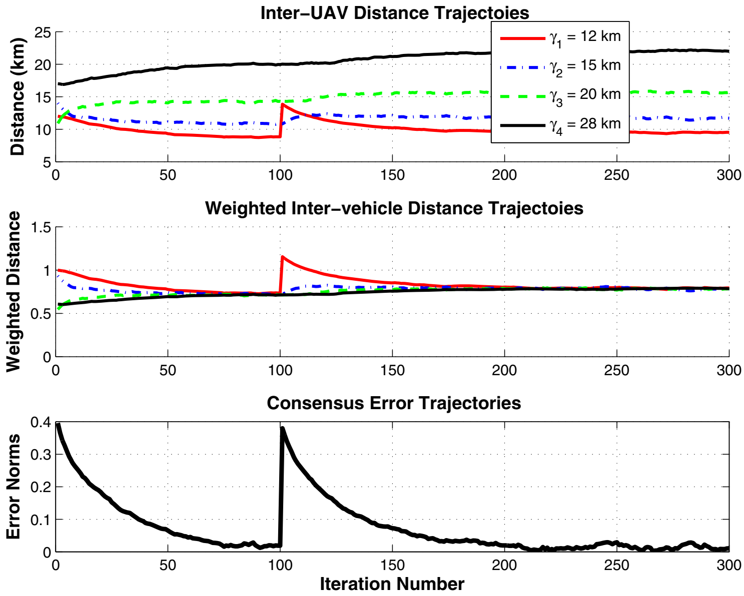

Fig. 7.8 Robustness of UAV distance control against terrain changes

Example 4. We now use the system in Example 2 to illustrate the effectiveness of robustness against load disruptions. Suppose that at the iteration step $n = 100$, a sudden increase of the total length by 5 km occurs to the surveillance mission. Consensus control then distributes it fairly according to the terrain conditions (represented by γ). Figure 7.8 shows the inter-UAV distance trajectories. The distance distributions converge to the weighted consensus.

7.5.2 Scalability

A team of UAVs often encounters dynamic changes in its team members or its information topology. When an UAV was damaged, it must retreat from the mission. Conversely, an enhancement of the team by additional UAVs changes the team composition and topologies. Both cases entail re-distribution of inter-UAV distances. In a centralized control scheme in which all information on UAVs is used by a central controller, the control strategy must be adapted for the entire system each time the UAV team topology changes. In our neighborhood-based network control method, an addition or deletion of an UAV will only affect its neighboring UAVs. In fact, all other UAVs will never be aware of changes in other parts of the team. However, by iterative control, an additional distance will be properly distributed throughout the entire team formation.

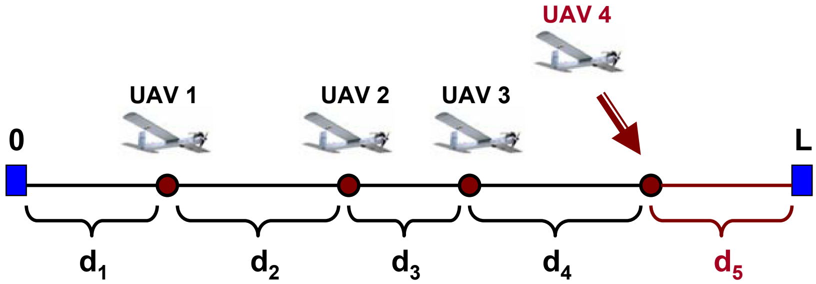

Fig. 7.9 An expanded UAV team from three members to four members

To be more concrete, suppose that a new UAV, labeled $r+1$, is added to the team with terrain factor γ^{r+1}. Assume that the new UAV is linked to UAV r only. Then, by (7.3), the control of UAV r will be modified from the original

$$x_{n+1}^r = x_n^r - \sum_{(r,j)\in\mathcal{G}} p_n^{rj} + \sum_{(j,r)\in\mathcal{G}} p_n^{jr}$$

to a slightly modified scheme

$$x_{n+1}^r = x_n^r - \sum_{(r,j)\in\mathcal{G}} p_n^{rj} + \sum_{(j,r)\in\mathcal{G}} p_n^{jr} - p_n^{r,r+1} + p_n^{r+1,r}$$

with all other bus control functions unchanged. The additional consumption of communication resources will be limited to the communication channel between UAV r and UAV $r+1$. This distributed and scalable control strategy is essential for UAV operations in reducing communication requirements and control complexity.

Example 5. Consider the same system as in Example 2. Suppose that at the iteration step $n = 100$, a new UAV becomes available. The new UAV 4 has weighting 10 and observes d_4 only. This addition results in a network topology change, leading to the new matrices derived below. The expanded network is shown in Figure 7.9. For systematic system analysis and presentation, we treat this at the system level with new M and W matrices. Note that for implementation of the consensus algorithms, only the control action of UAV 3 is affected with an additional term representing the link between D_4 and d_5.

Capacity factors are now expanded to $\gamma = [12, 15, 20, 28, 10]'$. The grid network set is expanded to

$$\mathcal{G} = \{(1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(4,5),(5,4)\}.$$

The nework state vector is

$$x = [P^1, P^2, P^3, P^4, P^5]',$$
$$\Psi = \mathrm{diag}[1/120, 1/150, 1/200, 1/280, 1/100].$$

Since $L = 10 + 12 + 8.9 + 23 = 53.9$ is unchanged, we have the new weighted consensus

$$\beta = \frac{L}{\gamma^1 + \gamma^2 + \gamma^3 + \gamma^4 + \gamma^5} = 0.6341,$$

and the new weighted power distribution

$$x = 0.6341\Psi^{-1}\mathbb{1} = [7.609, 9.512, 12.682, 17.755, 6.341]'.$$

By choosing the order for the links as $(1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4)$, we have

$$\widetilde{x} = [\widehat{x}^{12}, \widehat{x}^{21}, \widehat{x}^{23}, \widehat{x}^{32}, \widehat{x}^{34}, \widehat{x}^{43}, \widehat{x}^{45}, \widehat{x}^{54}]'$$

and

$$H_1 = \begin{bmatrix} 0\,1\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,0\,0\,1\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,1\,0 \end{bmatrix}; H_2 = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,0\,1\,0 \\ 0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,1 \end{bmatrix}.$$

It follows that $\widetilde{\Psi} = \text{diag}[1/15, 1/12, 1/20, 1/15, 1/28, 1/20, 1/10, 1/28]$ and

$$\begin{aligned} H &= H_2\Psi - \widetilde{\Psi}H_1 \\ &= \begin{bmatrix} 0.083 & -0.067 & 0 & 0 & 0 \\ -0.083 & 0.067 & 0 & 0 & 0 \\ 0 & 0.067 & -0.050 & 0 & 0 \\ 0 & -0.067 & 0.050 & 0 & 0 \\ 0 & 0 & 0.050 & -0.036 & 0 \\ 0 & 0 & -0.050 & 0.036 & 0 \\ 0 & 0 & 0 & 0.036 & -0.100 \\ 0 & 0 & 0 & -0.036 & 0.100 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} J &= H_2 - H_1 \\ &= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \end{aligned}$$

Suppose the control gains on the new link are $g_{45} = g_{54} = 10$. Then $G = \text{diag}[3, 3, 7, 7, 9, 9, 10, 10]$. It follows that

$$M = -J'GH = \begin{bmatrix} -0.5 & 0.4 & 0 & 0 & 0 \\ 0.5 & -1.333 & 0.7 & 0 & 0 \\ 0 & 0.933 & -1.6 & 0.643 & 0 \\ 0 & 0 & 0.9 & -1.357 & 2 \\ 0 & 0 & 0 & 0.714 & -2 \end{bmatrix}$$

$$W = J'G\widetilde{\Psi} = \begin{bmatrix} 0.2 & -0.25 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.2 & 0.25 & 0.35 & -0.467 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.35 & 0.467 & 0.321 & -0.45 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.321 & 0.45 & 1 & -0.357 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.357 \end{bmatrix}.$$

After the new UAV is added into the system, consensus control distributes inter-UAV distances according to the terrain factors of all distances. Figure 7.10 shows the distance trajectories. Initially, the new UAV reduces its neighbor's (UAV 4) distances d_4, due to its direct link to it. But, afterward the control adjusts distances among other UAVs throughout the entire team. The distance distributions converge to the new weighted consensus.

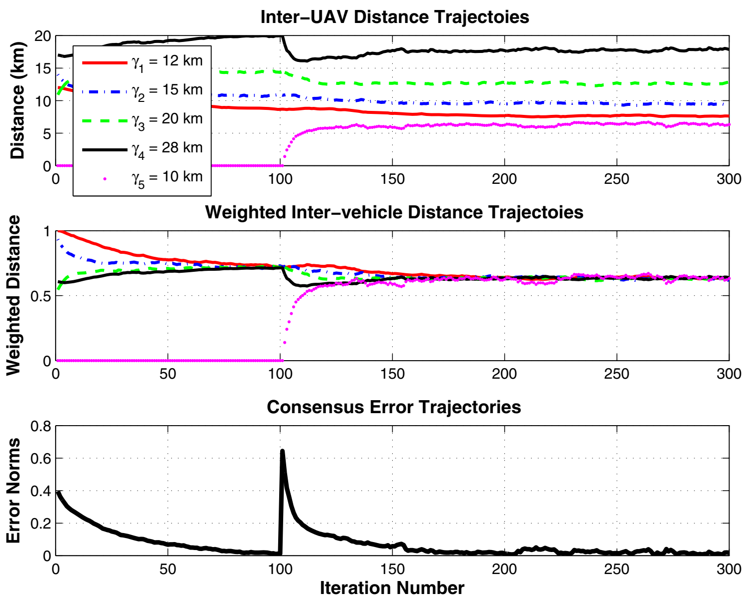

Fig. 7.10 UAV deployment network topology changes

7.6 Performance Optimization

7.6.1 Control Gains and Performance Optimization

Due to the C-R lower bound, by (7.26) the asymptotic estimation errors that can be achieved by any algorithm are explicitly given by

$$nEe_n' e_n \to \operatorname{tr}(D\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})').$$

In other words, the best convergence rates are in terms of mean-squares errors are $\operatorname{tr}(D\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})')/n$. Consequently, to improve convergence rates, it is desirable to reduce $\operatorname{tr}(D\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})')$.

Since $M = -J'GH$, $W = J'G\widetilde{\Psi}$, $D = I_{r-1} + \mathbb{1}_{r-1}\mathbb{1}_{r-1}'$,

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, W = \begin{bmatrix} \widetilde{W} \\ W_1, \end{bmatrix}, \widetilde{M} = M_{11} - M_{12}\mathbb{1}_{r-1}',$$

it is clear that other than the structural variables that are determined by the network topology, the gain matrix G is the only design variable in this expression. By Assumption A0, G is a diagonal matrix with all diagonal elements positive. This section will explore optimization of G for different performance measures. In this section, we use the notation $G = \operatorname{diag}[g_i]$, $i = 1, \ldots, l_s$.

Denote $\eta(G) = \operatorname{tr}(D\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})')$. We note that η_{opt} is invariant under scaling of G. This is due to the fact that for any $c > 0$, $\eta(cG) = \eta(G)$. This observation point to a *design separation principle*: The gain matrix G and the step size μ_n in (7.9) can be designed separately. In other words, change in the step size μ_n will affect convergence but not the C-R lower bound which depends on G. As a result, in this section, we will concentrate on designing G to minimize certain performance measures.

7.6.2 Convergence Rate: Optimization

We start with design of G to minimize the MS estimation errors so that the convergence rate is optimized. The convergence rate optimization amounts to

$$\eta_{opt} = \min_{g_i > 0, i=1,\ldots,l_s} \eta(G). \tag{7.27}$$

Since for any $c > 0$, $\eta(cG) = \eta(G)$, in search of the optimal G, we may limit the search range to $1 \geq g_i > 0$,

$$\eta_{opt} = \min_{1 \geq g_i > 0, i=1,\ldots,l_s} \eta(G). \tag{7.28}$$

Example 6. Consider the same system as in Example 5. The selected gain matrix in Example 5 is $G = \operatorname{diag}[3, 3, 7, 7, 9, 9, 10, 10]$, which may be equivalent scaled to $0.05G = \text{diag}[0.15, 0.15, 0.35, 0.35, 0.45, 0.45, 0.5, 0.5]$ without affecting $\eta(G)$. This choice of G results in $\eta(G) = 1.5915$.

A Monte Carlo search is performed to find η_{opt} in (7.28). G is searched over 5000 randomly-selected sample points with

$$G = \text{diag}[\text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}],$$

where rand is a random variable of uniform distribution in $(0, 1]$. The minimum $\eta(G)$ over this set is $\eta(G_0) = 1.3298$ which is achieved by

$$G_0 = \text{diag}[0.2078, 0.6030, 0.4909, 0.7039, 0.4047, 0.8076, 0.6913, 0.7920].$$

Due to performance optimization, this design exceeds the performance of the control in Example 5.

7.6.3 Min-Max Optimization

In UAV missions, communication systems are subject to uncertainties due to terrain conditions, inter-UAV distance changes, and adversary signal jammer. Consequently, accurate channel noise characterizations are often difficult. This implies that the matrix Σ in (7.26) may not be known. Here we assume that Σ belongs to a set Ω of potential noise characterizations. In this case, we should use $\eta(G, \Sigma)$ to indicate its dependence on Σ. To ensure reliable control performance, we seek the optimal gain matrix design under a worst-case scenario. Mathematically, this means the following min-max design problem

$$\eta_{opt} = \min_{1 \geq g_i > 0, i=1,\ldots,l_s} \max_{\Sigma \in \Omega} \eta(G, \Sigma). \tag{7.29}$$

Example 7. Consider the system in Example 6. In addition, we assume that one adversary signal jammer with unknown location can corrupt one communication, resulting in a substantially increased noise level from variance 1 to variance 100. In this case, Ω contains 8 possible values of Σ_i, $i = 1, \ldots, 8$ such that Σ_i is the diagonal matrix will all diagonal elements equal to 1,, except a 100 at the ith position.

A Monte Carlo search is performed to find η_{opt} in (7.29). G is searched over 5000 sample points with

$$G = \text{diag}[\text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}, \text{rand}],$$

where *rand* is a random variable of uniform distribution in $(0, 1]$. The worst-case

$$\max_{\Sigma_i, i=1,\ldots,8} \eta(G, \Sigma)$$

is calculated for each G. The minimum $\eta(G)$ over this set is $\eta(G_0) = 25.9553$ which is achieved by

$$G_0 = \text{diag}[0.5418, 0.0213, 0.7086, 0.4075, 0.8299, 0.7014, 0.8112, 0.0561].$$

It is observed that due to the worst-case situation, the control design is conservative and the achievable performance is significantly affected. This is to be expected in such a robust control design.

7.7 Concluding Remarks

This paper introduces a new control methodology for UAV coordination. The methodology is based on weighted and constrained consensus control that can take into considerations of terrain conditions. The scalability of the framework permits dynamic expansion of team UAVs without increasing communication, control, and computation complexities of network control functions.

This paper is a first attempt in this new direction. We have left many open issues. For instance, this paper considers communication uncertainty in terms of additive noises. Other types of communication uncertainties such as latency, gains, quantization errors, data compression, and packet losses, will be of interests in development of UAV technology. Integration of consensus control with dynamic control of subsystems, fault detection, dynamic stability of UAV flight control will be of values in gaining better control strategies for UAV coordination.

References

1. Cortes, J., Bullo, F.: Coordination and geometric optimization via distributed dynamical systems. SIAM J. Control Optim. (5), 1543–1574 (May 2005)
2. Huang, M., Manton, J.H.: Coordination and consensus of networked agents with noisy measurements: stochastic algorithms and asymptotic behavior. SIAM J. Control Optim. 48(1), 134–161 (2009)
3. Huang, M., Dey, S., Nair, G.N., Manton, J.H.: Stochastic consensus over noisy networks with Markovian and arbitrary switches. Automatica 46, 1571–1583 (2010)
4. Jadbabaie, A., Lin, J., Morse, A.S.: Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans. Automat. Contr. 48, 988–1000 (2003)
5. Kar, S., Moura, J.M.F.: Distributed consensus algorithms in sensor networks with imperfect communication: Link failures and channel noise. IEEE Trans. Signal Processing 57(1), 355–369 (2009)
6. Karlin, S., Taylor, H.M.: A First Course in Stochastic Processes, 2nd edn. Academic Press, New York (1975)
7. Kothari, D.P., Nagrath, I.J.: Modern Power System Analysis. McGraw Hill Higher Education (2008)
8. Kushner, H.J., Yin, G.: Stochastic Approximation and Recursive Algorithms and Applications, 2nd edn. Springer, New York (2003)

9. Moreau, L.: Stability of multiagent systems with time- dependent communication links. IEEE Trans. Autom. Control 50(2), 169–182 (2005)
10. Nelson, M.B., Khasminskii, R.Z.: Stochastic Approximation and Recursive Estimation. Amer. Math. Soc., Providence (1976)
11. Olfati-Saber, R., Fax, J.A., Murray, R.M.: Consensus and cooperation in networked multi-agent systems. IEEE Proc. 95(1), 215–233 (2007)
12. Ogren, P., Fiorelli, E., Leonard, N.E.: Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed environment. IEEE Trans. Autom. Control 49(8), 1292–1302 (2005)
13. Polyak, B.T.: New method of stochastic approximation type. Automation Remote Control 7, 937–946 (1991)
14. Rau, N.S., Wan, Y.-H.: Optimum location of resources in distributed planning. IEEE Trans. Power Syst. 9, 2014–2020 (1994)
15. Ren, W., Beard, R.W.: Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans. Automat. Control 50(5), 655–661 (2005)
16. Ruppert, D.: Stochastic approximation. In: Ghosh, B.K., Sen, P.K. (eds.) Handbook in Sequential Analysis, pp. 503–529. Marcel Dekker, New York (1991)
17. Tenti, P., Paredes, H.K.M., Mattavelli, P.: Conservative Power Theory, a Framework to Approach Control and Accountability Issues in Smart Microgrids. IEEE Transactions on Power Electronics 26(3), 664–673 (2011)
18. Tsitsiklis, J.N., Bertsekas, D.P., Athans, M.: Distributed asynchronous deterministic and stochastic gradient optimization algorithms. IEEE Trans. Automat. Control 31(9), 803–812 (1986)
19. Wang, L.Y., Wang, C., Yin, G.: Weighted and Constrained Consensus for Distributed Power Flow Control. In: PMAPS 2012, Istanbul, Turkey, June 10-14 (2012)
20. Wang, L.Y., Syed, A., Yin, G., Pandya, A., Zhang, H.: Control of Vehicle Platoons for Highway Safety and Efficient Utility: Consensus with Switching Communication Network Topologies and Vehicle Dynamics (submitted)
21. Xiao, L., Boyd, S., Kim, S.J.: Distributed average consensus with least-mean-square deviation. Journal of Parallel and Distributed Computing 67, 33–46 (2007)
22. Yin, G.: On extensions of Polyak's averaging approach to stochastic approximation. Stochastics Stochastics Rep. 36, 245–264 (1991)
23. Yin, G., Sun, Y., Wang, L.Y.: Asymptotic properties of consensus-type algorithms for networked systems with regime-switching topologies. Automatica 47, 1366–1378 (2011)
24. Yin, G., Wang, L.Y., Sun, Y., Casbeer, D., Holsapple, R., Kingston, D.: Iterate averaging for asymptotic optimality in stochastic approximation algorithms of consensus type. To Appear in J. Control Theory Appl.
25. Yin, G., Zhang, Q.: Continuous-Time Markov Chains and Applications: A Singular Perturbation Approach. Springer, New York (1998)
26. Zhang, P., Guo, Y.: A novel spectral partitioning method for large-scale distribution system feeder reconfiguration. Automation of Electric Power Systems 26(18), 25–29 (2002)